Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/22-1.1.2.5-a+b-x^2-^p-c+d-x^2-^q-e+f-x^2-^r

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [115]. This is test number [22].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (115)	0.00 (0)
Mathematica	99.13 (114)	0.87 (1)
Maple	93.04 (107)	6.96 (8)
Fricas	30.43 (35)	69.57 (80)
Giac	26.96 (31)	73.04 (84)
Mupad	23.48 (27)	76.52 (88)
Maxima	23.48 (27)	76.52 (88)
Sympy	22.61 (26)	77.39 (89)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

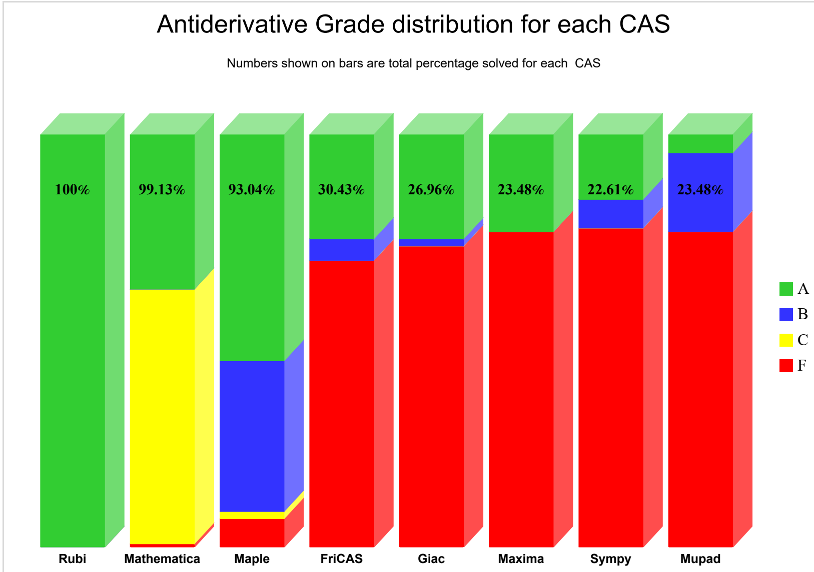
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

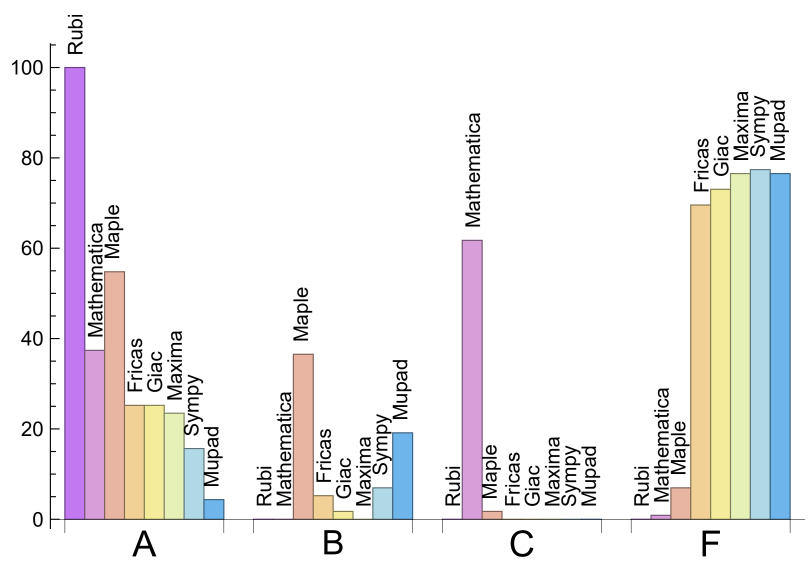
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Maple	54.78	36.52	1.74	6.96
Mathematica	37.39	0.00	61.74	0.87
Fricas	25.22	5.22	0.00	69.57
Giac	25.22	1.74	0.00	73.04
Maxima	23.48	0.00	0.00	76.52
Sympy	15.65	6.96	0.00	77.39
Mupad	N/A	19.13	0.00	76.52

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	8	100.00 %	0.00 %	0.00 %
Fricas	80	13.75 %	43.75 %	42.50 %
Giac	84	95.24 %	0.00 %	4.76 %
Maxima	88	100.00 %	0.00 %	0.00 %
Sympy	89	93.26 %	6.74 %	0.00 %
Mupad	88	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

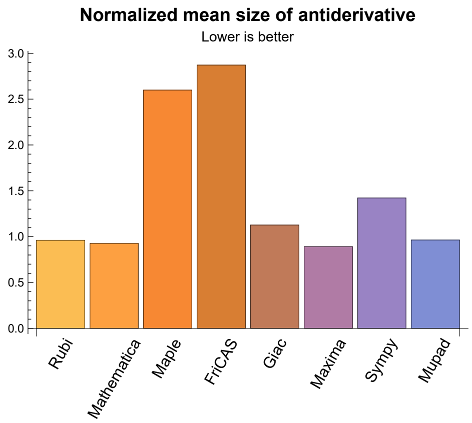
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

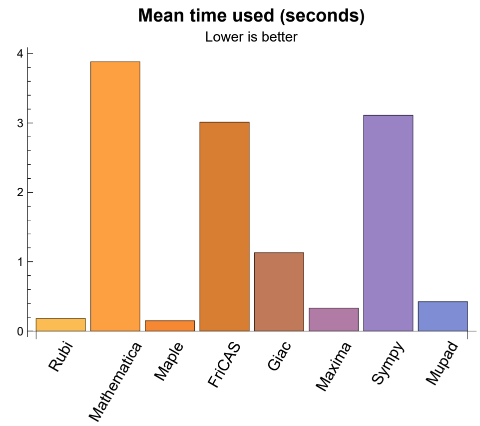
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	283.78	0.96	242.00	1.00
Mathematica	3.88	261.06	0.93	212.00	0.94
Maple	0.15	843.15	2.60	370.00	1.60
Maxima	0.33	163.59	0.89	161.00	1.08
Fricas	3.01	476.89	2.87	313.00	2.56
Sympy	3.11	245.08	1.42	211.00	1.35
Giac	1.13	191.71	1.13	178.00	1.25
Mupad	0.42	179.07	0.96	158.00	1.05

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
4
3
2
1
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{103, 107, 110, 112, 115\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {63}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

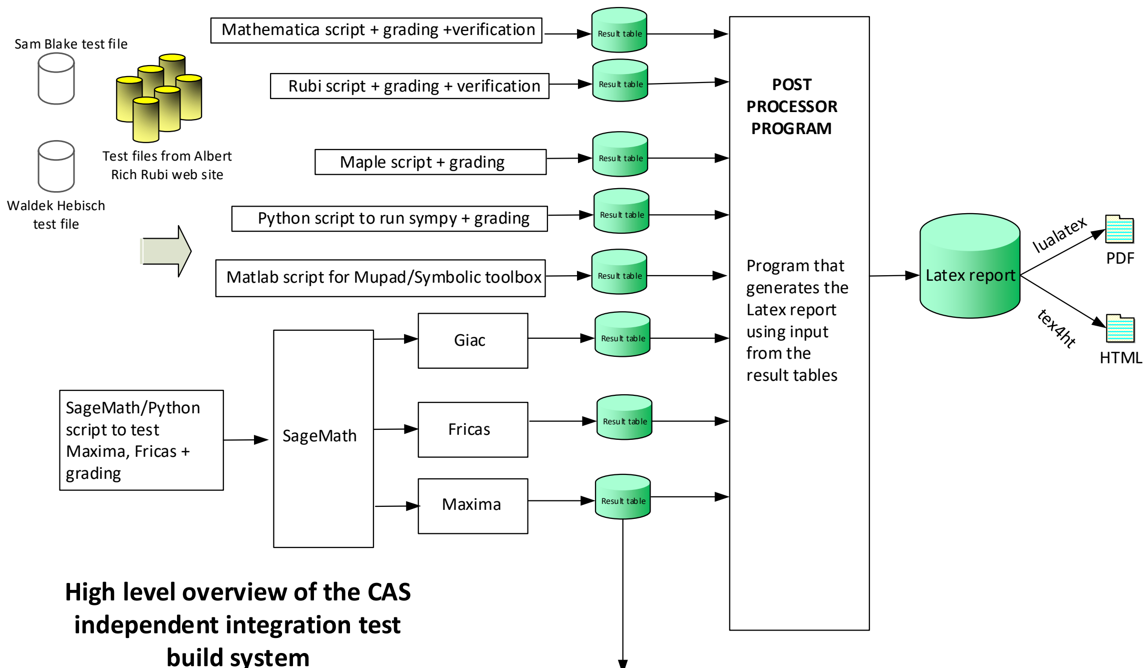
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 57, 58, 59, 60, 61, 62, 96, 97, 98, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115 }

B grade: { }

C grade: { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102 }

F grade: { 108 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 31, 37, 38, 39, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 66, 67, 68, 73, 77, 78, 79, 80, 81, 83, 85, 86, 87, 91, 92, 94, 95, 96, 97, 100, 103, 107, 110, 112, 115 }

B grade: { 23, 24, 27, 28, 29, 30, 32, 33, 34, 35, 36, 40, 41, 42, 47, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 74, 75, 76, 82, 84, 88, 93, 98, 99, 101, 102 }

C grade: { 89, 90 }

F grade: { 104, 105, 106, 108, 109, 111, 113, 114 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 103, 107, 110, 112, 115 }

B grade: { }

C grade: { }

F grade: { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 57, 58, 59, 97, 103, 107, 110, 112, 115 }

B grade: { 15, 22, 60, 61, 62, 98 }

C grade: { }

F grade: { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 6, 8, 9, 10, 11, 16, 17, 18, 97, 103, 107, 110, 112, 115 }

B grade: { 5, 7, 12, 13, 14, 19, 20, 21 }

C grade: { }

F grade: { 15, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 61, 62, 103, 107, 110, 112, 115 }

B grade: { 63, 98 }

C grade: { }

F grade: { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

2.1.8 Mupad

A grade: { 103, 107, 110, 112, 115 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22 }

C grade: { }

F grade: { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	172	176	175	179	236	210	182
N.S.	1	1.00	1.00	1.02	1.02	1.04	1.37	1.22	1.06
time (sec)	N/A	0.135	0.064	0.154	0.264	0.968	0.022	1.430	0.102

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	130	135	135	139	173	161	143
N.S.	1	1.00	1.00	1.04	1.04	1.07	1.33	1.24	1.10
time (sec)	N/A	0.087	0.048	0.157	0.268	0.835	0.018	1.348	0.838

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	96	94	95	99	121	114	99
N.S.	1	1.00	1.02	1.00	1.01	1.05	1.29	1.21	1.05
time (sec)	N/A	0.056	0.027	0.146	0.272	0.988	0.015	0.966	0.047

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	55	59	63	66	54
N.S.	1	1.00	1.00	0.95	0.98	1.05	1.12	1.18	0.96
time (sec)	N/A	0.026	0.010	0.127	0.260	0.921	0.009	1.321	0.048

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	72	74	73	191	206	80	108
N.S.	1	1.00	0.89	0.91	0.90	2.36	2.54	0.99	1.33
time (sec)	N/A	0.053	0.039	0.138	0.519	0.954	0.387	1.181	0.115

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	95	97	96	313	190	95	95
N.S.	1	1.00	0.88	0.90	0.89	2.90	1.76	0.88	0.88
time (sec)	N/A	0.057	0.047	0.133	0.476	0.668	0.735	1.279	0.167

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	130	132	140	469	246	135	136
N.S.	1	1.00	1.00	1.02	1.08	3.61	1.89	1.04	1.05
time (sec)	N/A	0.074	0.058	0.139	0.560	0.961	1.623	1.148	0.968

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	171	163	189	647	313	184	176
N.S.	1	1.00	1.00	0.95	1.11	3.78	1.83	1.08	1.03
time (sec)	N/A	0.109	0.073	0.139	0.518	0.937	3.554	0.690	0.177

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	226	237	240	242	304	283	233
N.S.	1	1.00	1.00	1.05	1.06	1.07	1.35	1.25	1.03
time (sec)	N/A	0.144	0.060	0.167	0.268	0.882	0.027	0.982	0.857

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	158	169	172	175	216	202	158
N.S.	1	1.00	1.00	1.07	1.09	1.11	1.37	1.28	1.00
time (sec)	N/A	0.114	0.044	0.158	0.358	1.191	0.024	1.158	0.069

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	96	101	104	108	121	120	99
N.S.	1	1.00	1.02	1.07	1.11	1.15	1.29	1.28	1.05
time (sec)	N/A	0.055	0.022	0.148	0.271	1.422	0.014	1.244	0.047

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	115	170	161	364	347	178	203
N.S.	1	1.00	0.81	1.20	1.13	2.56	2.44	1.25	1.43
time (sec)	N/A	0.141	0.046	0.135	0.509	0.781	0.580	0.636	0.857

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	134	182	186	552	483	195	257
N.S.	1	1.00	0.82	1.11	1.13	3.37	2.95	1.19	1.57
time (sec)	N/A	0.152	0.066	0.155	0.492	1.003	1.501	0.672	0.167

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	183	230	233	776	400	238	243
N.S.	1	1.00	0.88	1.11	1.13	3.75	1.93	1.15	1.17
time (sec)	N/A	0.160	0.088	0.167	0.504	2.478	7.023	0.633	0.976

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	242	289	301	1033	0	311	303
N.S.	1	1.00	1.01	1.20	1.25	4.30	0.00	1.30	1.26
time (sec)	N/A	0.183	0.114	0.163	0.515	1.223	0.000	0.633	0.976

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	310	339	333	337	423	401	335
N.S.	1	1.00	1.00	1.09	1.07	1.09	1.36	1.29	1.08
time (sec)	N/A	0.215	0.089	0.169	0.280	0.808	0.033	0.548	0.116

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	226	244	246	245	304	289	233
N.S.	1	1.00	1.00	1.08	1.09	1.08	1.35	1.28	1.03
time (sec)	N/A	0.146	0.059	0.168	0.279	1.063	0.026	0.581	0.846

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	130	149	152	153	173	173	143
N.S.	1	1.00	1.00	1.15	1.17	1.18	1.33	1.33	1.10
time (sec)	N/A	0.092	0.036	0.148	0.271	1.133	0.018	0.769	0.815

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	179	300	272	581	508	307	312
N.S.	1	1.00	0.79	1.32	1.20	2.56	2.24	1.35	1.37
time (sec)	N/A	0.252	0.067	0.137	0.501	1.872	0.919	1.003	0.849

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	176	308	298	824	661	321	389
N.S.	1	1.00	0.73	1.27	1.23	3.40	2.73	1.33	1.61
time (sec)	N/A	0.275	0.089	0.166	0.487	1.182	2.654	1.246	0.977

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	219	344	345	1104	865	371	495
N.S.	1	1.00	0.75	1.18	1.19	3.79	2.97	1.27	1.70
time (sec)	N/A	0.270	0.109	0.171	0.570	1.470	59.616	1.681	0.223

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	295	417	416	1424	0	447	444
N.S.	1	1.00	0.85	1.20	1.20	4.09	0.00	1.28	1.28
time (sec)	N/A	0.286	0.143	0.181	0.493	1.448	0.000	1.264	1.187

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	373	1332	0	0	0	0	-1
N.S.	1	1.00	0.69	2.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.453	3.365	0.176	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	267	865	0	0	0	0	-1
N.S.	1	1.00	0.70	2.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.243	1.704	0.132	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	212	394	0	0	0	0	-1
N.S.	1	1.00	0.75	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.123	1.530	0.127	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	192	328	0	0	0	0	-1
N.S.	1	1.00	0.71	1.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.116	2.677	0.127	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	297	1237	0	0	0	0	-1
N.S.	1	1.00	1.08	4.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.137	3.738	0.140	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	379	2861	0	0	0	0	-1
N.S.	1	1.00	0.98	7.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.261	4.135	0.154	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	372	1331	0	0	0	0	-1
N.S.	1	1.00	0.69	2.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.429	3.345	0.139	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	275	870	0	0	0	0	-1
N.S.	1	1.00	0.69	2.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	3.273	0.147	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	248	671	0	0	0	0	-1
N.S.	1	1.00	0.67	1.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.269	5.056	0.173	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	296	1225	0	0	0	0	-1
N.S.	1	1.00	0.79	3.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.260	5.855	0.143	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	382	2860	0	0	0	0	-1
N.S.	1	1.00	1.02	7.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.275	6.238	0.138	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	545	5113	0	0	0	0	-1
N.S.	1	1.00	1.03	9.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.423	6.714	0.159	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	386	1386	0	0	0	0	-1
N.S.	1	1.00	0.70	2.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.431	4.275	0.142	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	279	924	0	0	0	0	-1
N.S.	1	1.00	0.70	2.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	3.302	0.136	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	215	501	0	0	0	0	-1
N.S.	1	1.00	0.76	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.125	1.590	0.127	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	131	158	0	0	0	0	-1
N.S.	1	1.00	0.64	0.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.069	2.953	0.115	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	206	334	0	0	0	0	-1
N.S.	1	1.00	0.99	1.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.057	7.296	0.120	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	302	1352	0	0	0	0	-1
N.S.	1	1.00	1.06	4.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.139	9.826	0.125	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	393	3039	0	0	0	0	-1
N.S.	1	1.00	0.98	7.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.279	10.920	0.133	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	369	1169	0	0	0	0	-1
N.S.	1	1.00	0.74	2.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.403	6.194	0.180	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	260	750	0	0	0	0	-1
N.S.	1	1.00	0.73	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	5.213	0.149	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	208	393	0	0	0	0	-1
N.S.	1	1.00	0.81	1.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.114	2.751	0.123	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	212	349	0	0	0	0	-1
N.S.	1	1.00	1.01	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.058	7.029	0.136	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	262	581	0	0	0	0	-1
N.S.	1	1.00	0.96	2.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.146	10.510	0.131	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	428	1742	0	0	0	0	-1
N.S.	1	1.00	1.14	4.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.265	11.492	0.141	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	212	349	0	0	0	0	-1
N.S.	1	1.00	1.01	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.060	7.326	0.129	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	220	349	0	0	0	0	-1
N.S.	1	1.00	0.89	1.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.157	7.537	0.125	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	213	333	0	0	0	0	-1
N.S.	1	1.00	0.90	1.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.146	7.380	0.125	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	221	338	0	0	0	0	-1
N.S.	1	1.00	0.91	1.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.162	7.436	0.117	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	81	105	0	0	0	0	-1
N.S.	1	1.00	0.42	0.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	2.740	0.130	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	142	367	0	0	0	0	-1
N.S.	1	1.00	0.54	1.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.115	1.253	0.141	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	186	775	0	0	0	0	-1
N.S.	1	1.00	0.52	2.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.214	1.261	0.134	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	104	2538	0	0	0	0	-1
N.S.	1	1.00	0.92	22.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	2.564	0.470	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	526	526	203	2477	0	0	0	0	-1
N.S.	1	1.00	0.39	4.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.427	2.600	0.318	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	141	707	0	786	0	0	-1
N.S.	1	1.00	1.10	5.52	0.00	6.14	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.382	0.175	0.000	2.253	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	214	591	0	1772	0	0	-1
N.S.	1	1.00	0.70	1.94	0.00	5.83	0.00	0.00	-0.00
time (sec)	N/A	0.211	0.590	0.175	0.000	10.185	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	151	449	0	1166	0	0	-1
N.S.	1	1.00	0.91	2.70	0.00	7.02	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.315	0.136	0.000	2.525	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	111	352	0	782	0	0	-1
N.S.	1	1.00	1.22	3.87	0.00	8.59	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.250	0.109	0.000	1.727	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	70	306	0	258	0	74	-1
N.S.	1	1.00	1.43	6.24	0.00	5.27	0.00	1.51	-0.02
time (sec)	N/A	0.014	0.085	0.112	0.000	0.954	0.000	1.029	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	153	782	0	1377	0	173	-1
N.S.	1	1.00	1.25	6.41	0.00	11.29	0.00	1.42	-0.01
time (sec)	N/A	0.078	0.381	0.123	0.000	58.266	0.000	1.267	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	531	1400	0	0	0	479	-1
N.S.	1	1.00	2.62	6.90	0.00	0.00	0.00	2.36	-0.00
time (sec)	N/A	0.189	14.002	0.118	0.000	0.000	0.000	7.963	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	776	456	1891	0	0	0	0	-1
N.S.	1	1.28	0.75	3.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.521	5.621	0.228	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	346	1059	0	0	0	0	-1
N.S.	1	1.00	0.86	2.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.195	4.058	0.194	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	184	340	0	0	0	0	-1
N.S.	1	1.00	0.57	1.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.123	1.725	0.139	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	143	191	0	0	0	0	-1
N.S.	1	1.00	1.40	1.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	1.847	0.133	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	347	390	0	0	0	0	-1
N.S.	1	1.00	1.66	1.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.069	3.790	0.156	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	427	2068	0	0	0	0	-1
N.S.	1	1.00	1.06	5.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.247	6.451	0.156	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	630	630	584	6245	0	0	0	0	-1
N.S.	1	1.00	0.93	9.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.484	6.062	0.175	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	659	784	445	1939	0	0	0	0	-1
N.S.	1	1.19	0.68	2.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.504	6.816	0.233	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	739	1028	0	0	0	0	-1
N.S.	1	1.00	1.83	2.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.194	3.710	0.179	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	184	300	0	0	0	0	-1
N.S.	1	1.00	0.56	0.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.125	3.440	0.141	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	492	630	0	0	0	0	-1
N.S.	1	1.00	2.20	2.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.073	6.354	0.155	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	999	1876	0	0	0	0	-1
N.S.	1	1.00	2.55	4.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.262	7.219	0.160	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	639	639	570	6211	0	0	0	0	-1
N.S.	1	1.00	0.89	9.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.512	7.900	0.164	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	621	350	988	0	0	0	0	-1
N.S.	1	1.00	0.56	1.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.307	4.976	0.189	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	197	341	0	0	0	0	-1
N.S.	1	1.00	0.62	1.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.119	3.511	0.141	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	143	191	0	0	0	0	-1
N.S.	1	1.00	1.40	1.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	1.822	0.139	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	101	118	0	0	0	0	-1
N.S.	1	1.00	1.01	1.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	2.250	0.134	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	365	413	0	0	0	0	-1
N.S.	1	1.00	1.06	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.147	4.404	0.145	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	433	2062	0	0	0	0	-1
N.S.	1	1.00	1.00	4.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.371	6.688	0.155	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	980	980	352	1063	0	0	0	0	-1
N.S.	1	1.00	0.36	1.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.752	7.094	0.160	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	304	594	0	0	0	0	-1
N.S.	1	1.00	1.36	2.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.079	6.169	0.151	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	207	285	0	0	0	0	-1
N.S.	1	1.00	0.99	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.068	3.600	0.144	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	221	303	0	0	0	0	-1
N.S.	1	1.00	0.64	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.147	4.283	0.146	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	539	539	418	956	0	0	0	0	-1
N.S.	1	1.00	0.78	1.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	8.419	0.172	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	814	814	1645	4115	0	0	0	0	-1
N.S.	1	1.00	2.02	5.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.631	10.037	0.189	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	239	204	370	0	0	0	0	-1
N.S.	1	0.99	0.84	1.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.095	2.223	0.219	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	71	121	0	0	0	0	-1
N.S.	1	1.00	0.37	0.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.900	0.131	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	50	64	0	0	0	0	-1
N.S.	1	1.00	0.86	1.10	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.874	0.141	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	122	147	0	0	0	0	-1
N.S.	1	1.00	1.01	1.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	2.219	0.146	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	357	477	0	0	0	0	-1
N.S.	1	1.00	1.66	2.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.093	3.077	0.156	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	134	293	0	0	0	0	-1
N.S.	1	1.00	0.45	0.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.118	1.566	0.131	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	94	133	0	0	0	0	-1
N.S.	1	1.00	1.01	1.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.024	1.630	0.138	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	53	0	0	0	0	-1
N.S.	1	1.00	1.00	1.08	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	1.862	0.137	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	35	0	13	19	0	-1
N.S.	1	1.00	1.03	0.97	0.00	0.36	0.53	0.00	-0.03
time (sec)	N/A	0.017	0.488	0.165	0.000	0.380	2.067	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	131	848	0	523	0	336	-1
N.S.	1	1.00	1.16	7.50	0.00	4.63	0.00	2.97	-0.01
time (sec)	N/A	0.078	0.552	0.120	0.000	3.918	0.000	2.320	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	422	773	0	0	0	0	-1
N.S.	1	1.00	1.18	2.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.210	3.424	0.161	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	401	765	0	0	0	0	-1
N.S.	1	1.00	1.05	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.192	3.100	0.155	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	617	1077	0	0	0	0	-1
N.S.	1	1.00	1.45	2.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	6.200	0.147	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	587	1078	0	0	0	0	-1
N.S.	1	1.00	1.21	2.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.234	4.474	0.149	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.036	15.336	0.021	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	503	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.317	3.518	0.037	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	162	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	2.823	0.044	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	148	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	4.904	0.051	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.038	17.982	0.043	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.508	24.962	0.036	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	148	0	0	0	0	0	-1
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.310	5.025	0.041	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.039	18.553	0.040	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	541	512	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	3.709	0.035	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.037	5.131	0.046	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	159	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	2.786	0.039	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	148	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	3.318	0.039	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.039	10.132	0.041	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [55] had the largest ratio of [87]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	24	0.042
2	A	2	1	1.00	24	0.042
3	A	2	1	1.00	24	0.042
4	A	2	1	1.00	22	0.045
5	A	3	3	1.00	24	0.125
6	A	3	3	1.00	24	0.125
7	A	3	3	1.00	24	0.125
8	A	4	4	1.00	24	0.167
9	A	2	1	1.00	26	0.038
10	A	2	1	1.00	26	0.038
11	A	2	1	1.00	24	0.042
12	A	4	3	1.00	26	0.115
13	A	4	4	1.00	26	0.154
14	A	4	3	1.00	26	0.115
15	A	4	3	1.00	26	0.115
16	A	2	1	1.00	26	0.038
17	A	2	1	1.00	26	0.038
18	A	2	1	1.00	24	0.042
19	A	5	3	1.00	26	0.115
20	A	5	4	1.00	26	0.154
21	A	5	4	1.00	26	0.154
22	A	5	3	1.00	26	0.115
23	A	7	5	1.00	30	0.167
24	A	6	5	1.00	30	0.167
25	A	5	5	1.00	30	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	5	5	1.00	30	0.167
27	A	4	4	1.00	30	0.133
28	A	5	5	1.00	30	0.167
29	A	7	5	1.00	30	0.167
30	A	6	5	1.00	30	0.167
31	A	6	6	1.00	30	0.200
32	A	6	5	1.00	30	0.167
33	A	5	4	1.00	30	0.133
34	A	6	5	1.00	30	0.167
35	A	7	5	1.00	30	0.167
36	A	6	5	1.00	30	0.167
37	A	5	5	1.00	30	0.167
38	A	4	4	1.00	30	0.133
39	A	3	3	1.00	30	0.100
40	A	4	4	1.00	30	0.133
41	A	5	4	1.00	30	0.133
42	A	7	6	1.00	30	0.200
43	A	6	6	1.00	30	0.200
44	A	5	5	1.00	30	0.167
45	A	3	3	1.00	30	0.100
46	A	4	4	1.00	30	0.133
47	A	5	4	1.00	30	0.133
48	A	3	3	1.00	30	0.100
49	A	8	7	1.00	31	0.226
50	A	8	7	1.00	31	0.226
51	A	8	7	1.00	32	0.219
52	A	4	4	1.00	30	0.133
53	A	5	5	1.00	30	0.167
54	A	6	5	1.00	30	0.167
55	A	2	2	1.00	87	0.023
56	A	5	5	1.00	81	0.062
57	A	6	6	1.00	28	0.214
58	A	14	8	1.00	30	0.267
59	A	9	7	1.00	30	0.233
60	A	5	5	1.00	28	0.179

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	2	1.00	21	0.095
62	A	5	3	1.00	30	0.100
63	A	7	5	1.00	30	0.167
64	A	14	9	1.28	32	0.281
65	A	7	7	1.00	32	0.219
66	A	6	6	1.00	32	0.188
67	A	1	1	1.00	32	0.031
68	A	3	3	1.00	32	0.094
69	A	6	6	1.00	32	0.188
70	A	9	8	1.00	32	0.250
71	A	14	9	1.19	32	0.281
72	A	7	7	1.00	32	0.219
73	A	6	6	1.00	32	0.188
74	A	3	3	1.00	32	0.094
75	A	6	6	1.00	32	0.188
76	A	9	8	1.00	32	0.250
77	A	12	8	1.00	32	0.250
78	A	6	6	1.00	32	0.188
79	A	1	1	1.00	32	0.031
80	A	3	2	1.00	32	0.062
81	A	5	5	1.00	32	0.156
82	A	8	7	1.00	32	0.219
83	A	14	9	1.00	32	0.281
84	A	3	3	1.00	32	0.094
85	A	3	3	1.00	32	0.094
86	A	5	5	1.00	32	0.156
87	A	8	7	1.00	32	0.219
88	A	11	6	1.00	32	0.188
89	A	7	7	0.99	28	0.250
90	A	6	6	1.00	28	0.214
91	A	1	1	1.00	28	0.036
92	A	3	3	1.00	28	0.107
93	A	6	6	1.00	28	0.214
94	A	6	6	1.00	32	0.188
95	A	1	1	1.00	32	0.031

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	1	1	1.00	32	0.031
97	A	3	3	1.00	30	0.100
98	A	4	4	1.00	28	0.143
99	A	11	9	1.00	33	0.273
100	A	8	7	1.00	32	0.219
101	A	11	9	1.00	33	0.273
102	A	8	7	1.00	32	0.219
103	A	0	0	0.00	0	0.000
104	A	7	7	1.00	34	0.206
105	A	2	2	1.00	34	0.059
106	A	2	2	1.00	34	0.059
107	A	0	0	0.00	0	0.000
108	A	8	8	1.00	34	0.235
109	A	5	5	1.00	34	0.147
110	A	0	0	0.00	0	0.000
111	A	7	7	1.00	34	0.206
112	A	0	0	0.00	0	0.000
113	A	2	2	1.00	34	0.059
114	A	2	2	1.00	34	0.059
115	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int (a+bx^2)(c+dx^2)(e+fx^2)^4 \, dx$

Optimal. Leaf size=172

$$ace^4x+\frac{1}{3}e^3(bce+ade+4acf)x^3+\frac{1}{5}e^2(2af(2de+3cf)+be(de+4cf))x^5+\frac{2}{7}ef(af(3de+2cf)+be(2de+3cf))x^7+$$

```
[Out] a*c*e^4*x+1/3*e^3*(4*a*c*f+a*d*e+b*c*e)*x^3+1/5*e^2*(2*a*f*(3*c*f+2*d*e)+b*
e*(4*c*f+d*e))*x^5+2/7*e*f*(a*f*(2*c*f+3*d*e)+b*e*(3*c*f+2*d*e))*x^7+1/9*f^
2*(a*f*(c*f+4*d*e)+2*b*e*(2*c*f+3*d*e))*x^9+1/11*f^3*(a*d*f+b*c*f+4*b*d*e)*
x^11+1/13*b*d*f^4*x^13
```

Rubi [A]
time = 0.13, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {535}

$$\frac{1}{3}e^3x^3(4acf+ade+bce)+\frac{1}{5}e^2x^5(2af(3cf+2de)+be(4cf+de))+\frac{1}{11}f^3x^{11}(adf+bcf+4bde)+\frac{1}{9}f^2x^9(af(cf+4de)+2be(2cf+3de))+\frac{2}{7}efx^7(af(2cf+3de)+be(3cf+2de))+ace^4x+\frac{1}{13}bdf^4x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^4,x]
```

```
[Out] a*c*e^4*x + (e^3*(b*c*e + a*d*e + 4*a*c*f)*x^3)/3 + (e^2*(2*a*f*(2*d*e + 3*
c*f) + b*e*(d*e + 4*c*f))*x^5)/5 + (2*e*f*(a*f*(3*d*e + 2*c*f) + b*e*(2*d*e
 + 3*c*f))*x^7)/7 + (f^2*(a*f*(4*d*e + c*f) + 2*b*e*(3*d*e + 2*c*f))*x^9)/9
 + (f^3*(4*b*d*e + b*c*f + a*d*f)*x^11)/11 + (b*d*f^4*x^13)/13
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)(c+dx^2)(e+fx^2)^4 \, dx &= \int (ace^4+e^3(bce+ade+4acf)x^2+e^2(2af(2de+3cf)+be(de+4\\ &= ace^4x+\frac{1}{3}e^3(bce+ade+4acf)x^3+\frac{1}{5}e^2(2af(2de+3cf)+be(de+4\end{aligned}$$

Mathematica [A]
time = 0.06, size = 172, normalized size = 1.00

$$ace^4x+\frac{1}{3}e^3(bce+ade+4acf)x^3+\frac{1}{5}e^2(2af(2de+3cf)+be(de+4cf))x^5+\frac{2}{7}ef(af(3de+2cf)+be(2de+3cf))x^7+\frac{1}{9}f^2(af(4de+cf)+2be(3de+2cf))x^9+\frac{1}{11}f^3(4bde+bcf+adf)x^{11}+\frac{1}{13}bdf^4x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^4,x]
```

```
[Out] a*c*e^4*x + (e^3*(b*c*e + a*d*e + 4*a*c*f)*x^3)/3 + (e^2*(2*a*f*(2*d*e + 3*
c*f) + b*e*(d*e + 4*c*f))*x^5)/5 + (2*e*f*(a*f*(3*d*e + 2*c*f) + b*e*(2*d*e
 + 3*c*f))*x^7)/7 + (f^2*(a*f*(4*d*e + c*f) + 2*b*e*(3*d*e + 2*c*f))*x^9)/9
 + (f^3*(4*b*d*e + b*c*f + a*d*f)*x^11)/11 + (b*d*f^4*x^13)/13
```

Maple [A]
time = 0.15, size = 176, normalized size = 1.02

method	result
default	$\frac{bd\,f^4x^{13}}{13}+\frac{((ad+bc)f^4+4bde\,f^3)x^{11}}{11}+\frac{(ac\,f^4+4(ad+bc)e\,f^3+6bd\,e^2f^2)x^9}{9}+\frac{(4ace\,f^3+6(ad+bc)e^2f^2+4bd\,e^3f)x^7}{7}+\frac{(6ac\,e$
norman	$\frac{bd\,f^4x^{13}}{13}+\left(\frac{1}{11}ad\,f^4+\frac{1}{11}bc\,f^4+\frac{4}{11}bde\,f^3\right)x^{11}+\left(\frac{1}{9}ac\,f^4+\frac{4}{9}ade\,f^3+\frac{4}{9}bce\,f^3+\frac{2}{3}bd\,e^2f^2\right)x^9+\left(\frac{4}{7}\right.$
gosper	$\frac{1}{13}bd\,f^4x^{13}+\frac{1}{11}x^{11}ad\,f^4+\frac{1}{11}x^{11}bc\,f^4+\frac{4}{11}x^{11}bde\,f^3+\frac{1}{9}x^9ac\,f^4+\frac{4}{9}x^9ade\,f^3+\frac{4}{9}x^9bce\,f^3+\frac{2}{3}x^9b$
risch	$\frac{1}{13}bd\,f^4x^{13}+\frac{1}{11}x^{11}ad\,f^4+\frac{1}{11}x^{11}bc\,f^4+\frac{4}{11}x^{11}bde\,f^3+\frac{1}{9}x^9ac\,f^4+\frac{4}{9}x^9ade\,f^3+\frac{4}{9}x^9bce\,f^3+\frac{2}{3}x^9b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b*d*f^4*x^13+1/11*((a*d+b*c)*f^4+4*b*d*e*f^3)*x^11+1/9*(a*c*f^4+4*(a*d
+b*c)*e*f^3+6*b*d*e^2*f^2)*x^9+1/7*(4*a*c*e*f^3+6*(a*d+b*c)*e^2*f^2+4*b*d*e
^3*f)*x^7+1/5*(6*a*c*e^2*f^2+4*(a*d+b*c)*e^3*f+b*d*e^4)*x^5+1/3*(4*a*c*e^3*
f+(a*d+b*c)*e^4)*x^3+a*c*e^4*x
```

Maxima [A]
time = 0.26, size = 175, normalized size = 1.02

$$\frac{1}{13}\,bdf^4x^{13}+\frac{1}{11}\,\left(4\,bdf^3e+(bc+ad)\,f^4\right)x^{11}+\frac{1}{9}\,\left(acf^4+6\,bdf^2e^2+4\,(bce+ade)\,f^3\right)x^9+\frac{2}{7}\,\left(2\,acf^3e+2\,bdfe^3+3\,\left(bce^2+ade^2\right)f^2\right)x^7+\frac{1}{5}\,\left(6\,acf^2e^2+bde^4+4\,\left(bce^3+ade^3\right)f\right)x^5+\frac{1}{3}\,\left(4\,acfe^3+bce^4+ade^4\right)x^3+acxe^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x, algorithm="maxima")
```

```
[Out] 1/13*b*d*f^4*x^13 + 1/11*(4*b*d*f^3*e + (b*c + a*d)*f^4)*x^11 + 1/9*(a*c*f^
4 + 6*b*d*f^2*e^2 + 4*(b*c*e + a*d*e)*f^3)*x^9 + 2/7*(2*a*c*f^3*e + 2*b*d*f
*e^3 + 3*(b*c*e^2 + a*d*e^2)*f^2)*x^7 + 1/5*(6*a*c*f^2*e^2 + b*d*e^4 + 4*(b
*c*e^3 + a*d*e^3)*f)*x^5 + 1/3*(4*a*c*f*e^3 + b*c*e^4 + a*d*e^4)*x^3 + a*c*
x*e^4
```

Fricas [A]
time = 0.97, size = 179, normalized size = 1.04

$$\frac{1}{13}\,bdf^4x^{13}+\frac{1}{11}\,(bc+ad)f^4x^{11}+\frac{1}{9}\,acf^4x^9+\frac{1}{15}\,\left(3\,bdx^5+5\,(bc+ad)x^3+15\,acx\right)e^4+\frac{4}{105}\,\left(15\,bdfx^7+21\,(bc+ad)fx^5+35\,acfx^3\right)e^3+\frac{2}{105}\,\left(35\,bdf^2x^9+45\,(bc+ad)f^2x^7+63\,acf^2x^5\right)e^2+\frac{4}{693}\,\left(63\,bdf^3x^{11}+77\,(bc+ad)f^3x^9+99\,acf^3x^7\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x, algorithm="fricas")
```

```
[Out] 1/13*b*d*f^4*x^13 + 1/11*(b*c + a*d)*f^4*x^11 + 1/9*a*c*f^4*x^9 + 1/15*(3*b
*d*x^5 + 5*(b*c + a*d)*x^3 + 15*a*c*x)*e^4 + 4/105*(15*b*d*f*x^7 + 21*(b*c
+ a*d)*f*x^5 + 35*a*c*f*x^3)*e^3 + 2/105*(35*b*d*f^2*x^9 + 45*(b*c + a*d)*f
^2*x^7 + 63*a*c*f^2*x^5)*e^2 + 4/693*(63*b*d*f^3*x^11 + 77*(b*c + a*d)*f^3*
x^9 + 99*a*c*f^3*x^7)*e
```

Sympy [A]
time = 0.02, size = 236, normalized size = 1.37

$$ace^4x + \frac{bdf^4x^{13}}{13} + x^{11}\left(\frac{adf^4}{11} + \frac{bcf^4}{11} + \frac{4bdef^3}{11}\right) + x^9\left(\frac{acf^4}{9} + \frac{4adef^3}{9} + \frac{4bcef^3}{9} + \frac{2bde^2f^2}{3}\right) + x^7\cdot\left(\frac{4acef^3}{7} + \frac{6ade^2f^2}{7} + \frac{6bce^2f^2}{7} + \frac{4bde^3f}{7}\right) + x^5\cdot\left(\frac{6ace^2f^2}{5} + \frac{4ade^3f}{5} + \frac{4bce^3f}{5} + \frac{bde^4}{5}\right) + x^3\cdot\left(\frac{4ace^3f}{3} + \frac{ade^4}{3} + \frac{bce^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**4,x)
```

```
[Out] a*c*e**4*x + b*d*f**4*x**13/13 + x**11*(a*d*f**4/11 + b*c*f**4/11 + 4*b*d*e
*f**3/11) + x**9*(a*c*f**4/9 + 4*a*d*e*f**3/9 + 4*b*c*e*f**3/9 + 2*b*d*e**2
*f**2/3) + x**7*(4*a*c*e*f**3/7 + 6*a*d*e**2*f**2/7 + 6*b*c*e**2*f**2/7 + 4
*b*d*e**3*f/7) + x**5*(6*a*c*e**2*f**2/5 + 4*a*d*e**3*f/5 + 4*b*c*e**3*f/5
+ b*d*e**4/5) + x**3*(4*a*c*e**3*f/3 + a*d*e**4/3 + b*c*e**4/3)
```

Giac [A]
time = 1.43, size = 210, normalized size = 1.22

$$\frac{1}{13}bdf^4x^{13} + \frac{1}{11}bcf^4x^{11} + \frac{1}{11}adf^4x^{11} + \frac{4}{11}bdf^3x^{11}e + \frac{1}{9}acf^4x^9 + \frac{4}{9}bcf^3x^9e + \frac{4}{9}adf^3x^9e + \frac{2}{3}bdf^2x^9e^2 + \frac{4}{7}acf^3x^7e + \frac{6}{7}bcf^2x^7e^2 + \frac{6}{7}adf^2x^7e^2 + \frac{4}{7}bdfx^7e^3 + \frac{6}{5}acf^2x^5e^2 + \frac{4}{5}bcfx^5e^3 + \frac{4}{5}adfx^5e^3 + \frac{1}{5}bdx^5e^4 + \frac{4}{3}acfx^3e^3 + \frac{1}{3}bcx^3e^4 + \frac{1}{3}adx^3e^4 + acxe^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x, algorithm="giac")
```

```
[Out] 1/13*b*d*f^4*x^13 + 1/11*b*c*f^4*x^11 + 1/11*a*d*f^4*x^11 + 4/11*b*d*f^3*x^
11*e + 1/9*a*c*f^4*x^9 + 4/9*b*c*f^3*x^9*e + 4/9*a*d*f^3*x^9*e + 2/3*b*d*f^
2*x^9*e^2 + 4/7*a*c*f^3*x^7*e + 6/7*b*c*f^2*x^7*e^2 + 6/7*a*d*f^2*x^7*e^2 +
 4/7*b*d*f*x^7*e^3 + 6/5*a*c*f^2*x^5*e^2 + 4/5*b*c*f*x^5*e^3 + 4/5*a*d*f*x^
5*e^3 + 1/5*b*d*x^5*e^4 + 4/3*a*c*f*x^3*e^3 + 1/3*b*c*x^3*e^4 + 1/3*a*d*x^3
*e^4 + a*c*x*e^4
```

Mupad [B]
time = 0.10, size = 182, normalized size = 1.06

$$x^3\left(\frac{a\,d\,e^4}{3} + \frac{b\,c\,e^4}{3} + \frac{4\,a\,c\,e^3\,f}{3}\right) + x^{11}\left(\frac{a\,d\,f^4}{11} + \frac{b\,c\,f^4}{11} + \frac{4\,b\,d\,e\,f^3}{11}\right) + x^5\left(\frac{b\,d\,e^4}{5} + \frac{4\,a\,d\,e^3\,f}{5} + \frac{4\,b\,c\,e^3\,f}{5} + \frac{6\,a\,c\,e^2\,f^2}{5}\right) + x^9\left(\frac{a\,c\,f^4}{9} + \frac{4\,a\,d\,e\,f^3}{9} + \frac{4\,b\,c\,e\,f^3}{9} + \frac{2\,b\,d\,e^2\,f^2}{3}\right) + \frac{2\,e\,f\,x^7\,\left(2\,a\,c\,f^2 + 2\,b\,d\,e^2 + 3\,a\,d\,e\,f + 3\,b\,c\,e\,f\right)}{7} + a\,c\,e^4\,x + \frac{b\,d\,f^4\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^4,x)
```

```
[Out] x^3*((a*d*e^4)/3 + (b*c*e^4)/3 + (4*a*c*e^3*f)/3) + x^11*((a*d*f^4)/11 + (b
*c*f^4)/11 + (4*b*d*e*f^3)/11) + x^5*((b*d*e^4)/5 + (4*a*d*e^3*f)/5 + (4*b*
c*e^3*f)/5 + (6*a*c*e^2*f^2)/5) + x^9*((a*c*f^4)/9 + (4*a*d*e*f^3)/9 + (4*b
*c*e*f^3)/9 + (2*b*d*e^2*f^2)/3) + (2*e*f*x^7*(2*a*c*f^2 + 2*b*d*e^2 + 3*a*
d*e*f + 3*b*c*e*f))/7 + a*c*e^4*x + (b*d*f^4*x^13)/13
```

3.2 $\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx$

Optimal. Leaf size=130

$$ace^3x+\frac{1}{3}e^2(bce+ade+3acf)x^3+\frac{1}{5}e(3af(de+cf)+be(de+3cf))x^5+\frac{1}{7}f(3be(de+cf)+af(3de+cf))x^7+\frac{1}{9}f^2(3b$$

```
[Out] a*c*e^3*x+1/3*e^2*(3*a*c*f+a*d*e+b*c*e)*x^3+1/5*e*(3*a*f*(c*f+d*e)+b*e*(3*c
*f+d*e))*x^5+1/7*f*(3*b*e*(c*f+d*e)+a*f*(c*f+3*d*e))*x^7+1/9*f^2*(a*d*f+b*c
*f+3*b*d*e)*x^9+1/11*b*d*f^3*x^11
```

Rubi [A]
time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {535}

$$\frac{1}{3}e^2x^3(3acf+ade+bce)+\frac{1}{9}f^2x^9(adf+bcf+3bde)+\frac{1}{7}fx^7(af(cf+3de)+3be(cf+de))+\frac{1}{5}ex^5(3af(cf+de)+be(3cf+de))+ace^3x+\frac{1}{11}bdf^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3,x]
```

```
[Out] a*c*e^3*x + (e^2*(b*c*e + a*d*e + 3*a*c*f)*x^3)/3 + (e*(3*a*f*(d*e + c*f) +
 b*e*(d*e + 3*c*f))*x^5)/5 + (f*(3*b*e*(d*e + c*f) + a*f*(3*d*e + c*f))*x^7
)/7 + (f^2*(3*b*d*e + b*c*f + a*d*f)*x^9)/9 + (b*d*f^3*x^11)/11
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx &= \int \left(ace^3+e^2(bce+ade+3acf)x^2+e(3af(de+cf)+be(de+3cf)\right.\\ &= ace^3x+\frac{1}{3}e^2(bce+ade+3acf)x^3+\frac{1}{5}e(3af(de+cf)+be(de+3cf\end{aligned}$$

Mathematica [A]
time = 0.05, size = 130, normalized size = 1.00

$$ace^3x+\frac{1}{3}e^2(bce+ade+3acf)x^3+\frac{1}{5}e(3af(de+cf)+be(de+3cf))x^5+\frac{1}{7}f(3be(de+cf)+af(3de+cf))x^7+\frac{1}{9}f^2(3bde+bcf+adf)x^9+\frac{1}{11}bdf^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3,x]
```

```
[Out] a*c*e^3*x + (e^2*(b*c*e + a*d*e + 3*a*c*f)*x^3)/3 + (e*(3*a*f*(d*e + c*f) +
 b*e*(d*e + 3*c*f))*x^5)/5 + (f*(3*b*e*(d*e + c*f) + a*f*(3*d*e + c*f))*x^7
)/7 + (f^2*(3*b*d*e + b*c*f + a*d*f)*x^9)/9 + (b*d*f^3*x^11)/11
```

Maple [A]
time = 0.16, size = 135, normalized size = 1.04

method	result
default	$\frac{bd\,f^3x^{11}}{11}+\frac{((ad+bc)f^3+3bde\,f^2)x^9}{9}+\frac{(ac\,f^3+3(ad+bc)e\,f^2+3bd\,e^2f)x^7}{7}+\frac{(3ace\,f^2+3(ad+bc)e^2f+bd\,e^3)x^5}{5}+$
norman	$\frac{bd\,f^3x^{11}}{11}+\left(\frac{1}{9}ad\,f^3+\frac{1}{9}bc\,f^3+\frac{1}{3}bde\,f^2\right)x^9+\left(\frac{1}{7}ac\,f^3+\frac{3}{7}ade\,f^2+\frac{3}{7}bce\,f^2+\frac{3}{7}bd\,e^2f\right)x^7+$
gosper	$\frac{1}{11}bd\,f^3x^{11}+\frac{1}{9}x^9ad\,f^3+\frac{1}{9}x^9bc\,f^3+\frac{1}{3}x^9bde\,f^2+\frac{1}{7}x^7ac\,f^3+\frac{3}{7}x^7ade\,f^2+\frac{3}{7}x^7bce\,f^2+\frac{3}{7}x^7bd\,e^2f$
risch	$\frac{1}{11}bd\,f^3x^{11}+\frac{1}{9}x^9ad\,f^3+\frac{1}{9}x^9bc\,f^3+\frac{1}{3}x^9bde\,f^2+\frac{1}{7}x^7ac\,f^3+\frac{3}{7}x^7ade\,f^2+\frac{3}{7}x^7bce\,f^2+\frac{3}{7}x^7bd\,e^2f$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b*d*f^3*x^11+1/9*((a*d+b*c)*f^3+3*b*d*e*f^2)*x^9+1/7*(a*c*f^3+3*(a*d+b
*c)*e*f^2+3*b*d*e^2*f)*x^7+1/5*(3*a*c*e*f^2+3*(a*d+b*c)*e^2*f+b*d*e^3)*x^5+
1/3*(3*a*c*e^2*f+(a*d+b*c)*e^3)*x^3+a*c*e^3*x
```

Maxima [A]
time = 0.27, size = 135, normalized size = 1.04

$$\frac{1}{11}\,bdf^3x^{11}+\frac{1}{9}\,(3\,bdf^2e+(bc+ad)f^3)x^9+\frac{1}{7}\,(acf^3+3\,bdfe^2+3\,(bce+ade)f^2)x^7+\frac{1}{5}\,(3\,acf^2e+bde^3+3\,(bce^2+ade^2)\,f)x^5+\frac{1}{3}\,(3\,acfe^2+bce^3+ade^3)x^3+acxe^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b*d*f^3*x^11 + 1/9*(3*b*d*f^2*e + (b*c + a*d)*f^3)*x^9 + 1/7*(a*c*f^3
+ 3*b*d*f*e^2 + 3*(b*c*e + a*d*e)*f^2)*x^7 + 1/5*(3*a*c*f^2*e + b*d*e^3 + 3
*(b*c*e^2 + a*d*e^2)*f)*x^5 + 1/3*(3*a*c*f*e^2 + b*c*e^3 + a*d*e^3)*x^3 + a
*c*x*e^3
```

Fricas [A]
time = 0.83, size = 139, normalized size = 1.07

$$\frac{1}{11}\,bdf^3x^{11}+\frac{1}{9}\,(bc+ad)f^3x^9+\frac{1}{7}\,acf^3x^7+\frac{1}{15}\,(3\,bdx^5+5\,(bc+ad)x^3+15\,acx)e^3+\frac{1}{35}\,(15\,bdfx^7+21\,(bc+ad)fx^5+35\,acfx^3)e^2+\frac{1}{105}\,(35\,bdf^2x^9+45\,(bc+ad)f^2x^7+63\,acf^2x^5)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] 1/11*b*d*f^3*x^11 + 1/9*(b*c + a*d)*f^3*x^9 + 1/7*a*c*f^3*x^7 + 1/15*(3*b*d
*x^5 + 5*(b*c + a*d)*x^3 + 15*a*c*x)*e^3 + 1/35*(15*b*d*f*x^7 + 21*(b*c + a
*d)*f*x^5 + 35*a*c*f*x^3)*e^2 + 1/105*(35*b*d*f^2*x^9 + 45*(b*c + a*d)*f^2*
x^7 + 63*a*c*f^2*x^5)*e
```

Sympy [A]
time = 0.02, size = 173, normalized size = 1.33

$$ace^3x + \frac{bdf^3x^{11}}{11} + x^9\left(\frac{adf^3}{9} + \frac{bcf^3}{9} + \frac{bdef^2}{3}\right) + x^7\left(\frac{acf^3}{7} + \frac{3adef^2}{7} + \frac{3bcef^2}{7} + \frac{3bde^2f}{7}\right) + x^5\cdot\left(\frac{3acef^2}{5} + \frac{3ade^2f}{5} + \frac{3bce^2f}{5} + \frac{bde^3}{5}\right) + x^3\left(ace^2f + \frac{ade^3}{3} + \frac{bce^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**3,x)
```

```
[Out] a*c*e**3*x + b*d*f**3*x**11/11 + x**9*(a*d*f**3/9 + b*c*f**3/9 + b*d*e*f**2
/3) + x**7*(a*c*f**3/7 + 3*a*d*e*f**2/7 + 3*b*c*e*f**2/7 + 3*b*d*e**2*f/7)
+ x**5*(3*a*c*e*f**2/5 + 3*a*d*e**2*f/5 + 3*b*c*e**2*f/5 + b*d*e**3/5) + x*
*3*(a*c*e**2*f + a*d*e**3/3 + b*c*e**3/3)
```

Giac [A]
time = 1.35, size = 161, normalized size = 1.24

$$\frac{1}{11}bdf^3x^{11} + \frac{1}{9}bcf^3x^9 + \frac{1}{9}adf^3x^9 + \frac{1}{3}bdf^2x^9e + \frac{1}{7}acf^3x^7 + \frac{3}{7}bcf^2x^7e + \frac{3}{7}adf^2x^7e + \frac{3}{7}bdfx^7e^2 + \frac{3}{5}acf^2x^5e + \frac{3}{5}bcfx^5e^2 + \frac{3}{5}adfx^5e^2 + \frac{1}{5}bdx^5e^3 + acfx^3e^2 + \frac{1}{3}bcx^3e^3 + \frac{1}{3}adx^3e^3 + acxe^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] 1/11*b*d*f^3*x^11 + 1/9*b*c*f^3*x^9 + 1/9*a*d*f^3*x^9 + 1/3*b*d*f^2*x^9*e +
 1/7*a*c*f^3*x^7 + 3/7*b*c*f^2*x^7*e + 3/7*a*d*f^2*x^7*e + 3/7*b*d*f*x^7*e^
2 + 3/5*a*c*f^2*x^5*e + 3/5*b*c*f*x^5*e^2 + 3/5*a*d*f*x^5*e^2 + 1/5*b*d*x^5
*e^3 + a*c*f*x^3*e^2 + 1/3*b*c*x^3*e^3 + 1/3*a*d*x^3*e^3 + a*c*x*e^3
```

Mupad [B]
time = 0.84, size = 143, normalized size = 1.10

$$x^5\left(\frac{b\,d\,e^3}{5} + \frac{3\,a\,c\,e\,f^2}{5} + \frac{3\,a\,d\,e^2\,f}{5} + \frac{3\,b\,c\,e^2\,f}{5}\right) + x^7\left(\frac{a\,c\,f^3}{7} + \frac{3\,a\,d\,e\,f^2}{7} + \frac{3\,b\,c\,e\,f^2}{7} + \frac{3\,b\,d\,e^2\,f}{7}\right) + x^3\left(\frac{a\,d\,e^3}{3} + \frac{b\,c\,e^3}{3} + a\,c\,e^2\,f\right) + x^9\left(\frac{a\,d\,f^3}{9} + \frac{b\,c\,f^3}{9} + \frac{b\,d\,e\,f^2}{3}\right) + a\,c\,e^3\,x + \frac{b\,d\,f^3\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3,x)
```

```
[Out] x^5*((b*d*e^3)/5 + (3*a*c*e*f^2)/5 + (3*a*d*e^2*f)/5 + (3*b*c*e^2*f)/5) + x
^7*((a*c*f^3)/7 + (3*a*d*e*f^2)/7 + (3*b*c*e*f^2)/7 + (3*b*d*e^2*f)/7) + x^
3*((a*d*e^3)/3 + (b*c*e^3)/3 + a*c*e^2*f) + x^9*((a*d*f^3)/9 + (b*c*f^3)/9
+ (b*d*e*f^2)/3) + a*c*e^3*x + (b*d*f^3*x^11)/11
```

3.3 $\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx$

Optimal. Leaf size=94

$$ace^2x+\frac{1}{3}e(bce+ade+2acf)x^3+\frac{1}{5}(af(2de+cf)+be(de+2cf))x^5+\frac{1}{7}f(2bde+bcf+adf)x^7+\frac{1}{9}bdf^2x^9$$

```
[Out] a*c*e^2*x+1/3*e*(2*a*c*f+a*d*e+b*c*e)*x^3+1/5*(a*f*(c*f+2*d*e)+b*e*(2*c*f+d
*e))*x^5+1/7*f*(a*d*f+b*c*f+2*b*d*e)*x^7+1/9*b*d*f^2*x^9
```

Rubi [A]
time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{535\}$

$$\frac{1}{7}fx^7(adf+bcf+2bde)+\frac{1}{5}x^5(af(cf+2de)+be(2cf+de))+\frac{1}{3}ex^3(2acf+ade+bce)+ace^2x+\frac{1}{9}bdf^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2,x]
```

```
[Out] a*c*e^2*x + (e*(b*c*e + a*d*e + 2*a*c*f)*x^3)/3 + ((a*f*(2*d*e + c*f) + b*e
*(d*e + 2*c*f))*x^5)/5 + (f*(2*b*d*e + b*c*f + a*d*f)*x^7)/7 + (b*d*f^2*x^9
)/9
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx &= \int \left(ace^2+e(bce+ade+2acf)x^2+(af(2de+cf)+be(de+2cf)\right)\\ &= ace^2x+\frac{1}{3}e(bce+ade+2acf)x^3+\frac{1}{5}(af(2de+cf)+be(de+2cf)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 96, normalized size = 1.02

$$ace^2x+\frac{1}{3}e(bce+ade+2acf)x^3+\frac{1}{5}\left(bde^2+2bcef+2adef+acf^2\right)x^5+\frac{1}{7}f(2bde+bcf+adf)x^7+\frac{1}{9}bdf^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2,x]
```

```
[Out] a*c*e^2*x + (e*(b*c*e + a*d*e + 2*a*c*f)*x^3)/3 + ((b*d*e^2 + 2*b*c*e*f + 2
*a*d*e*f + a*c*f^2)*x^5)/5 + (f*(2*b*d*e + b*c*f + a*d*f)*x^7)/7 + (b*d*f^2
*x^9)/9
```

Maple [A]
time = 0.15, size = 94, normalized size = 1.00

method	result
default	$\frac{bd\,f^2x^9}{9}+\frac{((ad+bc)f^2+2bdef)x^7}{7}+\frac{(ac\,f^2+2(ad+bc)ef+bd\,e^2)x^5}{5}+\frac{(2acef+(ad+bc)e^2)x^3}{3}+ac\,e^2x$
norman	$\frac{bd\,f^2x^9}{9}+\left(\frac{1}{7}ad\,f^2+\frac{1}{7}bc\,f^2+\frac{2}{7}bdef\right)x^7+\left(\frac{1}{5}ac\,f^2+\frac{2}{5}adef+\frac{2}{5}bcef+\frac{1}{5}bd\,e^2\right)x^5+\left(\frac{2}{3}acef+\frac{1}{3}ad\,e\right.$
gosper	$\frac{1}{9}bd\,f^2x^9+\frac{1}{7}x^7ad\,f^2+\frac{1}{7}x^7bc\,f^2+\frac{2}{7}x^7bdef+\frac{1}{5}x^5ac\,f^2+\frac{2}{5}x^5adef+\frac{2}{5}x^5bcef+\frac{1}{5}x^5bd\,e^2+\frac{2}{3}x^3ac$
risch	$\frac{1}{9}bd\,f^2x^9+\frac{1}{7}x^7ad\,f^2+\frac{1}{7}x^7bc\,f^2+\frac{2}{7}x^7bdef+\frac{1}{5}x^5ac\,f^2+\frac{2}{5}x^5adef+\frac{2}{5}x^5bcef+\frac{1}{5}x^5bd\,e^2+\frac{2}{3}x^3ac$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b*d*f^2*x^9+1/7*((a*d+b*c)*f^2+2*b*d*e*f)*x^7+1/5*(a*c*f^2+2*(a*d+b*c)*
e*f+b*d*e^2)*x^5+1/3*(2*a*c*e*f+(a*d+b*c)*e^2)*x^3+a*c*e^2*x
```

Maxima [A]
time = 0.27, size = 95, normalized size = 1.01

$$\frac{1}{9}\,bdf^2x^9+\frac{1}{7}\,(2\,bdfe+(bc+ad)\,f^2)x^7+\frac{1}{5}\,(acf^2+bde^2+2\,(bce+ade)\,f)\,x^5+\frac{1}{3}\,(2\,acfe+bce^2+ade^2)x^3+acxe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b*d*f^2*x^9 + 1/7*(2*b*d*f*e + (b*c + a*d)*f^2)*x^7 + 1/5*(a*c*f^2 + b*
d*e^2 + 2*(b*c*e + a*d*e)*f)*x^5 + 1/3*(2*a*c*f*e + b*c*e^2 + a*d*e^2)*x^3
+ a*c*x*e^2
```

Fricas [A]
time = 0.99, size = 99, normalized size = 1.05

$$\frac{1}{9}\,bdf^2x^9+\frac{1}{7}\,(bc+ad)\,f^2x^7+\frac{1}{5}\,acf^2x^5+\frac{1}{15}\,(3\,bdx^5+5\,(bc+ad)x^3+15\,acx)\,e^2+\frac{2}{105}\,(15\,bdfx^7+21\,(bc+ad)\,fx^5+35\,acfx^3)\,e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="fricas")
```

```
[Out] 1/9*b*d*f^2*x^9 + 1/7*(b*c + a*d)*f^2*x^7 + 1/5*a*c*f^2*x^5 + 1/15*(3*b*d*x
^5 + 5*(b*c + a*d)*x^3 + 15*a*c*x)*e^2 + 2/105*(15*b*d*f*x^7 + 21*(b*c + a*
d)*f*x^5 + 35*a*c*f*x^3)*e
```

Sympy [A]
time = 0.02, size = 121, normalized size = 1.29

$$ace^2x + \frac{bdf^2x^9}{9} + x^7\left(\frac{adf^2}{7} + \frac{bcf^2}{7} + \frac{2bdef}{7}\right) + x^5\left(\frac{acf^2}{5} + \frac{2adef}{5} + \frac{2bcef}{5} + \frac{bde^2}{5}\right) + x^3 \cdot \left(\frac{2acef}{3} + \frac{ade^2}{3} + \frac{bce^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**2,x)
```

```
[Out] a*c*e**2*x + b*d*f**2*x**9/9 + x**7*(a*d*f**2/7 + b*c*f**2/7 + 2*b*d*e*f/7)
 + x**5*(a*c*f**2/5 + 2*a*d*e*f/5 + 2*b*c*e*f/5 + b*d*e**2/5) + x**3*(2*a*c
*e*f/3 + a*d*e**2/3 + b*c*e**2/3)
```

Giac [A]
time = 0.97, size = 114, normalized size = 1.21

$$\frac{1}{9}bdf^2x^9 + \frac{1}{7}bcf^2x^7 + \frac{1}{7}adf^2x^7 + \frac{2}{7}bdfx^7e + \frac{1}{5}acf^2x^5 + \frac{2}{5}bcfx^5e + \frac{2}{5}adfx^5e + \frac{1}{5}bdx^5e^2 + \frac{2}{3}acfx^3e + \frac{1}{3}bcx^3e^2 + \frac{1}{3}adx^3e^2 + acxe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] 1/9*b*d*f^2*x^9 + 1/7*b*c*f^2*x^7 + 1/7*a*d*f^2*x^7 + 2/7*b*d*f*x^7*e + 1/5
*a*c*f^2*x^5 + 2/5*b*c*f*x^5*e + 2/5*a*d*f*x^5*e + 1/5*b*d*x^5*e^2 + 2/3*a*
c*f*x^3*e + 1/3*b*c*x^3*e^2 + 1/3*a*d*x^3*e^2 + a*c*x*e^2
```

Mupad [B]
time = 0.05, size = 99, normalized size = 1.05

$$x^5\left(\frac{a\,c\,f^2}{5} + \frac{b\,d\,e^2}{5} + \frac{2\,a\,d\,e\,f}{5} + \frac{2\,b\,c\,e\,f}{5}\right) + x^3\left(\frac{a\,d\,e^2}{3} + \frac{b\,c\,e^2}{3} + \frac{2\,a\,c\,e\,f}{3}\right) + x^7\left(\frac{a\,d\,f^2}{7} + \frac{b\,c\,f^2}{7} + \frac{2\,b\,d\,e\,f}{7}\right) + a\,c\,e^2\,x + \frac{b\,d\,f^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2,x)
```

```
[Out] x^5*((a*c*f^2)/5 + (b*d*e^2)/5 + (2*a*d*e*f)/5 + (2*b*c*e*f)/5) + x^3*((a*d
*e^2)/3 + (b*c*e^2)/3 + (2*a*c*e*f)/3) + x^7*((a*d*f^2)/7 + (b*c*f^2)/7 + (
2*b*d*e*f)/7) + a*c*e^2*x + (b*d*f^2*x^9)/9
```

3.4 $\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx$

Optimal. Leaf size=56

$$acex+\frac{1}{3}(bce+ade+acf)x^3+\frac{1}{5}(bde+bcf+adf)x^5+\frac{1}{7}bdfx^7$$

```
[Out] a*c*e*x+1/3*(a*c*f+a*d*e+b*c*e)*x^3+1/5*(a*d*f+b*c*f+b*d*e)*x^5+1/7*b*d*f*x
^7
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = $\{535\}$

$$\frac{1}{5}x^5(adf+bcf+bde)+\frac{1}{3}x^3(acf+ade+bce)+acex+\frac{1}{7}bdfx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2),x]
```

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^3)/3 + ((b*d*e + b*c*f + a*d*f)*x^5)/5
 + (b*d*f*x^7)/7
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx &= \int \left(ace+(bce+ade+acf)x^2+(bde+bcf+adf)x^4+bdfx^6\right)dx\\ &= acex+\frac{1}{3}(bce+ade+acf)x^3+\frac{1}{5}(bde+bcf+adf)x^5+\frac{1}{7}bdfx^7\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 1.00

$$acex+\frac{1}{3}(bce+ade+acf)x^3+\frac{1}{5}(bde+bcf+adf)x^5+\frac{1}{7}bdfx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2),x]
```

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^3)/3 + ((b*d*e + b*c*f + a*d*f)*x^5)/5
 + (b*d*f*x^7)/7
```

Maple [A]
time = 0.13, size = 53, normalized size = 0.95

method	result	size
default	$\frac{bdf\,x^7}{7}+\frac{((ad+bc)f+bde)x^5}{5}+\frac{(acf+(ad+bc)e)x^3}{3}+acex$	53
norman	$\frac{bdf\,x^7}{7}+\left(\frac{1}{5}adf+\frac{1}{5}bcf+\frac{1}{5}bde\right)x^5+\left(\frac{1}{3}acf+\frac{1}{3}ade+\frac{1}{3}bce\right)x^3+acex$	55
gosper	$\frac{1}{7}bdf\,x^7+\frac{1}{5}x^5adf+\frac{1}{5}x^5bcf+\frac{1}{5}x^5bde+\frac{1}{3}x^3acf+\frac{1}{3}x^3ade+\frac{1}{3}x^3bce+acex$	63
risch	$\frac{1}{7}bdf\,x^7+\frac{1}{5}x^5adf+\frac{1}{5}x^5bcf+\frac{1}{5}x^5bde+\frac{1}{3}x^3acf+\frac{1}{3}x^3ade+\frac{1}{3}x^3bce+acex$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b*d*f*x^7+1/5*((a*d+b*c)*f+b*d*e)*x^5+1/3*(a*c*f+(a*d+b*c)*e)*x^3+a*c*e
*x
```

Maxima [A]
time = 0.26, size = 55, normalized size = 0.98

$$\frac{1}{7}\,bdfx^7+\frac{1}{5}\,(bde+(bc+ad)f)x^5+\frac{1}{3}\,(acf+bce+ade)x^3+acxe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x, algorithm="maxima")
```

```
[Out] 1/7*b*d*f*x^7 + 1/5*(b*d*e + (b*c + a*d)*f)*x^5 + 1/3*(a*c*f + b*c*e + a*d*
e)*x^3 + a*c*x*e
```

Fricas [A]
time = 0.92, size = 59, normalized size = 1.05

$$\frac{1}{7}\,bdfx^7+\frac{1}{5}\,(bc+ad)fx^5+\frac{1}{3}\,acfx^3+\frac{1}{15}\,\left(3\,bdx^5+5\,(bc+ad)x^3+15\,acx\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x, algorithm="fricas")
```

```
[Out] 1/7*b*d*f*x^7 + 1/5*(b*c + a*d)*f*x^5 + 1/3*a*c*f*x^3 + 1/15*(3*b*d*x^5 + 5
*(b*c + a*d)*x^3 + 15*a*c*x)*e
```

Sympy [A]
time = 0.01, size = 63, normalized size = 1.12

$$acex+\frac{bdfx^7}{7}+x^5\left(\frac{adf}{5}+\frac{bcf}{5}+\frac{bde}{5}\right)+x^3\left(\frac{acf}{3}+\frac{ade}{3}+\frac{bce}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e),x)
```

```
[Out] a*c*e*x + b*d*f*x**7/7 + x**5*(a*d*f/5 + b*c*f/5 + b*d*e/5) + x**3*(a*c*f/3
 + a*d*e/3 + b*c*e/3)
```

Giac [A]
time = 1.32, size = 66, normalized size = 1.18

$$\frac{1}{7}bdfx^7 + \frac{1}{5}bcfx^5 + \frac{1}{5}adfx^5 + \frac{1}{5}bdx^5e + \frac{1}{3}acfx^3 + \frac{1}{3}bcx^3e + \frac{1}{3}adx^3e + acxe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x, algorithm="giac")
```

```
[Out] 1/7*b*d*f*x^7 + 1/5*b*c*f*x^5 + 1/5*a*d*f*x^5 + 1/5*b*d*x^5*e + 1/3*a*c*f*x
^3 + 1/3*b*c*x^3*e + 1/3*a*d*x^3*e + a*c*x*e
```

Mupad [B]
time = 0.05, size = 54, normalized size = 0.96

$$\frac{b\,d\,f\,x^7}{7} + \left(\frac{a\,d\,f}{5} + \frac{b\,c\,f}{5} + \frac{b\,d\,e}{5}\right)\,x^5 + \left(\frac{a\,c\,f}{3} + \frac{a\,d\,e}{3} + \frac{b\,c\,e}{3}\right)\,x^3 + a\,c\,e\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)*(e + f*x^2),x)
```

```
[Out] x^3*((a*c*f)/3 + (a*d*e)/3 + (b*c*e)/3) + x^5*((a*d*f)/5 + (b*c*f)/5 + (b*d
*e)/5) + a*c*e*x + (b*d*f*x^7)/7
```

3.5 $\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2} \, dx$

Optimal. Leaf size=81

$$-\frac{(3bde-3bcf-2adf)x}{3f^2}+\frac{dx(a+bx^2)}{3f}+\frac{(be-af)(de-cf)\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}\,f^{5/2}}$$

```
[Out] -1/3*(-2*a*d*f-3*b*c*f+3*b*d*e)*x/f^2+1/3*d*x*(b*x^2+a)/f+(-a*f+b*e)*(-c*f+
d*e)*arctan(x*f^(1/2)/e^(1/2))/f^(5/2)/e^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {542, 396, 211}

$$\frac{(be-af)\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(de-cf)}{\sqrt{e}\,f^{5/2}}-\frac{x(-2adf-3bcf+3bde)}{3f^2}+\frac{dx(a+bx^2)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2),x]
```

```
[Out] -1/3*((3*b*d*e - 3*b*c*f - 2*a*d*f)*x)/f^2 + (d*x*(a + b*x^2))/(3*f) + ((b*
e - a*f)*(d*e - c*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx &= \frac{dx(a+bx^2)}{3f}+\frac{\int \frac{-a(de-3cf)-(3bde-3bcf-2adf)x^2}{e+fx^2}\,dx}{3f}\\
&= -\frac{(3bde-3bcf-2adf)x}{3f^2}+\frac{dx(a+bx^2)}{3f}+\frac{((be-af)(de-cf))\int \frac{1}{e+fx^2}\,dx}{f^2}\\
&= -\frac{(3bde-3bcf-2adf)x}{3f^2}+\frac{dx(a+bx^2)}{3f}+\frac{(be-af)(de-cf)\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}\,f^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 0.89

$$
\frac{(-bde+bcf+adf)x}{f^2}+\frac{bdx^3}{3f}+\frac{(be-af)(de-cf)\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}\,f^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2),x]
[Out] ((-(b*d*e) + b*c*f + a*d*f)*x)/f^2 + (b*d*x^3)/(3*f) + ((b*e - a*f)*(d*e - c*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(5/2))
```

Maple [A]
time = 0.14, size = 74, normalized size = 0.91

method	result
default	$\frac{\frac{1}{3}bd\,x^3f+adfx+bcfx-bdex}{f^2}+\frac{(ac\,f^2-adef-bcef+bd\,e^2)\arctan\left(\frac{fx}{\sqrt{fe}}\right)}{f^2\sqrt{fe}}$
risch	$\frac{bd\,x^3}{3f}+\frac{adx}{f}+\frac{bcx}{f}-\frac{bdex}{f^2}-\frac{\ln\left(fx+\sqrt{-fe}\right)ac}{2\sqrt{-fe}}+\frac{\ln\left(fx+\sqrt{-fe}\right)ade}{2f\sqrt{-fe}}+\frac{\ln\left(fx+\sqrt{-fe}\right)bce}{2f\sqrt{-fe}}-\frac{\ln\left(fx+\sqrt{-f}\right.}{2f^2\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x,method=_RETURNVERBOSE)
[Out] 1/f^2*(1/3*b*d*x^3*f+a*d*f*x+b*c*f*x-b*d*e*x)+(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/f^2/(f*e)^(1/2)*arctan(f*x/(f*e)^(1/2))
```

Maxima [A]
time = 0.52, size = 73, normalized size = 0.90

$$\frac{(acf^2 + bde^2 - (bce + ade)f)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}\right)}}{f^{\frac{5}{2}}} + \frac{bdfx^3 - 3\,(bde - (bc + ad)f)x}{3\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")
```

```
[Out] (a*c*f^2 + b*d*e^2 - (b*c*e + a*d*e)*f)*arctan(sqrt(f)*x*e^(-1/2))*e^(-1/2)
/f^(5/2) + 1/3*(b*d*f*x^3 - 3*(b*d*e - (b*c + a*d)*f)*x)/f^2
```

Fricas [A]
time = 0.95, size = 191, normalized size = 2.36

$$\left[-\frac{\left(6\,bdfxe^2 + 3\,(acf^2 + bde^2 - (bc + ad)fe)\sqrt{-fe}\,\log\left(\frac{fx^2 - 2\,\sqrt{-fe}\,x - e}{fx^2 + e}\right) - 2\,(bdf^2x^3 + 3\,(bc + ad)f^2x)e\right)e^{(-1)}}{6\,f^3}, -\frac{\left(3\,bdfxe^2 - 3\,(acf^2 + bde^2 - (bc + ad)fe)\sqrt{f}\,\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\frac{1}{2}} - (bdf^2x^3 + 3\,(bc + ad)f^2x)e\right)e^{(-1)}}{3\,f^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")
```

```
[Out] [-1/6*(6*b*d*f*x*e^2 + 3*(a*c*f^2 + b*d*e^2 - (b*c + a*d)*f*e)*sqrt(-f*e)*l
og((f*x^2 - 2*sqrt(-f*e)*x - e)/(f*x^2 + e)) - 2*(b*d*f^2*x^3 + 3*(b*c + a*
d)*f^2*x)*e)*e^(-1)/f^3, -1/3*(3*b*d*f*x*e^2 - 3*(a*c*f^2 + b*d*e^2 - (b*c
+ a*d)*f*e)*sqrt(f)*arctan(sqrt(f)*x*e^(-1/2))*e^(1/2) - (b*d*f^2*x^3 + 3*(
b*c + a*d)*f^2*x)*e)*e^(-1)/f^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(73) = 146$.
time = 0.39, size = 206, normalized size = 2.54

$$\frac{bdx^3}{3f} + x\left(\frac{ad}{f} + \frac{bc}{f} - \frac{bde}{f^2}\right) - \frac{\sqrt{-\frac{1}{ef^5}}\,(af - be)\,(cf - de)\log\left(-\frac{ef^2\sqrt{-\frac{1}{ef^5}}\,(af - be)(cf - de)}{acf^2 - adef - bcef + bde^2} + x\right)}{2} + \frac{\sqrt{-\frac{1}{ef^5}}\,(af - be)\,(cf - de)\log\left(\frac{ef^2\sqrt{-\frac{1}{ef^5}}\,(af - be)(cf - de)}{acf^2 - adef - bcef + bde^2} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e),x)
```

```
[Out] b*d*x**3/(3*f) + x*(a*d/f + b*c/f - b*d*e/f**2) - sqrt(-1/(e*f**5))*(a*f -
b*e)*(c*f - d*e)*log(-e*f**2*sqrt(-1/(e*f**5))*(a*f - b*e)*(c*f - d*e)/(a*c
*f**2 - a*d*e*f - b*c*e*f + b*d*e**2) + x)/2 + sqrt(-1/(e*f**5))*(a*f - b*e
)*(c*f - d*e)*log(e*f**2*sqrt(-1/(e*f**5))*(a*f - b*e)*(c*f - d*e)/(a*c*f**
2 - a*d*e*f - b*c*e*f + b*d*e**2) + x)/2
```

Giac [A]
time = 1.18, size = 80, normalized size = 0.99

$$\frac{(acf^2 - bcfe - adfe + bde^2)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}\right)}}{f^{\frac{5}{2}}} + \frac{bdf^2x^3 + 3\,bcf^2x + 3\,adf^2x - 3\,bdfxe}{3\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x, algorithm="giac")
```

```
[Out] (a*c*f^2 - b*c*f*e - a*d*f*e + b*d*e^2)*arctan(sqrt(f)*x*e^(-1/2))*e^(-1/2)
/f^(5/2) + 1/3*(b*d*f^2*x^3 + 3*b*c*f^2*x + 3*a*d*f^2*x - 3*b*d*f*x*e)/f^3
```

Mupad [B]
time = 0.11, size = 108, normalized size = 1.33

$$x\left(\frac{a\,d+b\,c}{f}-\frac{b\,d\,e}{f^2}\right)+\frac{b\,d\,x^3}{3\,f}+\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(a\,f-b\,e)\,(c\,f-d\,e)}{\sqrt{e}\,(a\,c\,f^2+b\,d\,e^2-a\,d\,e\,f-b\,c\,e\,f)}\right)\,(a\,f-b\,e)\,(c\,f-d\,e)}{\sqrt{e}\,f^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2))/(e + f*x^2),x)
```

```
[Out] x*((a*d + b*c)/f - (b*d*e)/f^2) + (b*d*x^3)/(3*f) + (atan((f^(1/2)*x*(a*f -
 b*e)*(c*f - d*e))/(e^(1/2)*(a*c*f^2 + b*d*e^2 - a*d*e*f - b*c*e*f)))*(a*f
- b*e)*(c*f - d*e))/(e^(1/2)*f^(5/2))
```

3.6 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2} dx$

Optimal. Leaf size=108

$$\frac{b(3de-cf)x}{2ef^2} - \frac{(de-cf)x(a+bx^2)}{2ef(e+fx^2)} - \frac{(be(3de-cf)-af(de+cf))\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{2e^{3/2}f^{5/2}}$$

```
[Out] 1/2*b*(-c*f+3*d*e)*x/e/f^2-1/2*(-c*f+d*e)*x*(b*x^2+a)/e/f/(f*x^2+e)-1/2*(b*
e*(-c*f+3*d*e)-a*f*(c*f+d*e))*arctan(x*f^(1/2)/e^(1/2))/e^(3/2)/f^(5/2)
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {540, 396, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(be(3de-cf)-af(cf+de))}{2e^{3/2}f^{5/2}} - \frac{x(a+bx^2)(de-cf)}{2ef(e+fx^2)} + \frac{bx(3de-cf)}{2ef^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^2,x]
```

```
[Out] (b*(3*d*e - c*f)*x)/(2*e*f^2) - ((d*e - c*f)*x*(a + b*x^2))/(2*e*f*(e + f*x
^2)) - ((b*e*(3*d*e - c*f) - a*f*(d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(2*e^(3/2)*f^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
```

```
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx &= -\frac{(de-cf)x(a+bx^2)}{2ef(e+fx^2)} - \frac{\int \frac{-a(de+cf)-b(3de-cf)x^2}{e+fx^2}\,dx}{2ef} \\
&= \frac{b(3de-cf)x}{2ef^2} - \frac{(de-cf)x(a+bx^2)}{2ef(e+fx^2)} - \frac{(be(3de-cf)-af(de+cf))\int \frac{1}{e+fx^2}\,dx}{2ef^2} \\
&= \frac{b(3de-cf)x}{2ef^2} - \frac{(de-cf)x(a+bx^2)}{2ef(e+fx^2)} - \frac{(be(3de-cf)-af(de+cf))\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 95, normalized size = 0.88

$$
\frac{bdx}{f^2} + \frac{(be-af)(de-cf)x}{2ef^2(e+fx^2)} - \frac{(be(3de-cf)-af(de+cf))\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{2e^{3/2}f^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^2,x]
```

```
[Out] (b*d*x)/f^2 + ((b*e - a*f)*(d*e - c*f)*x)/(2*e*f^2*(e + f*x^2)) - ((b*e*(3*
d*e - c*f) - a*f*(d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(5/
2))
```

Maple [A]
time = 0.13, size = 97, normalized size = 0.90

method	result
default	$\frac{bdx}{f^2} + \frac{\frac{(ac f^2-adef-bcef+bd e^2)x}{2e(f x^2+e)} + \frac{(ac f^2+adef+bcef-3bd e^2)\arctan\left(\frac{fx}{\sqrt{fe}}\right)}{2e\sqrt{fe}}}{f^2}$
risch	$\frac{bdx}{f^2} + \frac{(ac f^2-adef-bcef+bd e^2)x}{2e f^2(f x^2+e)} - \frac{\ln\left(fx+\sqrt{-fe}\right)ac}{4\sqrt{-fe}\,e} - \frac{\ln\left(fx+\sqrt{-fe}\right)ad}{4f\sqrt{-fe}} - \frac{\ln\left(fx+\sqrt{-fe}\right)bc}{4f\sqrt{-fe}} + \frac{3e\ln\left(fx+\right.}{4f^2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*d/f^2*x+1/f^2*(1/2*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x/(f*x^2+e)+1/2*(a
*c*f^2+a*d*e*f+b*c*e*f-3*b*d*e^2)/e/(f*e)^(1/2)*arctan(f*x/(f*e)^(1/2)))
```

Maxima [**A**]
time = 0.48, size = 96, normalized size = 0.89

$$\frac{(acf^2+bde^2-(bce+ade)f)x}{2\,(f^3x^2e+f^2e^2)}+\frac{bdx}{f^2}+\frac{(acf^2-3\,bde^2+(bce+ade)f)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{3}{2}\right)}}{2\,f^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*c*f^2 + b*d*e^2 - (b*c*e + a*d*e)*f)*x/(f^3*x^2*e + f^2*e^2) + b*d*x
/f^2 + 1/2*(a*c*f^2 - 3*b*d*e^2 + (b*c*e + a*d*e)*f)*arctan(sqrt(f)*x*e^(-1
/2))*e^(-3/2)/f^(5/2)
```

Fricas [**A**]
time = 0.67, size = 313, normalized size = 2.90

$$\left[\frac{2\,acf^3xe+6\,bdfxe^3+(acf^3x^2-3\,bde^3-(3\,bdfx^2-(bc+ad)f)e^2+((bc+ad)f^2x^2+acf^2)e)\sqrt{-fe}\,\log\left(\frac{fx^2+2\,\sqrt{-fe}\,x-e}{fx^2+e}\right)+2\,(2\,bdf^2x^3-(bc+ad)f^2x)e^2}{4\,(f^4x^2e^2+f^3e^3)},\ \frac{acf^3xe+3\,bdfxe^3+(acf^3x^2-3\,bde^3-(3\,bdfx^2-(bc+ad)f)e^2+((bc+ad)f^2x^2+acf^2)e)\sqrt{f}\,\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\frac{1}{2}}+(2\,bdf^2x^3-(bc+ad)f^2x)e^2}{2\,(f^4x^2e^2+f^3e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*c*f^3*x*e + 6*b*d*f*x*e^3 + (a*c*f^3*x^2 - 3*b*d*e^3 - (3*b*d*f*x
^2 - (b*c + a*d)*f)*e^2 + ((b*c + a*d)*f^2*x^2 + a*c*f^2)*e)*sqrt(-f*e)*log
((f*x^2 + 2*sqrt(-f*e)*x - e)/(f*x^2 + e)) + 2*(2*b*d*f^2*x^3 - (b*c + a*d)
*f^2*x)*e^2)/(f^4*x^2*e^2 + f^3*e^3), 1/2*(a*c*f^3*x*e + 3*b*d*f*x*e^3 + (a
*c*f^3*x^2 - 3*b*d*e^3 - (3*b*d*f*x^2 - (b*c + a*d)*f)*e^2 + ((b*c + a*d)*f
^2*x^2 + a*c*f^2)*e)*sqrt(f)*arctan(sqrt(f)*x*e^(-1/2))*e^(1/2) + (2*b*d*f^
2*x^3 - (b*c + a*d)*f^2*x)*e^2)/(f^4*x^2*e^2 + f^3*e^3)]
```

Sympy [**A**]
time = 0.73, size = 190, normalized size = 1.76

$$\frac{bdx}{f^2}+\frac{x(acf^2-adef-bcef+bde^2)}{2e^2f^2+2ef^3x^2}-\frac{\sqrt{-\frac{1}{e^3f^5}}\,(acf^2+adef+bcef-3bde^2)\log\left(-e^2f^2\sqrt{-\frac{1}{e^3f^5}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{e^3f^5}}\,(acf^2+adef+bcef-3bde^2)\log\left(e^2f^2\sqrt{-\frac{1}{e^3f^5}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**2,x)
```

```
[Out] b*d*x/f**2 + x*(a*c*f**2 - a*d*e*f - b*c*e*f + b*d*e**2)/(2*e**2*f**2 + 2*e
*f**3*x**2) - sqrt(-1/(e**3*f**5))*(a*c*f**2 + a*d*e*f + b*c*e*f - 3*b*d*e*
*2)*log(-e**2*f**2*sqrt(-1/(e**3*f**5)) + x)/4 + sqrt(-1/(e**3*f**5))*(a*c*
f**2 + a*d*e*f + b*c*e*f - 3*b*d*e**2)*log(e**2*f**2*sqrt(-1/(e**3*f**5)) +
 x)/4
```

Giac [A]
time = 1.28, size = 95, normalized size = 0.88

$$\frac{bdx}{f^2} + \frac{(acf^2 + bcfe + adfe - 3\,bde^2)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{3}{2}\right)}}{2\,f^{\frac{5}{2}}} + \frac{(acf^2x - bcfxe - adfxe + bdxe^2)e^{(-1)}}{2\,(fx^2 + e)f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] b*d*x/f^2 + 1/2*(a*c*f^2 + b*c*f*e + a*d*f*e - 3*b*d*e^2)*arctan(sqrt(f)*x*
e^(-1/2))*e^(-3/2)/f^(5/2) + 1/2*(a*c*f^2*x - b*c*f*x*e - a*d*f*x*e + b*d*x
*e^2)*e^(-1)/((f*x^2 + e)*f^2)
```

Mupad [B]
time = 0.17, size = 95, normalized size = 0.88

$$\frac{b\,d\,x}{f^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,\left(a\,c\,f^2 - 3\,b\,d\,e^2 + a\,d\,e\,f + b\,c\,e\,f\right)}{2\,e^{3/2}\,f^{5/2}} + \frac{x\,\left(a\,c\,f^2 + b\,d\,e^2 - a\,d\,e\,f - b\,c\,e\,f\right)}{2\,e\,\left(f^3\,x^2 + e\,f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^2,x)
```

```
[Out] (b*d*x)/f^2 + (atan((f^(1/2)*x)/e^(1/2))*(a*c*f^2 - 3*b*d*e^2 + a*d*e*f + b
*c*e*f))/(2*e^(3/2)*f^(5/2)) + (x*(a*c*f^2 + b*d*e^2 - a*d*e*f - b*c*e*f))/
(2*e*(e*f^2 + f^3*x^2))
```

3.7 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3} dx$

Optimal. Leaf size=130

$$-\frac{(de-cf)x(a+bx^2)}{4ef(e+fx^2)^2}-\frac{(be(3de+cf)-af(de+3cf))x}{8e^2f^2(e+fx^2)}+\frac{(be(3de+cf)+af(de+3cf))\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{8e^{5/2}f^{5/2}}$$

```
[Out] -1/4*(-c*f+d*e)*x*(b*x^2+a)/e/f/(f*x^2+e)^2-1/8*(b*e*(c*f+3*d*e)-a*f*(3*c*f
+d*e))*x/e^2/f^2/(f*x^2+e)+1/8*(b*e*(c*f+3*d*e)+a*f*(3*c*f+d*e))*arctan(x*f
^(1/2)/e^(1/2))/e^(5/2)/f^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {540, 393, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(af(3cf+de)+be(cf+3de))}{8e^{5/2}f^{5/2}}-\frac{x(be(cf+3de)-af(3cf+de))}{8e^2f^2(e+fx^2)}-\frac{x(a+bx^2)(de-cf)}{4ef(e+fx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^3,x]
```

```
[Out] -1/4*((d*e - c*f)*x*(a + b*x^2))/(e*f*(e + f*x^2)^2) - ((b*e*(3*d*e + c*f)
- a*f*(d*e + 3*c*f))*x)/(8*e^2*f^2*(e + f*x^2)) + ((b*e*(3*d*e + c*f) + a*f
*(d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx = -\frac{(de-cf)x(a+bx^2)}{4ef(e+fx^2)^2} - \frac{\int \frac{-a(de+3cf)-b(3de+cf)x^2}{(e+fx^2)^2}\,dx}{4ef}$$

$$= -\frac{(de-cf)x(a+bx^2)}{4ef(e+fx^2)^2} - \frac{(be(3de+cf)-af(de+3cf))x}{8e^2f^2(e+fx^2)} + \frac{(be(3de+cf)+af}{8}$$

$$= -\frac{(de-cf)x(a+bx^2)}{4ef(e+fx^2)^2} - \frac{(be(3de+cf)-af(de+3cf))x}{8e^2f^2(e+fx^2)} + \frac{(be(3de+cf)+af}{}$$

Mathematica [A]
time = 0.06, size = 130, normalized size = 1.00

$$\frac{(be-af)(de-cf)x}{4ef^2(e+fx^2)^2} + \frac{(be(-5de+cf)+af(de+3cf))x}{8e^2f^2(e+fx^2)} + \frac{(be(3de+cf)+af(de+3cf))\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{8e^{5/2}f^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^3,x]
[Out] ((b*e - a*f)*(d*e - c*f)*x)/(4*e*f^2*(e + f*x^2)^2) + ((b*e*(-5*d*e + c*f)
+ a*f*(d*e + 3*c*f))*x)/(8*e^2*f^2*(e + f*x^2)) + ((b*e*(3*d*e + c*f) + a*f
*(d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(5/2))
```

Maple [A]
time = 0.14, size = 132, normalized size = 1.02

method	result
default	$\frac{\frac{(3acf^2+adef+bcef-5bde^2)x^3}{8e^2f}+\frac{(5acf^2-adef-bcef-3bde^2)x}{8ef^2}}{(fx^2+e)^2} + \frac{(3acf^2+adef+bcef+3bde^2)\arctan\left(\frac{fx}{\sqrt{fe}}\right)}{8e^2f^2\sqrt{fe}}$
risch	$\frac{\frac{(3acf^2+adef+bcef-5bde^2)x^3}{8e^2f}+\frac{(5acf^2-adef-bcef-3bde^2)x}{8ef^2}}{(fx^2+e)^2} - \frac{3\ln\left(fx+\sqrt{-fe}\right)ac}{16\sqrt{-fe}\,e^2} - \frac{\ln\left(fx+\sqrt{-fe}\right)ad}{16\sqrt{-fe}\,fe} - \frac{\ln\left(fx+\sqrt{-\right.}{16\sqrt{-f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*a*c*f^2+a*d*e*f+b*c*e*f-5*b*d*e^2)/e^2/f*x^3+1/8*(5*a*c*f^2-a*d*e*f
-b*c*e*f-3*b*d*e^2)/e/f^2*x)/(f*x^2+e)^2+1/8*(3*a*c*f^2+a*d*e*f+b*c*e*f+3*b
*d*e^2)/e^2/f^2/(f*e)^(1/2)*arctan(f*x/(f*e)^(1/2))
```

Maxima [**A**]
time = 0.56, size = 140, normalized size = 1.08

$$\frac{\left(3\,acf^{2}+3\,bde^{2}+\left(bce+ade\right)f\right)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{5}{2}\right)}}{8\,f^{\frac{5}{2}}}+\frac{\left(3\,acf^{3}-5\,bdfe^{2}+\left(bce+ade\right)f^{2}\right)x^{3}+\left(5\,acf^{2}e-3\,bde^{3}-\left(bce^{2}+ade^{2}\right)f\right)x}{8\,\left(f^{4}x^{4}e^{2}+2\,f^{3}x^{2}e^{3}+f^{2}e^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*a*c*f^2 + 3*b*d*e^2 + (b*c*e + a*d*e)*f)*arctan(sqrt(f)*x*e^(-1/2))*
e^(-5/2)/f^(5/2) + 1/8*((3*a*c*f^3 - 5*b*d*f*e^2 + (b*c*e + a*d*e)*f^2)*x^3
 + (5*a*c*f^2*e - 3*b*d*e^3 - (b*c*e^2 + a*d*e^2)*f)*x)/(f^4*x^4*e^2 + 2*f^
3*x^2*e^3 + f^2*e^4)
```

Fricas [**A**]
time = 0.96, size = 469, normalized size = 3.61

$$\left[\frac{6\,acf^{4}x^{3}e-6\,bdfxe^{4}-\left(3\,acf^{4}x^{4}+3\,bde^{4}+\left(6\,bdfx^{2}+\left(bc+ad\right)f\right)e^{3}+\left(3\,bdf^{2}x^{4}+2\,\left(bc+ad\right)f^{2}x^{2}+3\,acf^{2}\right)e^{2}+\left(\left(bc+ad\right)f^{3}x^{4}+6\,acf^{3}x^{2}\right)e\right)\sqrt{-fe}\,\log\left(\frac{fx^{2}-2\,\sqrt{-fe}\,x-e}{fx^{2}+e}\right)-2\,\left(5\,bdf^{2}x^{3}+\left(bc+ad\right)f^{2}x\right)e^{3}+2\,\left(\left(bc+ad\right)f^{3}x^{3}+5\,acf^{3}x\right)e^{2}}{16\,\left(f^{5}x^{4}e^{3}+2\,f^{4}x^{2}e^{4}+f^{3}e^{5}\right)},\ \frac{3\,acf^{4}x^{3}e-3\,bdfxe^{4}+\left(3\,acf^{4}x^{4}+3\,bde^{4}+\left(6\,bdfx^{2}+\left(bc+ad\right)f\right)e^{3}+\left(3\,bdf^{2}x^{4}+2\,\left(bc+ad\right)f^{2}x^{2}+3\,acf^{2}\right)e^{2}+\left(\left(bc+ad\right)f^{3}x^{4}+6\,acf^{3}x^{2}\right)e\right)\sqrt{f}\,\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\frac{1}{2}}-\left(5\,bdf^{2}x^{3}+\left(bc+ad\right)f^{2}x\right)e^{3}+\left(\left(bc+ad\right)f^{3}x^{3}+5\,acf^{3}x\right)e^{2}}{8\,\left(f^{5}x^{4}e^{3}+2\,f^{4}x^{2}e^{4}+f^{3}e^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*c*f^4*x^3*e - 6*b*d*f*x*e^4 - (3*a*c*f^4*x^4 + 3*b*d*e^4 + (6*b*
d*f*x^2 + (b*c + a*d)*f)*e^3 + (3*b*d*f^2*x^4 + 2*(b*c + a*d)*f^2*x^2 + 3*a
*c*f^2)*e^2 + ((b*c + a*d)*f^3*x^4 + 6*a*c*f^3*x^2)*e)*sqrt(-f*e)*log((f*x^
2 - 2*sqrt(-f*e)*x - e)/(f*x^2 + e)) - 2*(5*b*d*f^2*x^3 + (b*c + a*d)*f^2*x
)*e^3 + 2*((b*c + a*d)*f^3*x^3 + 5*a*c*f^3*x)*e^2)/(f^5*x^4*e^3 + 2*f^4*x^2
*e^4 + f^3*e^5), 1/8*(3*a*c*f^4*x^3*e - 3*b*d*f*x*e^4 + (3*a*c*f^4*x^4 + 3*
b*d*e^4 + (6*b*d*f*x^2 + (b*c + a*d)*f)*e^3 + (3*b*d*f^2*x^4 + 2*(b*c + a*d
)*f^2*x^2 + 3*a*c*f^2)*e^2 + ((b*c + a*d)*f^3*x^4 + 6*a*c*f^3*x^2)*e)*sqrt(
f)*arctan(sqrt(f)*x*e^(-1/2))*e^(1/2) - (5*b*d*f^2*x^3 + (b*c + a*d)*f^2*x)
*e^3 + ((b*c + a*d)*f^3*x^3 + 5*a*c*f^3*x)*e^2)/(f^5*x^4*e^3 + 2*f^4*x^2*e^
4 + f^3*e^5)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(122) = 244$.
time = 1.62, size = 246, normalized size = 1.89

$$-\frac{\sqrt{-\frac{1}{e^{5}f^{5}}}\cdot\left(3acf^{2}+adef+bcef+3bde^{2}\right)\log\left(-e^{3}f^{2}\sqrt{-\frac{1}{e^{5}f^{5}}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{e^{5}f^{5}}}\cdot\left(3acf^{2}+adef+bcef+3bde^{2}\right)\log\left(e^{3}f^{2}\sqrt{-\frac{1}{e^{5}f^{5}}}+x\right)}{16}+\frac{x^{3}\cdot\left(3acf^{3}+adef^{2}+bcef^{2}-5bde^{2}f\right)+x\left(5acef^{2}-ade^{2}f-bce^{2}f-3bde^{3}\right)}{8e^{4}f^{2}+16e^{3}f^{3}x^{2}+8e^{2}f^{4}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**3,x)
```

```
[Out] -sqrt(-1/(e**5*f**5))*(3*a*c*f**2 + a*d*e*f + b*c*e*f + 3*b*d*e**2)*log(-e*
*3*f**2*sqrt(-1/(e**5*f**5)) + x)/16 + sqrt(-1/(e**5*f**5))*(3*a*c*f**2 + a
*d*e*f + b*c*e*f + 3*b*d*e**2)*log(e**3*f**2*sqrt(-1/(e**5*f**5)) + x)/16 +
 (x**3*(3*a*c*f**3 + a*d*e*f**2 + b*c*e*f**2 - 5*b*d*e**2*f) + x*(5*a*c*e*f
**2 - a*d*e**2*f - b*c*e**2*f - 3*b*d*e**3))/(8*e**4*f**2 + 16*e**3*f**3*x*
*2 + 8*e**2*f**4*x**4)
```

Giac [A]
time = 1.15, size = 135, normalized size = 1.04

$$\frac{(3\,acf^2 + bcfe + adfe + 3\,bde^2)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{5}{2}\right)}}{8\,f^{\frac{5}{2}}} + \frac{(3\,acf^3x^3 + bcf^2x^3e + adf^2x^3e - 5\,bdfx^3e^2 + 5\,acf^2xe - bcfxe^2 - adfxe^2 - 3\,bdxe^3)e^{(-2)}}{8\,(fx^2+e)^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*a*c*f^2 + b*c*f*e + a*d*f*e + 3*b*d*e^2)*arctan(sqrt(f)*x*e^(-1/2))*
e^(-5/2)/f^(5/2) + 1/8*(3*a*c*f^3*x^3 + b*c*f^2*x^3*e + a*d*f^2*x^3*e - 5*b
*d*f*x^3*e^2 + 5*a*c*f^2*x*e - b*c*f*x*e^2 - a*d*f*x*e^2 - 3*b*d*x*e^3)*e^(
-2)/((f*x^2 + e)^2*f^2)
```

Mupad [B]
time = 0.97, size = 136, normalized size = 1.05

$$\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,\left(3\,a\,c\,f^2+3\,b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f\right)}{8\,e^{5/2}\,f^{5/2}}-\frac{\frac{x\,\left(3\,b\,d\,e^2-5\,a\,c\,f^2+a\,d\,e\,f+b\,c\,e\,f\right)}{8\,e\,f^2}-\frac{x^3\,\left(3\,a\,c\,f^2-5\,b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f\right)}{8\,e^2\,f}}{e^2+2\,e\,f\,x^2+f^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^3,x)
```

```
[Out] (atan((f^(1/2)*x)/e^(1/2))*(3*a*c*f^2 + 3*b*d*e^2 + a*d*e*f + b*c*e*f))/(8*
e^(5/2)*f^(5/2)) - ((x*(3*b*d*e^2 - 5*a*c*f^2 + a*d*e*f + b*c*e*f))/(8*e*f^
2) - (x^3*(3*a*c*f^2 - 5*b*d*e^2 + a*d*e*f + b*c*e*f))/(8*e^2*f))/(e^2 + f^
2*x^4 + 2*e*f*x^2)
```

3.8 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx$

Optimal. Leaf size=171

$$-\frac{(de-cf)x(a+bx^2)}{6ef(e+fx^2)^3}-\frac{(3be(de+cf)-af(de+5cf))x}{24e^2f^2(e+fx^2)^2}+\frac{(be(de+cf)+af(de+5cf))x}{16e^3f^2(e+fx^2)}+\frac{(be(de+cf)+$$

```
[Out] -1/6*(-c*f+d*e)*x*(b*x^2+a)/e/f/(f*x^2+e)^3-1/24*(3*b*e*(c*f+d*e)-a*f*(5*c*
f+d*e))*x/e^2/f^2/(f*x^2+e)^2+1/16*(b*e*(c*f+d*e)+a*f*(5*c*f+d*e))*x/e^3/f^
2/(f*x^2+e)+1/16*(b*e*(c*f+d*e)+a*f*(5*c*f+d*e))*arctan(x*f^(1/2)/e^(1/2))/
e^(7/2)/f^(5/2)
```

Rubi [A]
time = 0.11, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {540, 393, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(5cf+de)+be(cf+de))}{16e^{7/2}f^{5/2}}+\frac{x(af(5cf+de)+be(cf+de))}{16e^3f^2(e+fx^2)}-\frac{x(3be(cf+de)-af(5cf+de))}{24e^2f^2(e+fx^2)^2}-\frac{x(a+bx^2)(de-cf)}{6ef(e+fx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^4,x]
```

```
[Out] -1/6*((d*e - c*f)*x*(a + b*x^2))/(e*f*(e + f*x^2)^3) - ((3*b*e*(d*e + c*f)
- a*f*(d*e + 5*c*f))*x)/(24*e^2*f^2*(e + f*x^2)^2) + ((b*e*(d*e + c*f) + a*
f*(d*e + 5*c*f))*x)/(16*e^3*f^2*(e + f*x^2)) + ((b*e*(d*e + c*f) + a*f*(d*e
 + 5*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(7/2)*f^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = -\frac{(de-cf)x(a+bx^2)}{6ef(e+fx^2)^3} - \frac{\int \frac{-a(de+5cf)-3b(de+cf)x^2}{(e+fx^2)^3}\,dx}{6ef}$$

$$= -\frac{(de-cf)x(a+bx^2)}{6ef(e+fx^2)^3} - \frac{(3be(de+cf)-af(de+5cf))x}{24e^2f^2(e+fx^2)^2} + \frac{(be(de+cf)+af(\ldots}{8\ldots}$$

$$= -\frac{(de-cf)x(a+bx^2)}{6ef(e+fx^2)^3} - \frac{(3be(de+cf)-af(de+5cf))x}{24e^2f^2(e+fx^2)^2} + \frac{(be(de+cf)+af(\ldots}{16e^3f^2(e+\ldots}$$

$$= -\frac{(de-cf)x(a+bx^2)}{6ef(e+fx^2)^3} - \frac{(3be(de+cf)-af(de+5cf))x}{24e^2f^2(e+fx^2)^2} + \frac{(be(de+cf)+af(\ldots}{16e^3f^2(e+\ldots}$$

Mathematica [A]
time = 0.07, size = 171, normalized size = 1.00

$$\frac{(be-af)(de-cf)x}{6ef^2(e+fx^2)^3} + \frac{(be(-7de+cf)+af(de+5cf))x}{24e^2f^2(e+fx^2)^2} + \frac{(be(de+cf)+af(de+5cf))x}{16e^3f^2(e+fx^2)} + \frac{(be(de+cf)+af(de+5cf))\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{16e^{7/2}f^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^4,x]
```

```
[Out] ((b*e - a*f)*(d*e - c*f)*x)/(6*e*f^2*(e + f*x^2)^3) + ((b*e*(-7*d*e + c*f)
+ a*f*(d*e + 5*c*f))*x)/(24*e^2*f^2*(e + f*x^2)^2) + ((b*e*(d*e + c*f) + a*
f*(d*e + 5*c*f))*x)/(16*e^3*f^2*(e + f*x^2)) + ((b*e*(d*e + c*f) + a*f*(d*e
 + 5*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(7/2)*f^(5/2))
```

Maple [A]
time = 0.14, size = 163, normalized size = 0.95

method	result
default	$\frac{\frac{\left(5ac\,f^{2}+adef+bcef+bd\,e^{2}\right)x^{5}}{16e^{3}}+\frac{\left(5ac\,f^{2}+adef+bcef-bd\,e^{2}\right)x^{3}}{6e^{2}f}+\frac{\left(11ac\,f^{2}-adef-bcef-bd\,e^{2}\right)x}{16e\,f^{2}}}{\left(f\,x^{2}+e\right)^{3}}+\frac{\left(5ac\,f^{2}+adef+bcef+bd\,e^{2}\right)\arctan\left(\right.}{16e^{3}f^{2}\sqrt{fe}}$
risch	$\frac{\frac{\left(5ac\,f^{2}+adef+bcef+bd\,e^{2}\right)x^{5}}{16e^{3}}+\frac{\left(5ac\,f^{2}+adef+bcef-bd\,e^{2}\right)x^{3}}{6e^{2}f}+\frac{\left(11ac\,f^{2}-adef-bcef-bd\,e^{2}\right)x}{16e\,f^{2}}}{\left(f\,x^{2}+e\right)^{3}}-\frac{5\ln\left(fx+\sqrt{-fe}\right)ac}{32\sqrt{-fe}\,e^{3}}-\frac{\ln\left(fx+\right.}{32\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(5*a*c*f^2+a*d*e*f+b*c*e*f+b*d*e^2)/e^3*x^5+1/6*(5*a*c*f^2+a*d*e*f+b*
c*e*f-b*d*e^2)/e^2/f*x^3+1/16*(11*a*c*f^2-a*d*e*f-b*c*e*f-b*d*e^2)/e/f^2*x)
/(f*x^2+e)^3+1/16*(5*a*c*f^2+a*d*e*f+b*c*e*f+b*d*e^2)/e^3/f^2/(f*e)^(1/2)*a
rctan(f*x/(f*e)^(1/2))
```

Maxima [A]
time = 0.52, size = 189, normalized size = 1.11

$$\frac{\left(5\,acf^{2}+bde^{2}+\left(bce+ade\right)f\right)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{7}{2}\right)}}{16\,f^{\frac{5}{2}}}+\frac{3\left(5\,acf^{4}+bdf^{2}e^{2}+\left(bce+ade\right)f^{3}\right)x^{5}+8\left(5\,acf^{3}e-bdfe^{3}+\left(bce^{2}+ade^{2}\right)f^{2}\right)x^{3}+3\left(11\,acf^{2}e^{2}-bde^{4}-\left(bce^{3}+ade^{3}\right)f\right)x}{48\left(f^{5}x^{6}e^{3}+3\,f^{4}x^{4}e^{4}+3\,f^{3}x^{2}e^{5}+f^{2}e^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="maxima")
```

```
[Out] 1/16*(5*a*c*f^2 + b*d*e^2 + (b*c*e + a*d*e)*f)*arctan(sqrt(f)*x*e^(-1/2))*e
^(-7/2)/f^(5/2) + 1/48*(3*(5*a*c*f^4 + b*d*f^2*e^2 + (b*c*e + a*d*e)*f^3)*x
^5 + 8*(5*a*c*f^3*e - b*d*f*e^3 + (b*c*e^2 + a*d*e^2)*f^2)*x^3 + 3*(11*a*c*
f^2*e^2 - b*d*e^4 - (b*c*e^3 + a*d*e^3)*f)*x)/(f^5*x^6*e^3 + 3*f^4*x^4*e^4
+ 3*f^3*x^2*e^5 + f^2*e^6)
```

Fricas [A]
time = 0.94, size = 647, normalized size = 3.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(30*a*c*f^5*x^5*e - 6*b*d*f*x*e^5 - 3*(5*a*c*f^5*x^6 + b*d*e^5 + (3*b
*d*f*x^2 + (b*c + a*d)*f)*e^4 + (3*b*d*f^2*x^4 + 3*(b*c + a*d)*f^2*x^2 + 5*
a*c*f^2)*e^3 + (b*d*f^3*x^6 + 3*(b*c + a*d)*f^3*x^4 + 15*a*c*f^3*x^2)*e^2 +
 ((b*c + a*d)*f^4*x^6 + 15*a*c*f^4*x^4)*e)*sqrt(-f*e)*log((f*x^2 - 2*sqrt(-
f*e)*x - e)/(f*x^2 + e)) - 2*(8*b*d*f^2*x^3 + 3*(b*c + a*d)*f^2*x)*e^4 + 2*
(3*b*d*f^3*x^5 + 8*(b*c + a*d)*f^3*x^3 + 33*a*c*f^3*x)*e^3 + 2*(3*(b*c + a*
```

```
d)*f^4*x^5 + 40*a*c*f^4*x^3)*e^2)/(f^6*x^6*e^4 + 3*f^5*x^4*e^5 + 3*f^4*x^2*
e^6 + f^3*e^7), 1/48*(15*a*c*f^5*x^5*e - 3*b*d*f*x*e^5 + 3*(5*a*c*f^5*x^6 +
 b*d*e^5 + (3*b*d*f*x^2 + (b*c + a*d)*f)*e^4 + (3*b*d*f^2*x^4 + 3*(b*c + a*
d)*f^2*x^2 + 5*a*c*f^2)*e^3 + (b*d*f^3*x^6 + 3*(b*c + a*d)*f^3*x^4 + 15*a*c
*f^3*x^2)*e^2 + ((b*c + a*d)*f^4*x^6 + 15*a*c*f^4*x^4)*e)*sqrt(f)*arctan(sq
rt(f)*x*e^(-1/2))*e^(1/2) - (8*b*d*f^2*x^3 + 3*(b*c + a*d)*f^2*x)*e^4 + (3*
b*d*f^3*x^5 + 8*(b*c + a*d)*f^3*x^3 + 33*a*c*f^3*x)*e^3 + (3*(b*c + a*d)*f^
4*x^5 + 40*a*c*f^4*x^3)*e^2)/(f^6*x^6*e^4 + 3*f^5*x^4*e^5 + 3*f^4*x^2*e^6 +
 f^3*e^7)]
```

Sympy [A]
time = 3.55, size = 313, normalized size = 1.83

$$-\frac{\sqrt{-\frac{1}{e^7f^5}}\cdot(5acf^2+adef+bcef+bde^2)\log\left(-e^4f^2\sqrt{-\frac{1}{e^7f^5}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{e^7f^5}}\cdot(5acf^2+adef+bcef+bde^2)\log\left(e^4f^2\sqrt{-\frac{1}{e^7f^5}}+x\right)}{32}+\frac{x^5\cdot(15acf^4+3adef^3+3bcef^3+3bde^2f^2)+x^3\cdot(40acef^3+8ade^2f^2+8bce^2f^2-8bde^3f)+x(33ace^2f^2-3ade^3f-3bce^3f-3bde^4)}{48e^6f^2+144e^5f^3x^2+144e^4f^4x^4+48e^3f^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**4,x)
```

```
[Out] -sqrt(-1/(e**7*f**5))*(5*a*c*f**2 + a*d*e*f + b*c*e*f + b*d*e**2)*log(-e**4
*f**2*sqrt(-1/(e**7*f**5)) + x)/32 + sqrt(-1/(e**7*f**5))*(5*a*c*f**2 + a*d
*e*f + b*c*e*f + b*d*e**2)*log(e**4*f**2*sqrt(-1/(e**7*f**5)) + x)/32 + (x*
*5*(15*a*c*f**4 + 3*a*d*e*f**3 + 3*b*c*e*f**3 + 3*b*d*e**2*f**2) + x**3*(40
*a*c*e*f**3 + 8*a*d*e**2*f**2 + 8*b*c*e**2*f**2 - 8*b*d*e**3*f) + x*(33*a*c
*e**2*f**2 - 3*a*d*e**3*f - 3*b*c*e**3*f - 3*b*d*e**4))/(48*e**6*f**2 + 144
*e**5*f**3*x**2 + 144*e**4*f**4*x**4 + 48*e**3*f**5*x**6)
```

Giac [A]
time = 0.69, size = 184, normalized size = 1.08

$$\frac{(5\,acf^2+bcfe+adfe+bde^2)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{7}{2}\right)}}{16\,f^{\frac{5}{2}}}+\frac{(15\,acf^4x^5+3\,bcf^3x^5e+3\,adf^3x^5e+3\,bdf^2x^5e^2+40\,acf^3x^3e+8\,bcf^2x^3e^2+8\,adf^2x^3e^2-8\,bdfx^3e^3+33\,acf^2xe^2-3\,bcfxe^3-3\,adfxe^3-3\,bdxe^4)e^{(-3)}}{48\,(fx^2+e)^3f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="giac")
```

```
[Out] 1/16*(5*a*c*f^2 + b*c*f*e + a*d*f*e + b*d*e^2)*arctan(sqrt(f)*x*e^(-1/2))*e
^(-7/2)/f^(5/2) + 1/48*(15*a*c*f^4*x^5 + 3*b*c*f^3*x^5*e + 3*a*d*f^3*x^5*e
+ 3*b*d*f^2*x^5*e^2 + 40*a*c*f^3*x^3*e + 8*b*c*f^2*x^3*e^2 + 8*a*d*f^2*x^3*
e^2 - 8*b*d*f*x^3*e^3 + 33*a*c*f^2*x*e^2 - 3*b*c*f*x*e^3 - 3*a*d*f*x*e^3 -
3*b*d*x*e^4)*e^(-3)/((f*x^2 + e)^3*f^2)
```

Mupad [B]
time = 0.18, size = 176, normalized size = 1.03

$$\frac{\frac{x^5\,\left(5\,a\,c\,f^2+b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f\right)}{16\,e^3}-\frac{x\,\left(b\,d\,e^2-11\,a\,c\,f^2+a\,d\,e\,f+b\,c\,e\,f\right)}{16\,e\,f^2}+\frac{x^3\,\left(5\,a\,c\,f^2-b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f\right)}{6\,e^2\,f}}{e^3+3\,e^2\,f\,x^2+3\,e\,f^2\,x^4+f^3\,x^6}+\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,\left(5\,a\,c\,f^2+b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f\right)}{16\,e^{7/2}\,f^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^4,x)
```

```
[Out] ((x^5*(5*a*c*f^2 + b*d*e^2 + a*d*e*f + b*c*e*f))/(16*e^3) - (x*(b*d*e^2 - 1
1*a*c*f^2 + a*d*e*f + b*c*e*f))/(16*e*f^2) + (x^3*(5*a*c*f^2 - b*d*e^2 + a*
d*e*f + b*c*e*f))/(6*e^2*f))/(e^3 + f^3*x^6 + 3*e^2*f*x^2 + 3*e*f^2*x^4) +
(atan((f^(1/2)*x)/e^(1/2))*(5*a*c*f^2 + b*d*e^2 + a*d*e*f + b*c*e*f))/(16*e
^(7/2)*f^(5/2))
```

3.9 $\int (a+bx^2)(c+dx^2)^2(e+fx^2)^3\,dx$

Optimal. Leaf size=226

$$ac^2e^3x+\frac{1}{3}ce^2(bce+2ade+3acf)x^3+\frac{1}{5}e\left(bce(2de+3cf)+a\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^5+\frac{1}{7}\left(af\left(3d^2e^2+6cdef\right.\right.$$

```
[Out] a*c^2*e^3*x+1/3*c*e^2*(3*a*c*f+2*a*d*e+b*c*e)*x^3+1/5*e*(b*c*e*(3*c*f+2*d*e
)+a*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x^5+1/7*(a*f*(c^2*f^2+6*c*d*e*f+3*d^2*e^
2)+b*e*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x^7+1/9*f*(a*d*f*(2*c*f+3*d*e)+b*(c^2
*f^2+6*c*d*e*f+3*d^2*e^2))*x^9+1/11*d*f^2*(a*d*f+2*b*c*f+3*b*d*e)*x^11+1/13
*b*d^2*f^3*x^13
```

Rubi [A]
time = 0.14, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {535}

$$\frac{1}{9}fx^9\left(adf(2cf+3de)+b\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{7}x^7\left(af\left(c^2f^2+6cdef+3d^2e^2\right)+be\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{5}ex^5\left(a\left(3c^2f^2+6cdef+d^2e^2\right)+bce(3cf+2de)\right)+\frac{1}{3}ce^2x^3(3acf+2ade+bce)+\frac{1}{11}df^2x^{11}(adf+2bcf+3bde)+ac^2e^3x+\frac{1}{13}bd^2f^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

```
[Out] a*c^2*e^3*x + (c*e^2*(b*c*e + 2*a*d*e + 3*a*c*f)*x^3)/3 + (e*(b*c*e*(2*d*e
+ 3*c*f) + a*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^5)/5 + ((a*f*(3*d^2*e^2 +
 6*c*d*e*f + c^2*f^2) + b*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 + (f*
(a*d*f*(3*d*e + 2*c*f) + b*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^9)/9 + (d*f
^2*(3*b*d*e + 2*b*c*f + a*d*f)*x^11)/11 + (b*d^2*f^3*x^13)/13
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx &= \int \left(ac^2e^3+ce^2(bce+2ade+3acf)x^2+e\left(bce(2de+3cf)+a\left(d^2e^2\right.\right.\right.\\ &= ac^2e^3x+\frac{1}{3}ce^2(bce+2ade+3acf)x^3+\frac{1}{5}e\left(bce(2de+3cf)+a\left(d^2e^2\right.\right.\end{aligned}$$

Mathematica [A]
time = 0.06, size = 226, normalized size = 1.00

$$ac^2e^3x+\frac{1}{3}ce^2(bce+2ade+3acf)x^3+\frac{1}{5}e\left(bce(2de+3cf)+a\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^5+\frac{1}{7}\left(af\left(3d^2e^2+6cdef+c^2f^2\right)+be\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^7+\frac{1}{9}f\left(adf(3de+2cf)+b\left(3d^2e^2+6cdef+c^2f^2\right)\right)x^9+\frac{1}{11}df^2(3bde+2bcf+adf)x^{11}+\frac{1}{13}bd^2f^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

```
[Out] a*c^2*e^3*x + (c*e^2*(b*c*e + 2*a*d*e + 3*a*c*f)*x^3)/3 + (e*(b*c*e*(2*d*e
+ 3*c*f) + a*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^5)/5 + ((a*f*(3*d^2*e^2 +
 6*c*d*e*f + c^2*f^2) + b*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 + (f*
(a*d*f*(3*d*e + 2*c*f) + b*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^9)/9 + (d*f
^2*(3*b*d*e + 2*b*c*f + a*d*f)*x^11)/11 + (b*d^2*f^3*x^13)/13
```

Maple [A]
time = 0.17, size = 237, normalized size = 1.05

method	result
default	$\frac{b\,d^2 f^3 x^{13}}{13}+\frac{\left(\left(a\,d^2+2bcd\right) f^3+3b\,d^2 e\,f^2\right) x^{11}}{11}+\frac{\left(\left(2acd+b\,c^2\right) f^3+3\left(a\,d^2+2bcd\right) e\,f^2+3b\,d^2 e^2 f\right) x^9}{9}+\frac{\left(c^2 a\,f^3+3\left(2acd+b\,c^2\right) e\,f^2\right.}{}$
norman	$\frac{b\,d^2 f^3 x^{13}}{13}+\left(\frac{1}{11}a\,d^2 f^3+\frac{2}{11}bcd\,f^3+\frac{3}{11}b\,d^2 e\,f^2\right) x^{11}+\left(\frac{2}{9}acd\,f^3+\frac{1}{3}a\,d^2 e\,f^2+\frac{1}{9}b\,c^2 f^3+\frac{2}{3}bcde\,f^2+\right.$
gosper	$\frac{1}{13}b\,d^2 f^3 x^{13}+\frac{1}{11}x^{11}a\,d^2 f^3+\frac{2}{11}x^{11}bcd\,f^3+\frac{3}{11}x^{11}b\,d^2 e\,f^2+\frac{2}{9}x^9 acd\,f^3+\frac{1}{3}x^9 a\,d^2 e\,f^2+\frac{1}{9}x^9 b\,c^2 f^3$
risch	$\frac{1}{13}b\,d^2 f^3 x^{13}+\frac{1}{11}x^{11}a\,d^2 f^3+\frac{2}{11}x^{11}bcd\,f^3+\frac{3}{11}x^{11}b\,d^2 e\,f^2+\frac{2}{9}x^9 acd\,f^3+\frac{1}{3}x^9 a\,d^2 e\,f^2+\frac{1}{9}x^9 b\,c^2 f^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b*d^2*f^3*x^13+1/11*((a*d^2+2*b*c*d)*f^3+3*b*d^2*e*f^2)*x^11+1/9*((2*a
*c*d+b*c^2)*f^3+3*(a*d^2+2*b*c*d)*e*f^2+3*b*d^2*e^2*f)*x^9+1/7*(c^2*a*f^3+3
*(2*a*c*d+b*c^2)*e*f^2+3*(a*d^2+2*b*c*d)*e^2*f+b*d^2*e^3)*x^7+1/5*(3*c^2*a*
e*f^2+3*(2*a*c*d+b*c^2)*e^2*f+(a*d^2+2*b*c*d)*e^3)*x^5+1/3*(3*c^2*a*e^2*f+(
2*a*c*d+b*c^2)*e^3)*x^3+a*c^2*e^3*x
```

Maxima [A]
time = 0.27, size = 240, normalized size = 1.06

$$\frac{1}{13}bd^2f^3x^{13}+\frac{1}{11}\left(3\,bd^2f^2e+\left(2\,bcd+ad^2\right)f^3\right)x^{11}+\frac{1}{9}\left(3\,bd^2fe^2+\left(bc^2+2\,acd\right)f^3+3\left(2\,bcde+ad^2e\right)f^2\right)x^9+\frac{1}{7}\left(ac^2f^3+bd^2e^3+3\left(bc^2e+2\,acde\right)f^2+3\left(2\,bcde^2+ad^2e^2\right)f\right)x^7+\frac{1}{5}\left(3\,ac^2f^2e+2\,bcde^3+ad^2e^3+3\left(bc^2e^2+2\,acde^2\right)f\right)x^5+ac^2xe^3+\frac{1}{3}\left(3\,ac^2fe^2+bc^2e^3+2\,acde^3\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^2*f^3*x^13 + 1/11*(3*b*d^2*f^2*e + (2*b*c*d + a*d^2)*f^3)*x^11 + 1
/9*(3*b*d^2*f*e^2 + (b*c^2 + 2*a*c*d)*f^3 + 3*(2*b*c*d*e + a*d^2*e)*f^2)*x^
9 + 1/7*(a*c^2*f^3 + b*d^2*e^3 + 3*(b*c^2*e + 2*a*c*d*e)*f^2 + 3*(2*b*c*d*e
```

```
^2 + a*d^2*e^2)*f)*x^7 + 1/5*(3*a*c^2*f^2*e + 2*b*c*d*e^3 + a*d^2*e^3 + 3*(
b*c^2*e^2 + 2*a*c*d*e^2)*f)*x^5 + a*c^2*x*e^3 + 1/3*(3*a*c^2*f*e^2 + b*c^2*
e^3 + 2*a*c*d*e^3)*x^3
```

Fricas [A]
time = 0.88, size = 242, normalized size = 1.07

$$\frac{1}{13}\,bd^2f^3x^{13}+\frac{1}{11}\left(2\,bcd+ad^2\right)f^3x^{11}+\frac{1}{7}\,ac^2f^3x^7+\frac{1}{9}\left(bc^2+2\,acd\right)f^3x^9+\frac{1}{105}\left(15\,bd^2x^7+21\left(2\,bcd+ad^2\right)x^5+105\,ac^2x+35\left(bc^2+2\,acd\right)x^3\right)e^3+\frac{1}{105}\left(35\,bd^2fx^9+45\left(2\,bcd+ad^2\right)fx^7+105\,ac^2fx^3+63\left(bc^2+2\,acd\right)fx^5\right)e^2+\frac{1}{1155}\left(315\,bd^2f^2x^{11}+385\left(2\,bcd+ad^2\right)f^2x^9+693\,ac^2f^2x^5+495\left(bc^2+2\,acd\right)f^2x^7\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] 1/13*b*d^2*f^3*x^13 + 1/11*(2*b*c*d + a*d^2)*f^3*x^11 + 1/7*a*c^2*f^3*x^7 +
 1/9*(b*c^2 + 2*a*c*d)*f^3*x^9 + 1/105*(15*b*d^2*x^7 + 21*(2*b*c*d + a*d^2)
*x^5 + 105*a*c^2*x + 35*(b*c^2 + 2*a*c*d)*x^3)*e^3 + 1/105*(35*b*d^2*f*x^9
+ 45*(2*b*c*d + a*d^2)*f*x^7 + 105*a*c^2*f*x^3 + 63*(b*c^2 + 2*a*c*d)*f*x^5
)*e^2 + 1/1155*(315*b*d^2*f^2*x^11 + 385*(2*b*c*d + a*d^2)*f^2*x^9 + 693*a*
c^2*f^2*x^5 + 495*(b*c^2 + 2*a*c*d)*f^2*x^7)*e
```

Sympy [A]
time = 0.03, size = 304, normalized size = 1.35

$$ac^2e^3x+\frac{bd^2f^3x^{13}}{13}+x^{11}\left(\frac{ad^2f^3}{11}+\frac{2bcdf^3}{11}+\frac{3bd^2ef^2}{11}\right)+x^9\cdot\left(\frac{2acdf^3}{9}+\frac{ad^2ef^2}{3}+\frac{bc^2f^3}{9}+\frac{2bcdef^2}{3}+\frac{bd^2e^2f}{3}\right)+x^7\left(\frac{ac^2f^3}{7}+\frac{6acdef^2}{7}+\frac{3ad^2e^2f}{7}+\frac{3bc^2ef^2}{7}+\frac{6bcde^2f}{7}+\frac{bd^2e^3}{7}\right)+x^5\cdot\left(\frac{3ac^2ef^2}{5}+\frac{6acde^2f}{5}+\frac{ad^2e^3}{5}+\frac{3bc^2e^2f}{5}+\frac{2bcde^3}{5}\right)+x^3\left(ac^2e^2f+\frac{2acde^3}{3}+\frac{bc^2e^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e)**3,x)
```

```
[Out] a*c**2*e**3*x + b*d**2*f**3*x**13/13 + x**11*(a*d**2*f**3/11 + 2*b*c*d*f**3
/11 + 3*b*d**2*e*f**2/11) + x**9*(2*a*c*d*f**3/9 + a*d**2*e*f**2/3 + b*c**2
*f**3/9 + 2*b*c*d*e*f**2/3 + b*d**2*e**2*f/3) + x**7*(a*c**2*f**3/7 + 6*a*c
*d*e*f**2/7 + 3*a*d**2*e**2*f/7 + 3*b*c**2*e*f**2/7 + 6*b*c*d*e**2*f/7 + b*
d**2*e**3/7) + x**5*(3*a*c**2*e*f**2/5 + 6*a*c*d*e**2*f/5 + a*d**2*e**3/5 +
 3*b*c**2*e**2*f/5 + 2*b*c*d*e**3/5) + x**3*(a*c**2*e**2*f + 2*a*c*d*e**3/3
 + b*c**2*e**3/3)
```

Giac [A]
time = 0.98, size = 283, normalized size = 1.25

$$\frac{1}{13}\,bd^2f^3x^{13}+\frac{2}{11}\,bcdf^3x^{11}+\frac{1}{11}\,ad^2f^3x^{11}+\frac{3}{11}\,bd^2f^2x^{11}e+\frac{1}{9}\,bc^2f^3x^9+\frac{2}{9}\,acdf^3x^9+\frac{2}{3}\,bcdf^2x^9e+\frac{1}{3}\,ad^2f^2x^9e+\frac{1}{3}\,bd^2fx^9e^2+\frac{1}{7}\,ac^2f^3x^7+\frac{3}{7}\,bc^2f^2x^7e+\frac{6}{7}\,acdf^2x^7e+\frac{6}{7}\,bcdfx^7e^2+\frac{3}{7}\,ad^2fx^7e^2+\frac{1}{7}\,bd^2x^7e^3+\frac{3}{5}\,ac^2f^2x^5e+\frac{3}{5}\,bc^2fx^5e^2+\frac{6}{5}\,acdfx^5e^2+\frac{2}{5}\,bcdx^5e^3+\frac{1}{5}\,ad^2x^5e^3+ac^2fx^3e^2+\frac{1}{3}\,bc^2x^3e^3+\frac{2}{3}\,acdx^3e^3+ac^2xe^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] 1/13*b*d^2*f^3*x^13 + 2/11*b*c*d*f^3*x^11 + 1/11*a*d^2*f^3*x^11 + 3/11*b*d^
2*f^2*x^11*e + 1/9*b*c^2*f^3*x^9 + 2/9*a*c*d*f^3*x^9 + 2/3*b*c*d*f^2*x^9*e
+ 1/3*a*d^2*f^2*x^9*e + 1/3*b*d^2*f*x^9*e^2 + 1/7*a*c^2*f^3*x^7 + 3/7*b*c^2
```

```
*f^2*x^7*e + 6/7*a*c*d*f^2*x^7*e + 6/7*b*c*d*f*x^7*e^2 + 3/7*a*d^2*f*x^7*e^
2 + 1/7*b*d^2*x^7*e^3 + 3/5*a*c^2*f^2*x^5*e + 3/5*b*c^2*f*x^5*e^2 + 6/5*a*c
*d*f*x^5*e^2 + 2/5*b*c*d*x^5*e^3 + 1/5*a*d^2*x^5*e^3 + a*c^2*f*x^3*e^2 + 1/
3*b*c^2*x^3*e^3 + 2/3*a*c*d*x^3*e^3 + a*c^2*x*e^3
```

Mupad [B]
time = 0.86, size = 233, normalized size = 1.03

$$x^5\left(\frac{3bc^2e^2f}{5}+\frac{3ac^2ef^2}{5}+\frac{2bcde^3}{5}+\frac{6acde^2f}{5}+\frac{ad^2e^3}{5}\right)+x^9\left(\frac{bc^2f^3}{9}+\frac{2bcdef^2}{3}+\frac{2acdf^3}{9}+\frac{bd^2e^2f}{3}+\frac{ad^2ef^2}{3}\right)+x^7\left(\frac{3bc^2ef^2}{7}+\frac{ac^2f^3}{7}+\frac{6bcde^2f}{7}+\frac{6acdef^2}{7}+\frac{bd^2e^3}{7}+\frac{3ad^2e^2f}{7}\right)+\frac{bd^2f^3x^{13}}{13}+\frac{ce^2x^3\left(3acf+2ade+bce\right)}{3}+\frac{df^2x^{11}\left(adf+2bcf+3bde\right)}{11}+ac^2e^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3,x)
```

```
[Out] x^5*((a*d^2*e^3)/5 + (2*b*c*d*e^3)/5 + (3*a*c^2*e*f^2)/5 + (3*b*c^2*e^2*f)/
5 + (6*a*c*d*e^2*f)/5) + x^9*((b*c^2*f^3)/9 + (2*a*c*d*f^3)/9 + (a*d^2*e*f^
2)/3 + (b*d^2*e^2*f)/3 + (2*b*c*d*e*f^2)/3) + x^7*((a*c^2*f^3)/7 + (b*d^2*e
^3)/7 + (3*a*d^2*e^2*f)/7 + (3*b*c^2*e*f^2)/7 + (6*a*c*d*e*f^2)/7 + (6*b*c*
d*e^2*f)/7) + (b*d^2*f^3*x^13)/13 + (c*e^2*x^3*(3*a*c*f + 2*a*d*e + b*c*e))
/3 + (d*f^2*x^11*(a*d*f + 2*b*c*f + 3*b*d*e))/11 + a*c^2*e^3*x
```

3.10 $\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^2\,dx$

Optimal. Leaf size=158

$$ac^2e^2x+\frac{1}{3}ce(bce+2a(de+cf))x^3+\frac{1}{5}\left(2bce(de+cf)+a\left(d^2e^2+4cdef+c^2f^2\right)\right)x^5+\frac{1}{7}\left(2adf(de+cf)+b\left(d^2e^2\right.\right.$$

```
[Out] a*c^2*e^2*x+1/3*c*e*(b*c*e+2*a*(c*f+d*e))*x^3+1/5*(2*b*c*e*(c*f+d*e)+a*(c^2
*f^2+4*c*d*e*f+d^2*e^2))*x^5+1/7*(2*a*d*f*(c*f+d*e)+b*(c^2*f^2+4*c*d*e*f+d^
2*e^2))*x^7+1/9*d*f*(a*d*f+2*b*(c*f+d*e))*x^9+1/11*b*d^2*f^2*x^11
```

Rubi [A]
time = 0.11, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {535}

$$\frac{1}{7}x^7\left(2adf(cf+de)+b\left(c^2f^2+4cdef+d^2e^2\right)\right)+\frac{1}{5}x^5\left(a\left(c^2f^2+4cdef+d^2e^2\right)+2bce(cf+de)\right)+\frac{1}{9}dfx^9(adf+2b(cf+de))+\frac{1}{3}cex^3(2a(cf+de)+bce)+ac^2e^2x+\frac{1}{11}bd^2f^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

```
[Out] a*c^2*e^2*x + (c*e*(b*c*e + 2*a*(d*e + c*f))*x^3)/3 + ((2*b*c*e*(d*e + c*f)
 + a*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + ((2*a*d*f*(d*e + c*f) + b*(d
^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^7)/7 + (d*f*(a*d*f + 2*b*(d*e + c*f))*x^9)
/9 + (b*d^2*f^2*x^11)/11
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\int\left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^2\,dx=\int\left(ac^2e^2+ce(bce+2a(de+cf))x^2+\left(2bce(de+cf)+a\left(d^2e^2+4\right.\right.\right.$$

$$=ac^2e^2x+\frac{1}{3}ce(bce+2a(de+cf))x^3+\frac{1}{5}\left(2bce(de+cf)+a\left(d^2e^2+\right.\right.$$

Mathematica [A]
time = 0.04, size = 158, normalized size = 1.00

$$ac^2e^2x+\frac{1}{3}ce(bce+2a(de+cf))x^3+\frac{1}{5}\left(2bce(de+cf)+a\left(d^2e^2+4cdef+c^2f^2\right)\right)x^5+\frac{1}{7}\left(2adf(de+cf)+b\left(d^2e^2+4cdef+c^2f^2\right)\right)x^7+\frac{1}{9}df(adf+2b(de+cf))x^9+\frac{1}{11}bd^2f^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

```
[Out] a*c^2*e^2*x + (c*e*(b*c*e + 2*a*(d*e + c*f))*x^3)/3 + ((2*b*c*e*(d*e + c*f)
 + a*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + ((2*a*d*f*(d*e + c*f) + b*(d
^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^7)/7 + (d*f*(a*d*f + 2*b*(d*e + c*f))*x^9)
/9 + (b*d^2*f^2*x^11)/11
```

Maple [A]
time = 0.16, size = 169, normalized size = 1.07

method	result
default	$\frac{b\,d^2 f^2 x^{11}}{11}+\frac{\left(\left(a\,d^2+2bcd\right)f^2+2b\,d^2 fe\right)x^9}{9}+\frac{\left(\left(2acd+b\,c^2\right)f^2+2\left(a\,d^2+2bcd\right)fe+b\,d^2 e^2\right)x^7}{7}+\frac{\left(c^2 a\,f^2+2\left(2acd+b\,c^2\right)fe+\left(a\,d^2+\right.\right.}{5}$
norman	$\frac{b\,d^2 f^2 x^{11}}{11}+\left(\frac{1}{9}a\,d^2 f^2+\frac{2}{9}bcd\,f^2+\frac{2}{9}b\,d^2 fe\right)x^9+\left(\frac{2}{7}acd\,f^2+\frac{2}{7}a\,d^2 ef+\frac{1}{7}b\,c^2 f^2+\frac{4}{7}bcdef+\frac{1}{7}b\,d^2 e^2\right.$
gosper	$\frac{1}{11}b\,d^2 f^2 x^{11}+\frac{1}{9}x^9 a\,d^2 f^2+\frac{2}{9}x^9 bcd\,f^2+\frac{2}{9}x^9 b\,d^2 fe+\frac{2}{7}x^7 acd\,f^2+\frac{2}{7}x^7 a\,d^2 ef+\frac{1}{7}x^7 b\,c^2 f^2+\frac{4}{7}x^7 bc$
risch	$\frac{1}{11}b\,d^2 f^2 x^{11}+\frac{1}{9}x^9 a\,d^2 f^2+\frac{2}{9}x^9 bcd\,f^2+\frac{2}{9}x^9 b\,d^2 fe+\frac{2}{7}x^7 acd\,f^2+\frac{2}{7}x^7 a\,d^2 ef+\frac{1}{7}x^7 b\,c^2 f^2+\frac{4}{7}x^7 bc$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b*d^2*f^2*x^11+1/9*((a*d^2+2*b*c*d)*f^2+2*b*d^2*f*e)*x^9+1/7*((2*a*c*d
+b*c^2)*f^2+2*(a*d^2+2*b*c*d)*f*e+b*d^2*e^2)*x^7+1/5*(c^2*a*f^2+2*(2*a*c*d+
b*c^2)*f*e+(a*d^2+2*b*c*d)*e^2)*x^5+1/3*(2*c^2*a*f*e+(2*a*c*d+b*c^2)*e^2)*x
^3+a*c^2*e^2*x
```

Maxima [A]
time = 0.36, size = 172, normalized size = 1.09

$$\frac{1}{11}\,bd^2f^2x^{11}+\frac{1}{9}\left(2\,bd^2fe+\left(2\,bcd+ad^2\right)f^2\right)x^9+\frac{1}{7}\left(bd^2e^2+\left(bc^2+2\,acd\right)f^2+2\left(2\,bcde+ad^2e\right)f\right)x^7+\frac{1}{5}\left(ac^2f^2+2\,bcde^2+ad^2e^2+2\left(bc^2e+2\,acde\right)f\right)x^5+ac^2xe^2+\frac{1}{3}\left(2\,ac^2fe+bc^2e^2+2\,acde^2\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b*d^2*f^2*x^11 + 1/9*(2*b*d^2*f*e + (2*b*c*d + a*d^2)*f^2)*x^9 + 1/7*(
b*d^2*e^2 + (b*c^2 + 2*a*c*d)*f^2 + 2*(2*b*c*d*e + a*d^2*e)*f)*x^7 + 1/5*(a
*c^2*f^2 + 2*b*c*d*e^2 + a*d^2*e^2 + 2*(b*c^2*e + 2*a*c*d*e)*f)*x^5 + a*c^2
*x*e^2 + 1/3*(2*a*c^2*f*e + b*c^2*e^2 + 2*a*c*d*e^2)*x^3
```

Fricas [A]
time = 1.19, size = 175, normalized size = 1.11

$$\frac{1}{11}\,bd^2f^2x^{11}+\frac{1}{9}\left(2\,bcd+ad^2\right)f^2x^9+\frac{1}{5}\,ac^2f^2x^5+\frac{1}{7}\left(bc^2+2\,acd\right)f^2x^7+\frac{1}{105}\left(15\,bd^2x^7+21\left(2\,bcd+ad^2\right)x^5+105\,ac^2x+35\left(bc^2+2\,acd\right)x^3\right)e^2+\frac{2}{315}\left(35\,bd^2fx^9+45\left(2\,bcd+ad^2\right)fx^7+105\,ac^2fx^3+63\left(bc^2+2\,acd\right)fx^5\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*b*d^2*f^2*x^11 + 1/9*(2*b*c*d + a*d^2)*f^2*x^9 + 1/5*a*c^2*f^2*x^5 + 1
/7*(b*c^2 + 2*a*c*d)*f^2*x^7 + 1/105*(15*b*d^2*x^7 + 21*(2*b*c*d + a*d^2)*x
^5 + 105*a*c^2*x + 35*(b*c^2 + 2*a*c*d)*x^3)*e^2 + 2/315*(35*b*d^2*f*x^9 +
45*(2*b*c*d + a*d^2)*f*x^7 + 105*a*c^2*f*x^3 + 63*(b*c^2 + 2*a*c*d)*f*x^5)*
e
```

Sympy [A]
time = 0.02, size = 216, normalized size = 1.37

$$ac^2e^2x + \frac{bd^2f^2x^{11}}{11} + x^9\left(\frac{ad^2f^2}{9} + \frac{2bcdf^2}{9} + \frac{2bd^2ef}{9}\right) + x^7\cdot\left(\frac{2acdf^2}{7} + \frac{2ad^2ef}{7} + \frac{bc^2f^2}{7} + \frac{4bcdef}{7} + \frac{bd^2e^2}{7}\right) + x^5\left(\frac{ac^2f^2}{5} + \frac{4acdef}{5} + \frac{ad^2e^2}{5} + \frac{2bc^2ef}{5} + \frac{2bcde^2}{5}\right) + x^3\cdot\left(\frac{2ac^2ef}{3} + \frac{2acde^2}{3} + \frac{bc^2e^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e)**2,x)
```

```
[Out] a*c**2*e**2*x + b*d**2*f**2*x**11/11 + x**9*(a*d**2*f**2/9 + 2*b*c*d*f**2/9
 + 2*b*d**2*e*f/9) + x**7*(2*a*c*d*f**2/7 + 2*a*d**2*e*f/7 + b*c**2*f**2/7
+ 4*b*c*d*e*f/7 + b*d**2*e**2/7) + x**5*(a*c**2*f**2/5 + 4*a*c*d*e*f/5 + a*
d**2*e**2/5 + 2*b*c**2*e*f/5 + 2*b*c*d*e**2/5) + x**3*(2*a*c**2*e*f/3 + 2*a
*c*d*e**2/3 + b*c**2*e**2/3)
```

Giac [A]
time = 1.16, size = 202, normalized size = 1.28

$$\frac{1}{11}bd^2f^2x^{11} + \frac{2}{9}bcdf^2x^9 + \frac{1}{9}ad^2f^2x^9 + \frac{2}{9}bd^2fx^9e + \frac{1}{7}bc^2f^2x^7 + \frac{2}{7}acdf^2x^7 + \frac{4}{7}bcdfx^7e + \frac{2}{7}ad^2fx^7e + \frac{1}{7}bd^2x^7e^2 + \frac{1}{5}ac^2f^2x^5 + \frac{2}{5}bc^2fx^5e + \frac{4}{5}acdfx^5e + \frac{2}{5}bcdx^5e^2 + \frac{1}{5}ad^2x^5e^2 + \frac{2}{3}ac^2fx^3e + \frac{1}{3}bc^2x^3e^2 + \frac{2}{3}acdx^3e^2 + ac^2xe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] 1/11*b*d^2*f^2*x^11 + 2/9*b*c*d*f^2*x^9 + 1/9*a*d^2*f^2*x^9 + 2/9*b*d^2*f*x
^9*e + 1/7*b*c^2*f^2*x^7 + 2/7*a*c*d*f^2*x^7 + 4/7*b*c*d*f*x^7*e + 2/7*a*d^
2*f*x^7*e + 1/7*b*d^2*x^7*e^2 + 1/5*a*c^2*f^2*x^5 + 2/5*b*c^2*f*x^5*e + 4/5
*a*c*d*f*x^5*e + 2/5*b*c*d*x^5*e^2 + 1/5*a*d^2*x^5*e^2 + 2/3*a*c^2*f*x^3*e
+ 1/3*b*c^2*x^3*e^2 + 2/3*a*c*d*x^3*e^2 + a*c^2*x*e^2
```

Mupad [B]
time = 0.07, size = 158, normalized size = 1.00

$$x^5\left(\frac{2\,b\,c^2\,e\,f}{5} + \frac{a\,c^2\,f^2}{5} + \frac{2\,b\,c\,d\,e^2}{5} + \frac{4\,a\,c\,d\,e\,f}{5} + \frac{a\,d^2\,e^2}{5}\right) + x^7\left(\frac{b\,c^2\,f^2}{7} + \frac{4\,b\,c\,d\,e\,f}{7} + \frac{2\,a\,c\,d\,f^2}{7} + \frac{b\,d^2\,e^2}{7} + \frac{2\,a\,d^2\,e\,f}{7}\right) + \frac{b\,d^2\,f^2\,x^{11}}{11} + a\,c^2\,e^2\,x + \frac{c\,e\,x^3\,\left(2\,a\,c\,f + 2\,a\,d\,e + b\,c\,e\right)}{3} + \frac{d\,f\,x^9\,\left(a\,d\,f + 2\,b\,c\,f + 2\,b\,d\,e\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2,x)
```

```
[Out] x^5*((a*c^2*f^2)/5 + (a*d^2*e^2)/5 + (2*b*c*d*e^2)/5 + (2*b*c^2*e*f)/5 + (4
*a*c*d*e*f)/5) + x^7*((b*c^2*f^2)/7 + (b*d^2*e^2)/7 + (2*a*c*d*f^2)/7 + (2*
a*d^2*e*f)/7 + (4*b*c*d*e*f)/7) + (b*d^2*f^2*x^11)/11 + a*c^2*e^2*x + (c*e*
x^3*(2*a*c*f + 2*a*d*e + b*c*e))/3 + (d*f*x^9*(a*d*f + 2*b*c*f + 2*b*d*e))/
9
```

3.11 $\int (a+bx^2)(c+dx^2)^2(e+fx^2)\,dx$

Optimal. Leaf size=94

$$ac^2ex+\frac{1}{3}c(bce+2ade+acf)x^3+\frac{1}{5}(bc(2de+cf)+ad(de+2cf))x^5+\frac{1}{7}d(bde+2bcf+adf)x^7+\frac{1}{9}bd^2fx^9$$

```
[Out] a*c^2*e*x+1/3*c*(a*c*f+2*a*d*e+b*c*e)*x^3+1/5*(b*c*(c*f+2*d*e)+a*d*(2*c*f+d
*e))*x^5+1/7*d*(a*d*f+2*b*c*f+b*d*e)*x^7+1/9*b*d^2*f*x^9
```

Rubi [A]
time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {535}

$$\frac{1}{7}dx^7(adf+2bcf+bde)+\frac{1}{5}x^5(ad(2cf+de)+bc(cf+2de))+\frac{1}{3}cx^3(acf+2ade+bce)+ac^2ex+\frac{1}{9}bd^2fx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2),x]
```

```
[Out] a*c^2*e*x + (c*(b*c*e + 2*a*d*e + a*c*f)*x^3)/3 + ((b*c*(2*d*e + c*f) + a*d
*(d*e + 2*c*f))*x^5)/5 + (d*(b*d*e + 2*b*c*f + a*d*f)*x^7)/7 + (b*d^2*f*x^9
)/9
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)\,dx = \int (ac^2e+c(bce+2ade+acf)x^2+(bc(2de+cf)+ad(de+2cf))$$
$$= ac^2ex+\frac{1}{3}c(bce+2ade+acf)x^3+\frac{1}{5}(bc(2de+cf)+ad(de+2cf)$$

Mathematica [A]
time = 0.02, size = 96, normalized size = 1.02

$$ac^2ex+\frac{1}{3}c(bce+2ade+acf)x^3+\frac{1}{5}\left(2bcde+ad^2e+bc^2f+2acdf\right)x^5+\frac{1}{7}d(bde+2bcf+adf)x^7+\frac{1}{9}bd^2fx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2),x]
```

```
[Out] a*c^2*e*x + (c*(b*c*e + 2*a*d*e + a*c*f)*x^3)/3 + ((2*b*c*d*e + a*d^2*e + b
*c^2*f + 2*a*c*d*f)*x^5)/5 + (d*(b*d*e + 2*b*c*f + a*d*f)*x^7)/7 + (b*d^2*f
*x^9)/9
```

Maple [A]
time = 0.15, size = 101, normalized size = 1.07

method	result
norman	$\frac{b\,d^2 f\,x^9}{9}+\left(\frac{1}{7}a\,d^2 f+\frac{2}{7}bcdf+\frac{1}{7}b\,d^2 e\right)x^7+\left(\frac{2}{5}acdf+\frac{1}{5}a\,d^2 e+\frac{1}{5}b\,c^2 f+\frac{2}{5}bcde\right)x^5+\left(\frac{1}{3}c^2 a f+\frac{2}{3}acde\right.$
default	$\frac{b\,d^2 f\,x^9}{9}+\frac{\left(\left(a\,d^2+2bcd\right)f+b\,d^2 e\right)x^7}{7}+\frac{\left(\left(2acd+b\,c^2\right)f+\left(a\,d^2+2bcd\right)e\right)x^5}{5}+\frac{\left(c^2 a f+\left(2acd+b\,c^2\right)e\right)x^3}{3}+a\,c^2 e x$
gosper	$\frac{1}{9}b\,d^2 f\,x^9+\frac{1}{7}x^7 a\,d^2 f+\frac{2}{7}x^7 bcdf+\frac{1}{7}x^7 b\,d^2 e+\frac{2}{5}x^5 acdf+\frac{1}{5}x^5 a\,d^2 e+\frac{1}{5}x^5 b\,c^2 f+\frac{2}{5}x^5 bcde+\frac{1}{3}x^3 c^2 a$
risch	$\frac{1}{9}b\,d^2 f\,x^9+\frac{1}{7}x^7 a\,d^2 f+\frac{2}{7}x^7 bcdf+\frac{1}{7}x^7 b\,d^2 e+\frac{2}{5}x^5 acdf+\frac{1}{5}x^5 a\,d^2 e+\frac{1}{5}x^5 b\,c^2 f+\frac{2}{5}x^5 bcde+\frac{1}{3}x^3 c^2 a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b*d^2*f*x^9+1/7*((a*d^2+2*b*c*d)*f+b*d^2*e)*x^7+1/5*((2*a*c*d+b*c^2)*f+
(a*d^2+2*b*c*d)*e)*x^5+1/3*(c^2*a*f+(2*a*c*d+b*c^2)*e)*x^3+a*c^2*e*x
```

Maxima [A]
time = 0.27, size = 104, normalized size = 1.11

$$\frac{1}{9}\,bd^2fx^9+\frac{1}{7}\left(bd^2e+\left(2\,bcd+ad^2\right)f\right)x^7+\frac{1}{5}\left(2\,bcde+ad^2e+\left(bc^2+2\,acd\right)f\right)x^5+ac^2xe+\frac{1}{3}\left(ac^2f+bc^2e+2\,acde\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x, algorithm="maxima")
```

```
[Out] 1/9*b*d^2*f*x^9 + 1/7*(b*d^2*e + (2*b*c*d + a*d^2)*f)*x^7 + 1/5*(2*b*c*d*e
+ a*d^2*e + (b*c^2 + 2*a*c*d)*f)*x^5 + a*c^2*x*e + 1/3*(a*c^2*f + b*c^2*e +
 2*a*c*d*e)*x^3
```

Fricas [A]
time = 1.42, size = 108, normalized size = 1.15

$$\frac{1}{9}\,bd^2fx^9+\frac{1}{7}\left(2\,bcd+ad^2\right)fx^7+\frac{1}{3}\,ac^2fx^3+\frac{1}{5}\left(bc^2+2\,acd\right)fx^5+\frac{1}{105}\left(15\,bd^2x^7+21\left(2\,bcd+ad^2\right)x^5+105\,ac^2x+35\left(bc^2+2\,acd\right)x^3\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x, algorithm="fricas")
```

```
[Out] 1/9*b*d^2*f*x^9 + 1/7*(2*b*c*d + a*d^2)*f*x^7 + 1/3*a*c^2*f*x^3 + 1/5*(b*c^
2 + 2*a*c*d)*f*x^5 + 1/105*(15*b*d^2*x^7 + 21*(2*b*c*d + a*d^2)*x^5 + 105*a
*c^2*x + 35*(b*c^2 + 2*a*c*d)*x^3)*e
```

Sympy [A]
time = 0.01, size = 121, normalized size = 1.29

$$ac^2ex + \frac{bd^2fx^9}{9} + x^7\left(\frac{ad^2f}{7} + \frac{2bcdf}{7} + \frac{bd^2e}{7}\right) + x^5\cdot\left(\frac{2acdf}{5} + \frac{ad^2e}{5} + \frac{bc^2f}{5} + \frac{2bcde}{5}\right) + x^3\left(\frac{ac^2f}{3} + \frac{2acde}{3} + \frac{bc^2e}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e),x)
```

```
[Out] a*c**2*e*x + b*d**2*f*x**9/9 + x**7*(a*d**2*f/7 + 2*b*c*d*f/7 + b*d**2*e/7)
 + x**5*(2*a*c*d*f/5 + a*d**2*e/5 + b*c**2*f/5 + 2*b*c*d*e/5) + x**3*(a*c**
2*f/3 + 2*a*c*d*e/3 + b*c**2*e/3)
```

Giac [A]
time = 1.24, size = 120, normalized size = 1.28

$$\frac{1}{9}bd^2fx^9 + \frac{2}{7}bcdfx^7 + \frac{1}{7}ad^2fx^7 + \frac{1}{7}bd^2x^7e + \frac{1}{5}bc^2fx^5 + \frac{2}{5}acdfx^5 + \frac{2}{5}bcdx^5e + \frac{1}{5}ad^2x^5e + \frac{1}{3}ac^2fx^3 + \frac{1}{3}bc^2x^3e + \frac{2}{3}acdx^3e + ac^2xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x, algorithm="giac")
```

```
[Out] 1/9*b*d^2*f*x^9 + 2/7*b*c*d*f*x^7 + 1/7*a*d^2*f*x^7 + 1/7*b*d^2*x^7*e + 1/5
*b*c^2*f*x^5 + 2/5*a*c*d*f*x^5 + 2/5*b*c*d*x^5*e + 1/5*a*d^2*x^5*e + 1/3*a*
c^2*f*x^3 + 1/3*b*c^2*x^3*e + 2/3*a*c*d*x^3*e + a*c^2*x*e
```

Mupad [B]
time = 0.05, size = 99, normalized size = 1.05

$$x^5\left(\frac{a\,d^2\,e}{5} + \frac{b\,c^2\,f}{5} + \frac{2\,a\,c\,d\,f}{5} + \frac{2\,b\,c\,d\,e}{5}\right) + x^3\left(\frac{a\,c^2\,f}{3} + \frac{b\,c^2\,e}{3} + \frac{2\,a\,c\,d\,e}{3}\right) + x^7\left(\frac{a\,d^2\,f}{7} + \frac{b\,d^2\,e}{7} + \frac{2\,b\,c\,d\,f}{7}\right) + a\,c^2\,e\,x + \frac{b\,d^2\,f\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2),x)
```

```
[Out] x^5*((a*d^2*e)/5 + (b*c^2*f)/5 + (2*a*c*d*f)/5 + (2*b*c*d*e)/5) + x^3*((a*c
^2*f)/3 + (b*c^2*e)/3 + (2*a*c*d*e)/3) + x^7*((a*d^2*f)/7 + (b*d^2*e)/7 + (
2*b*c*d*f)/7) + a*c^2*e*x + (b*d^2*f*x^9)/9
```

3.12 $\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx$

Optimal. Leaf size=142

$$-\frac{(5adf(3de-5cf)-b(15d^2e^2-25cdef+8c^2f^2))\,x}{15f^3}-\frac{(5bde-4bcf-5adf)x(c+dx^2)}{15f^2}+\frac{bx(c+dx^2)^2}{5f}-\frac{(be\cdots}{\cdots}$$

```
[Out] -1/15*(5*a*d*f*(-5*c*f+3*d*e)-b*(8*c^2*f^2-25*c*d*e*f+15*d^2*e^2))*x/f^3-1/
15*(-5*a*d*f-4*b*c*f+5*b*d*e)*x*(d*x^2+c)/f^2+1/5*b*x*(d*x^2+c)^2/f-(-a*f+b
*e)*(-c*f+d*e)^2*arctan(x*f^(1/2)/e^(1/2))/f^(7/2)/e^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {542, 396, 211}

$$-\frac{(be-af)\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(de-cf)^2}{\sqrt{e}\,f^{7/2}}-\frac{x(5adf(3de-5cf)-b(8c^2f^2-25cdef+15d^2e^2))}{15f^3}-\frac{x(c+dx^2)(-5adf-4bcf+5bde)}{15f^2}+\frac{bx(c+dx^2)^2}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2),x]
```

```
[Out] -1/15*((5*a*d*f*(3*d*e - 5*c*f) - b*(15*d^2*e^2 - 25*c*d*e*f + 8*c^2*f^2))*
x)/f^3 - ((5*b*d*e - 4*b*c*f - 5*a*d*f)*x*(c + d*x^2))/(15*f^2) + (b*x*(c +
 d*x^2)^2)/(5*f) - ((b*e - a*f)*(d*e - c*f)^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(Sqrt[e]*f^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
```

```
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx = \frac{bx(c+dx^2)^2}{5f} + \frac{\int \frac{(c+dx^2)(-c(be-5af)+(-5bde+4bcf+5adf)x^2)}{e+fx^2}\,dx}{5f}$$

$$= -\frac{(5bde-4bcf-5adf)x(c+dx^2)}{15f^2} + \frac{bx(c+dx^2)^2}{5f} + \frac{\int \frac{c(be(5de-7cf)-5af(de-3cf))}{}}{}$$

$$= -\frac{(5adf(3de-5cf)-b(15d^2e^2-25cdef+8c^2f^2))\,x}{15f^3} - \frac{(5bde-4bcf-5adf)}{15f^2}$$

$$= -\frac{(5adf(3de-5cf)-b(15d^2e^2-25cdef+8c^2f^2))\,x}{15f^3} - \frac{(5bde-4bcf-5adf)}{15f^2}$$

Mathematica [A]
time = 0.05, size = 115, normalized size = 0.81

$$\frac{(b(de-cf)^2+adf(-de+2cf))\,x}{f^3} + \frac{d(-bde+2bcf+adf)x^3}{3f^2} + \frac{bd^2x^5}{5f} - \frac{(be-af)(de-cf)^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}\,f^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2),x]
[Out] ((b*(d*e - c*f)^2 + a*d*f*(-(d*e) + 2*c*f))*x)/f^3 + (d*(-(b*d*e) + 2*b*c*f
 + a*d*f)*x^3)/(3*f^2) + (b*d^2*x^5)/(5*f) - ((b*e - a*f)*(d*e - c*f)^2*Arc
Tan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(7/2))
```

Maple [A]
time = 0.14, size = 170, normalized size = 1.20

method	result
default	$\frac{\frac{1}{5}bd^2x^5f^2+\frac{1}{3}ad^2f^2x^3+\frac{2}{3}bcdf^2x^3-\frac{1}{3}bd^2efx^3+2acdf^2x-ad^2efx+bc^2f^2x-2bcdefx+bd^2e^2x}{f^3} + \frac{(c^2af^3-2acdef^2+ad^2e^2f-b}{}$
risch	$\frac{bd^2x^5}{5f}+\frac{ad^2x^3}{3f}+\frac{2bcdx^3}{3f}-\frac{bd^2ex^3}{3f^2}+\frac{2acdx}{f}-\frac{ad^2ex}{f^2}+\frac{bc^2x}{f}-\frac{2bcdex}{f^2}+\frac{bd^2e^2x}{f^3}-\frac{\ln\left(fx+\sqrt{-fe}\right)c^2a}{2\sqrt{-fe}}+\frac{\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^3*(1/5*b*d^2*x^5*f^2+1/3*a*d^2*f^2*x^3+2/3*b*c*d*f^2*x^3-1/3*b*d^2*e*f*
x^3+2*a*c*d*f^2*x-a*d^2*e*f*x+b*c^2*f^2*x-2*b*c*d*e*f*x+b*d^2*e^2*x)+(a*c^2
*f^3-2*a*c*d*e*f^2+a*d^2*e^2*f-b*c^2*e*f^2+2*b*c*d*e^2*f-b*d^2*e^3)/f^3/(f*
e)^(1/2)*arctan(f*x/(f*e)^(1/2))
```

Maxima [A]
time = 0.51, size = 161, normalized size = 1.13

$$\frac{(ac^2f^3 - bd^2e^3 - (bc^2e + 2\,acde)f^2 + (2\,bcde^2 + ad^2e^2)f)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}\right)}}{f^{\frac{7}{2}}} + \frac{3\,bd^2f^2x^5 - 5\,(bd^2fe - (2\,bcd + ad^2)f^2)x^3 + 15\,(bd^2e^2 + (bc^2 + 2\,acd)f^2 - (2\,bcde + ad^2e)f)x}{15\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="maxima")
```

```
[Out] (a*c^2*f^3 - b*d^2*e^3 - (b*c^2*e + 2*a*c*d*e)*f^2 + (2*b*c*d*e^2 + a*d^2*e
^2)*f)*arctan(sqrt(f)*x*e^(-1/2))*e^(-1/2)/f^(7/2) + 1/15*(3*b*d^2*f^2*x^5
- 5*(b*d^2*f*e - (2*b*c*d + a*d^2)*f^2)*x^3 + 15*(b*d^2*e^2 + (b*c^2 + 2*a*
c*d)*f^2 - (2*b*c*d*e + a*d^2*e)*f)*x)/f^3
```

Fricas [A]
time = 0.78, size = 364, normalized size = 2.56

$$\left[\frac{\left(30\,bd^2fxe^3 + 15\,(ac^2f^3 - bd^2e^3 - (bc^2 + 2\,acd)f^2e + (2\,bcd + ad^2)fe^2)\sqrt{-fe}\log\left(\frac{fx^2 + 2\,\sqrt{-fe}\,x - e}{fx^2 + e}\right) - 10\,(bd^2f^2x^3 + 3\,(2\,bcd + ad^2)f^2x)e^2 + 2\,(3\,bd^2f^3x^5 + 5\,(2\,bcd + ad^2)f^3x^3 + 15\,(bc^2 + 2\,acd)f^3x)e\right)e^{(-1)}}{30\,f^4}, \frac{\left(15\,bd^2fxe^3 + 15\,(ac^2f^3 - bd^2e^3 - (bc^2 + 2\,acd)f^2e + (2\,bcd + ad^2)fe^2)\sqrt{f}\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\frac{1}{2}} - 5\,(bd^2f^2x^3 + 3\,(2\,bcd + ad^2)f^2x)e^2 + (3\,bd^2f^3x^5 + 5\,(2\,bcd + ad^2)f^3x^3 + 15\,(bc^2 + 2\,acd)f^3x)e\right)e^{(-1)}}{15\,f^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="fricas")
```

```
[Out] [1/30*(30*b*d^2*f*x*e^3 + 15*(a*c^2*f^3 - b*d^2*e^3 - (b*c^2 + 2*a*c*d)*f^2
*e + (2*b*c*d + a*d^2)*f*e^2)*sqrt(-f*e)*log((f*x^2 + 2*sqrt(-f*e)*x - e)/(
f*x^2 + e)) - 10*(b*d^2*f^2*x^3 + 3*(2*b*c*d + a*d^2)*f^2*x)*e^2 + 2*(3*b*d
^2*f^3*x^5 + 5*(2*b*c*d + a*d^2)*f^3*x^3 + 15*(b*c^2 + 2*a*c*d)*f^3*x)*e)*e
^(-1)/f^4, 1/15*(15*b*d^2*f*x*e^3 + 15*(a*c^2*f^3 - b*d^2*e^3 - (b*c^2 + 2*
a*c*d)*f^2*e + (2*b*c*d + a*d^2)*f*e^2)*sqrt(f)*arctan(sqrt(f)*x*e^(-1/2))*
e^(1/2) - 5*(b*d^2*f^2*x^3 + 3*(2*b*c*d + a*d^2)*f^2*x)*e^2 + (3*b*d^2*f^3*
x^5 + 5*(2*b*c*d + a*d^2)*f^3*x^3 + 15*(b*c^2 + 2*a*c*d)*f^3*x)*e)*e^(-1)/f
^4]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(138) = 276$.
time = 0.58, size = 347, normalized size = 2.44

$$\frac{bd^2x^5}{5f} + x^3\left(\frac{ad^2}{3f} + \frac{2bcd}{3f} - \frac{bd^2e}{3f^2}\right) + x\left(\frac{2acd}{f} - \frac{ad^2e}{f^2} + \frac{bc^2}{f} - \frac{2bcde}{f^2} + \frac{bd^2e^2}{f^3}\right) - \frac{\sqrt{-\frac{1}{ef^7}}\,(af - be)\,(cf - de)^2\log\left(-\frac{ef^3\sqrt{-\frac{1}{ef^7}}(af-be)(cf-de)^2}{ac^2f^3-2acdef^2+ad^2e^2f-bc^2ef^2+2bcde^2f-bd^2e^3} + x\right)}{2} + \frac{\sqrt{-\frac{1}{ef^7}}\,(af - be)\,(cf - de)^2\log\left(\frac{ef^3\sqrt{-\frac{1}{ef^7}}(af-be)(cf-de)^2}{ac^2f^3-2acdef^2+ad^2e^2f-bc^2ef^2+2bcde^2f-bd^2e^3} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e),x)
```

```
[Out] b*d**2*x**5/(5*f) + x**3*(a*d**2/(3*f) + 2*b*c*d/(3*f) - b*d**2*e/(3*f**2))
 + x*(2*a*c*d/f - a*d**2*e/f**2 + b*c**2/f - 2*b*c*d*e/f**2 + b*d**2*e**2/f
**3) - sqrt(-1/(e*f**7))*(a*f - b*e)*(c*f - d*e)**2*log(-e*f**3*sqrt(-1/(e*
f**7))*(a*f - b*e)*(c*f - d*e)**2/(a*c**2*f**3 - 2*a*c*d*e*f**2 + a*d**2*e*
*2*f - b*c**2*e*f**2 + 2*b*c*d*e**2*f - b*d**2*e**3) + x)/2 + sqrt(-1/(e*f*
*7))*(a*f - b*e)*(c*f - d*e)**2*log(e*f**3*sqrt(-1/(e*f**7))*(a*f - b*e)*(c
*f - d*e)**2/(a*c**2*f**3 - 2*a*c*d*e*f**2 + a*d**2*e**2*f - b*c**2*e*f**2
+ 2*b*c*d*e**2*f - b*d**2*e**3) + x)/2
```

Giac [**A**]
time = 0.64, size = 178, normalized size = 1.25

$$\frac{(ac^2f^3 - bc^2f^2e - 2\,acdf^2e + 2\,bcdfe^2 + ad^2fe^2 - bd^2e^3)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}\right)}}{f^{\frac{7}{2}}} + \frac{3\,bd^2f^4x^5 + 10\,bcdf^4x^3 + 5\,ad^2f^4x^3 - 5\,bd^2f^3x^3e + 15\,bc^2f^4x + 30\,acdf^4x - 30\,bcdf^3xe - 15\,ad^2f^3xe + 15\,bd^2f^2xe^2}{15\,f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="giac")
```

```
[Out] (a*c^2*f^3 - b*c^2*f^2*e - 2*a*c*d*f^2*e + 2*b*c*d*f*e^2 + a*d^2*f*e^2 - b*
d^2*e^3)*arctan(sqrt(f)*x*e^(-1/2))*e^(-1/2)/f^(7/2) + 1/15*(3*b*d^2*f^4*x^
5 + 10*b*c*d*f^4*x^3 + 5*a*d^2*f^4*x^3 - 5*b*d^2*f^3*x^3*e + 15*b*c^2*f^4*x
 + 30*a*c*d*f^4*x - 30*b*c*d*f^3*x*e - 15*a*d^2*f^3*x*e + 15*b*d^2*f^2*x*e^
2)/f^5
```

Mupad [**B**]
time = 0.86, size = 203, normalized size = 1.43

$$x^3\left(\frac{a\,d^2+2\,b\,c\,d}{3\,f}-\frac{b\,d^2\,e}{3\,f^2}\right)+x\left(\frac{b\,c^2+2\,a\,d\,c}{f}-\frac{e\,\left(\frac{a\,d^2+2\,b\,c\,d}{f}-\frac{b\,d^2\,e}{f^2}\right)}{f}\right)+\frac{b\,d^2\,x^5}{5\,f}+\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,\left(a\,f-b\,e\right)\,{\left(c\,f-d\,e\right)}^2}{\sqrt{e}\,\left(-b\,c^2\,e\,f^2+a\,c^2\,f^3+2\,b\,c\,d\,e^2\,f-2\,a\,c\,d\,e\,f^2-b\,d^2\,e^3+a\,d^2\,e^2\,f\right)}\right)\,\left(a\,f-b\,e\right)\,{\left(c\,f-d\,e\right)}^2}{\sqrt{e}\,f^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2),x)
```

```
[Out] x^3*((a*d^2 + 2*b*c*d)/(3*f) - (b*d^2*e)/(3*f^2)) + x*((b*c^2 + 2*a*c*d)/f
- (e*((a*d^2 + 2*b*c*d)/f - (b*d^2*e)/f^2))/f) + (b*d^2*x^5)/(5*f) + (atan(
(f^(1/2)*x*(a*f - b*e)*(c*f - d*e)^2)/(e^(1/2)*(a*c^2*f^3 - b*d^2*e^3 + a*d
^2*e^2*f - b*c^2*e*f^2 - 2*a*c*d*e*f^2 + 2*b*c*d*e^2*f)))*(a*f - b*e)*(c*f
- d*e)^2)/(e^(1/2)*f^(7/2))
```

3.13 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx$

Optimal. Leaf size=164

$$-\frac{d(be(15de-13cf)-3af(3de-cf))x}{6ef^3}+\frac{d(5be-3af)x(c+dx^2)}{6ef^2}-\frac{(be-af)x(c+dx^2)^2}{2ef\,(e+fx^2)}+\frac{(de-cf)(be(5de}{}$$

```
[Out] -1/6*d*(b*e*(-13*c*f+15*d*e)-3*a*f*(-c*f+3*d*e))*x/e/f^3+1/6*d*(-3*a*f+5*b*
e)*x*(d*x^2+c)/e/f^2-1/2*(-a*f+b*e)*x*(d*x^2+c)^2/e/f/(f*x^2+e)+1/2*(-c*f+d
*e)*(b*e*(-c*f+5*d*e)-a*f*(c*f+3*d*e))*arctan(x*f^(1/2)/e^(1/2))/e^(3/2)/f^
(7/2)
```

Rubi [A]
time = 0.15, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {540, 542, 396, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(de-cf)(be(5de-cf)-af(cf+3de))}{2e^{3/2}f^{7/2}}-\frac{dx(be(15de-13cf)-3af(3de-cf))}{6ef^3}+\frac{dx(c+dx^2)(5be-3af)}{6ef^2}-\frac{x(c+dx^2)^2(be-af)}{2ef\,(e+fx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

```
[Out] -1/6*(d*(b*e*(15*d*e - 13*c*f) - 3*a*f*(3*d*e - c*f))*x)/(e*f^3) + (d*(5*b*
e - 3*a*f)*x*(c + d*x^2))/(6*e*f^2) - ((b*e - a*f)*x*(c + d*x^2)^2)/(2*e*f*
(e + f*x^2)) + ((d*e - c*f)*(b*e*(5*d*e - c*f) - a*f*(3*d*e + c*f))*ArcTan[
(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx = -\frac{(be-af)x(c+dx^2)^2}{2ef(e+fx^2)} - \frac{\int \frac{(c+dx^2)(-c(be+af)-d(5be-3af)x^2)}{e+fx^2}\,dx}{2ef}$$

$$= \frac{d(5be-3af)x(c+dx^2)}{6ef^2} - \frac{(be-af)x(c+dx^2)^2}{2ef(e+fx^2)} - \frac{\int \frac{c(be(5de-3cf)-3af(de+cf))+d}{e+}}{6}$$

$$= -\frac{d(be(15de-13cf)-3af(3de-cf))x}{6ef^3} + \frac{d(5be-3af)x(c+dx^2)}{6ef^2} - \frac{(be-a}{2ef}$$

$$= -\frac{d(be(15de-13cf)-3af(3de-cf))x}{6ef^3} + \frac{d(5be-3af)x(c+dx^2)}{6ef^2} - \frac{(be-a}{2ef}$$

Mathematica [A]
time = 0.07, size = 134, normalized size = 0.82

$$\frac{d(-2bde+2bcf+adf)x}{f^3} + \frac{bd^2x^3}{3f^2} - \frac{(be-af)(de-cf)^2x}{2ef^3(e+fx^2)} + \frac{(de-cf)(be(5de-cf)-af(3de+cf))\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{2e^{3/2}f^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^2,x]
[Out] (d*(-2*b*d*e + 2*b*c*f + a*d*f)*x)/f^3 + (b*d^2*x^3)/(3*f^2) - ((b*e - a*f)
*(d*e - c*f)^2*x)/(2*e*f^3*(e + f*x^2)) + ((d*e - c*f)*(b*e*(5*d*e - c*f) -
 a*f*(3*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(7/2))
```

Maple [A]
time = 0.16, size = 182, normalized size = 1.11

method	result
default	$\frac{d\left(\frac{1}{3}bd\,x^3f+adfx+2bcfx-2bdex\right)}{f^3}+\frac{\frac{\left(c^2a\,f^3-2acde\,f^2+a\,d^2e^2f-b\,c^2e\,f^2+2bcd\,e^2f-b\,d^2e^3\right)x}{2e\left(f\,x^2+e\right)}+\frac{\left(c^2a\,f^3+2acde\,f^2-3a\,d^2e^2f+b\,c^2e\,f^2-6bcd\right.}{2e\sqrt{fe}}}{f^3}$
risch	$\frac{d^2b\,x^3}{3f^2}+\frac{d^2ax}{f^2}+\frac{2dbcx}{f^2}-\frac{2d^2bex}{f^3}+\frac{(c^2a\,f^3-2acde\,f^2+a\,d^2e^2f-b\,c^2e\,f^2+2bcd\,e^2f-b\,d^2e^3)x}{2e\,f^3(f\,x^2+e)}-\frac{\ln\left(fx+\sqrt{-fe}\right)c^2a}{4\sqrt{-fe}\;e}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d/f^3*(1/3*b*d*x^3*f+a*d*f*x+2*b*c*f*x-2*b*d*e*x)+1/f^3*(1/2*(a*c^2*f^3-2*a
*c*d*e*f^2+a*d^2*e^2*f-b*c^2*e*f^2+2*b*c*d*e^2*f-b*d^2*e^3)/e*x/(f*x^2+e)+1
/2*(a*c^2*f^3+2*a*c*d*e*f^2-3*a*d^2*e^2*f+b*c^2*e*f^2-6*b*c*d*e^2*f+5*b*d^2
*e^3)/e/(f*e)^(1/2)*arctan(f*x/(f*e)^(1/2)))
```

Maxima [A]
time = 0.49, size = 186, normalized size = 1.13

$$\frac{(ac^2f^3-bd^2e^3-(bc^2e+2\,acde)f^2+(2\,bcde^2+ad^2e^2)f)x}{2\,(f^4x^2e+f^3e^2)}+\frac{(ac^2f^3+5\,bd^2e^3+(bc^2e+2\,acde)f^2-3\,(2\,bcde^2+ad^2e^2)f)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{3}{2}\right)}}{2\,f^{\frac{7}{2}}}+\frac{bd^2fx^3-3\,(2\,bd^2e-(2\,bcd+ad^2)f)x}{3\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*c^2*f^3 - b*d^2*e^3 - (b*c^2*e + 2*a*c*d*e)*f^2 + (2*b*c*d*e^2 + a*d
^2*e^2)*f)*x/(f^4*x^2*e + f^3*e^2) + 1/2*(a*c^2*f^3 + 5*b*d^2*e^3 + (b*c^2*
e + 2*a*c*d*e)*f^2 - 3*(2*b*c*d*e^2 + a*d^2*e^2)*f)*arctan(sqrt(f)*x*e^(-1/
2))*e^(-3/2)/f^(7/2) + 1/3*(b*d^2*f*x^3 - 3*(2*b*d^2*e - (2*b*c*d + a*d^2)*
f)*x)/f^3
```

Fricas [A]
time = 1.00, size = 552, normalized size = 3.37

$$\left[\frac{6\,ac^2f^4xe-30\,bd^2fxe^4-3\,(ac^2f^4x^2+5\,bd^2e^4+(5\,bd^2fx^2-3\,(2\,bcd+ad^2)f)e^3-(3\,(2\,bcd+ad^2)f^2x^2-(bc^2+2\,acd)f^2)e^2+(ac^2f^3+(bc^2+2\,acd)f^3x^2)e)\sqrt{-fe}\,\log\left(\frac{fx^2-2\sqrt{-fe}\,x-e}{fx^2+e}\right)-2\,(10\,bd^2f^2x^3-9\,(2\,bcd+ad^2)f^2x)e^3+2\,(2\,bd^2f^3x^5+6\,(2\,bcd+ad^2)f^3x^3-3\,(bc^2+2\,acd)f^3x)e^2}{12\,(f^5x^2e^2+f^4e^3)},\frac{3\,ac^2f^4xe-15\,bd^2fxe^4+3\,(ac^2f^4x^2+5\,bd^2e^4+(5\,bd^2fx^2-3\,(2\,bcd+ad^2)f)e^3-(3\,(2\,bcd+ad^2)f^2x^2-(bc^2+2\,acd)f^2)e^2+(ac^2f^3+(bc^2+2\,acd)f^3x^2)e)\sqrt{f}\,\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\frac{1}{2}}-(10\,bd^2f^2x^3-9\,(2\,bcd+ad^2)f^2x)e^3+(2\,bd^2f^3x^5+6\,(2\,bcd+ad^2)f^3x^3-3\,(bc^2+2\,acd)f^3x)e^2}{6\,(f^5x^2e^2+f^4e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(6*a*c^2*f^4*x*e - 30*b*d^2*f*x*e^4 - 3*(a*c^2*f^4*x^2 + 5*b*d^2*e^4
+ (5*b*d^2*f*x^2 - 3*(2*b*c*d + a*d^2)*f)*e^3 - (3*(2*b*c*d + a*d^2)*f^2*x^
2 - (b*c^2 + 2*a*c*d)*f^2)*e^2 + (a*c^2*f^3 + (b*c^2 + 2*a*c*d)*f^3*x^2)*e)
*sqrt(-f*e)*log((f*x^2 - 2*sqrt(-f*e)*x - e)/(f*x^2 + e)) - 2*(10*b*d^2*f^2
*x^3 - 9*(2*b*c*d + a*d^2)*f^2*x)*e^3 + 2*(2*b*d^2*f^3*x^5 + 6*(2*b*c*d + a
```

```
*d^2)*f^3*x^3 - 3*(b*c^2 + 2*a*c*d)*f^3*x)*e^2)/(f^5*x^2*e^2 + f^4*e^3), 1/
6*(3*a*c^2*f^4*x*e - 15*b*d^2*f*x*e^4 + 3*(a*c^2*f^4*x^2 + 5*b*d^2*e^4 + (5
*b*d^2*f*x^2 - 3*(2*b*c*d + a*d^2)*f)*e^3 - (3*(2*b*c*d + a*d^2)*f^2*x^2 -
(b*c^2 + 2*a*c*d)*f^2)*e^2 + (a*c^2*f^3 + (b*c^2 + 2*a*c*d)*f^3*x^2)*e)*sqr
t(f)*arctan(sqrt(f)*x*e^(-1/2))*e^(1/2) - (10*b*d^2*f^2*x^3 - 9*(2*b*c*d +
a*d^2)*f^2*x)*e^3 + (2*b*d^2*f^3*x^5 + 6*(2*b*c*d + a*d^2)*f^3*x^3 - 3*(b*c
^2 + 2*a*c*d)*f^3*x)*e^2)/(f^5*x^2*e^2 + f^4*e^3)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(151) = 302$.
time = 1.50, size = 483, normalized size = 2.95

$$\frac{bd^2x^3}{3f^2} + x\left(\frac{ad^2}{f^2} + \frac{2bcd}{f^2} - \frac{2bd^2e}{f^3}\right) + \frac{x(ac^2f^3 - 2acdef^2 + ad^2e^2f - bc^2ef^2 + 2bcde^2f - bd^2e^3)}{2e^2f^3 + 2ef^4x^2} - \frac{\sqrt{-\frac{1}{e^3f^7}}(cf - de)(acf^2 + 3adef + bcef - 5bde^2)\log\left(-\frac{e^2f^3\sqrt{-\frac{1}{e^3f^7}}(cf-de)(acf^2+3adef+bcef-5bde^2)}{ac^2f^3+2acdef^2-3ad^2e^2f+bc^2ef^2-6bcde^2f+5bd^2e^3} + x\right)}{4} + \frac{\sqrt{-\frac{1}{e^3f^7}}(cf - de)(acf^2 + 3adef + bcef - 5bde^2)\log\left(\frac{e^2f^3\sqrt{-\frac{1}{e^3f^7}}(cf-de)(acf^2+3adef+bcef-5bde^2)}{ac^2f^3+2acdef^2-3ad^2e^2f+bc^2ef^2-6bcde^2f+5bd^2e^3} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**2,x)
```

```
[Out] b*d**2*x**3/(3*f**2) + x*(a*d**2/f**2 + 2*b*c*d/f**2 - 2*b*d**2*e/f**3) + x
*(a*c**2*f**3 - 2*a*c*d*e*f**2 + a*d**2*e**2*f - b*c**2*e*f**2 + 2*b*c*d*e*
*2*f - b*d**2*e**3)/(2*e**2*f**3 + 2*e*f**4*x**2) - sqrt(-1/(e**3*f**7))*(c
*f - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)*log(-e**2*f**3*sqrt
(-1/(e**3*f**7))*(c*f - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)/
(a*c**2*f**3 + 2*a*c*d*e*f**2 - 3*a*d**2*e**2*f + b*c**2*e*f**2 - 6*b*c*d*e
**2*f + 5*b*d**2*e**3) + x)/4 + sqrt(-1/(e**3*f**7))*(c*f - d*e)*(a*c*f**2
+ 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)*log(e**2*f**3*sqrt(-1/(e**3*f**7))*(c*f
 - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)/(a*c**2*f**3 + 2*a*c*
d*e*f**2 - 3*a*d**2*e**2*f + b*c**2*e*f**2 - 6*b*c*d*e**2*f + 5*b*d**2*e**3
) + x)/4
```

Giac [**A**]
time = 0.67, size = 195, normalized size = 1.19

$$\frac{(ac^2f^3 + bc^2f^2e + 2\,acdf^2e - 6\,bcdfe^2 - 3\,ad^2fe^2 + 5\,bd^2e^3)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{3}{2}\right)}}{2\,f^{\frac{7}{2}}} + \frac{(ac^2f^3x - bc^2f^2xe - 2\,acdf^2xe + 2\,bcdfxe^2 + ad^2fxe^2 - bd^2xe^3)e^{(-1)}}{2\,(fx^2 + e)f^3} + \frac{bd^2f^4x^3 + 6\,bcdf^4x + 3\,ad^2f^4x - 6\,bd^2f^3xe}{3\,f^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*c^2*f^3 + b*c^2*f^2*e + 2*a*c*d*f^2*e - 6*b*c*d*f*e^2 - 3*a*d^2*f*e^
2 + 5*b*d^2*e^3)*arctan(sqrt(f)*x*e^(-1/2))*e^(-3/2)/f^(7/2) + 1/2*(a*c^2*f
^3*x - b*c^2*f^2*x*e - 2*a*c*d*f^2*x*e + 2*b*c*d*f*x*e^2 + a*d^2*f*x*e^2 -
b*d^2*x*e^3)*e^(-1)/((f*x^2 + e)*f^3) + 1/3*(b*d^2*f^4*x^3 + 6*b*c*d*f^4*x
+ 3*a*d^2*f^4*x - 6*b*d^2*f^3*x*e)/f^6
```

Mupad [**B**]
time = 0.17, size = 257, normalized size = 1.57

$$x\left(\frac{a\,d^2 + 2\,b\,c\,d}{f^2} - \frac{2\,b\,d^2\,e}{f^3}\right) + \frac{b\,d^2\,x^3}{3\,f^2} + \frac{x\,(-b\,c^2\,e\,f^2 + a\,c^2\,f^3 + 2\,b\,c\,d\,e^2\,f - 2\,a\,c\,d\,e\,f^2 - b\,d^2\,e^3 + a\,d^2\,e^2\,f)}{2\,e\,(f^4\,x^2 + e\,f^3)} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(c\,f-d\,e)\,(a\,c\,f^2-5\,b\,d\,e^2+3\,a\,d\,e\,f+b\,c\,e\,f)}{\sqrt{e}\,(b\,c^2\,e\,f^2+a\,c^2\,f^3-6\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2+5\,b\,d^2\,e^3-3\,a\,d^2\,e^2\,f)}\right)(c\,f - d\,e)\,(a\,c\,f^2 - 5\,b\,d\,e^2 + 3\,a\,d\,e\,f + b\,c\,e\,f)}{2\,e^{3/2}\,f^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^2,x)
```

```
[Out] x*((a*d^2 + 2*b*c*d)/f^2 - (2*b*d^2*e)/f^3) + (b*d^2*x^3)/(3*f^2) + (x*(a*c
^2*f^3 - b*d^2*e^3 + a*d^2*e^2*f - b*c^2*e*f^2 - 2*a*c*d*e*f^2 + 2*b*c*d*e^
2*f))/(2*e*(e*f^3 + f^4*x^2)) + (atan((f^(1/2)*x*(c*f - d*e)*(a*c*f^2 - 5*b
*d*e^2 + 3*a*d*e*f + b*c*e*f))/(e^(1/2)*(a*c^2*f^3 + 5*b*d^2*e^3 - 3*a*d^2*
e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^2 - 6*b*c*d*e^2*f)))*(c*f - d*e)*(a*c*f^2
 - 5*b*d*e^2 + 3*a*d*e*f + b*c*e*f))/(2*e^(3/2)*f^(7/2))
```

3.14 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx$

Optimal. Leaf size=207

$$\frac{d(be(15de-cf)-3af(de+cf))x}{8e^2f^3}-\frac{(be-af)x(c+dx^2)^2}{4ef\left(e+fx^2\right)^2}-\frac{(be(5de-cf)-af(de+3cf))x(c+dx^2)}{8e^2f^2\left(e+fx^2\right)}-\frac{(be}{}$$

```
[Out] 1/8*d*(b*e*(-c*f+15*d*e)-3*a*f*(c*f+d*e))*x/e^2/f^3-1/4*(-a*f+b*e)*x*(d*x^2
+c)^2/e/f/(f*x^2+e)^2-1/8*(b*e*(-c*f+5*d*e)-a*f*(3*c*f+d*e))*x*(d*x^2+c)/e^
2/f^2/(f*x^2+e)-1/8*(b*e*(-c^2*f^2-6*c*d*e*f+15*d^2*e^2)-a*f*(3*c^2*f^2+2*c
*d*e*f+3*d^2*e^2))*arctan(x*f^(1/2)/e^(1/2))/e^(5/2)/f^(7/2)
```

Rubi [A]
time = 0.16, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {540, 396, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{8e^{5/2}f^{7/2}}+\frac{dx(be(15de-cf)-3af(cf+de))}{8e^2f^3}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3cf+de))}{8e^2f^2\left(e+fx^2\right)}-\frac{x\left(c+dx^2\right)^2(be-af)}{4ef\left(e+fx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

```
[Out] (d*(b*e*(15*d*e - c*f) - 3*a*f*(d*e + c*f))*x)/(8*e^2*f^3) - ((b*e - a*f)*x
*(c + d*x^2)^2)/(4*e*f*(e + f*x^2)^2) - ((b*e*(5*d*e - c*f) - a*f*(d*e + 3*
c*f))*x*(c + d*x^2))/(8*e^2*f^2*(e + f*x^2)) - ((b*e*(15*d^2*e^2 - 6*c*d*e*
f - c^2*f^2) - a*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/
Sqrt[e]])/(8*e^(5/2)*f^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx = -\frac{(be-af)x(c+dx^2)^2}{4ef(e+fx^2)^2} - \frac{\int \frac{(c+dx^2)(-c(be+3af)-d(5be-af)x^2)}{(e+fx^2)^2}\,dx}{4ef}$$

$$= -\frac{(be-af)x(c+dx^2)^2}{4ef(e+fx^2)^2} - \frac{(be(5de-cf)-af(de+3cf))x(c+dx^2)}{8e^2f^2(e+fx^2)} + \frac{\int \frac{-c(af(\ldots}{\ldots}}{\ldots}$$

$$= \frac{d(be(15de-cf)-3af(de+cf))x}{8e^2f^3} - \frac{(be-af)x(c+dx^2)^2}{4ef(e+fx^2)^2} - \frac{(be(5de-cf)-\ldots}{8e^2\ldots}$$

$$= \frac{d(be(15de-cf)-3af(de+cf))x}{8e^2f^3} - \frac{(be-af)x(c+dx^2)^2}{4ef(e+fx^2)^2} - \frac{(be(5de-cf)-\ldots}{8e^2\ldots}$$

Mathematica [A]
time = 0.09, size = 183, normalized size = 0.88

$$\frac{bd^2x}{f^3} - \frac{(be-af)(de-cf)^2x}{4ef^3(e+fx^2)^2} + \frac{(de-cf)(be(9de-cf)-af(5de+3cf))x}{8e^2f^3(e+fx^2)} - \frac{(be(15d^2e^2-6cdef-c^2f^2)-af(3d^2e^2+2cdef+3c^2f^2))\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{8e^{5/2}f^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

```
[Out] (b*d^2*x)/f^3 - ((b*e - a*f)*(d*e - c*f)^2*x)/(4*e*f^3*(e + f*x^2)^2) + ((d
*e - c*f)*(b*e*(9*d*e - c*f) - a*f*(5*d*e + 3*c*f))*x)/(8*e^2*f^3*(e + f*x^
2)) - ((b*e*(15*d^2*e^2 - 6*c*d*e*f - c^2*f^2) - a*f*(3*d^2*e^2 + 2*c*d*e*f
 + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(7/2))
```

Maple [A]
time = 0.17, size = 230, normalized size = 1.11

method	result
default	$\frac{b\,d^2x}{f^3} + \frac{\frac{\frac{f\left(3c^2a\,f^3+2acde\,f^2-5a\,d^2e^2f+b\,c^2e\,f^2-10bcd\,e^2f+9b\,d^2e^3\right)x^3}{8e^2}+\frac{\left(5c^2a\,f^3-2acde\,f^2-3a\,d^2e^2f-b\,c^2e\,f^2-6bcd\,e^2f+7b\,d^2e^3\right)x}{8e}}{\left(f\,x^2+e\right)^2}+\frac{(3c^2\ldots}{\ldots}}{f^3}$

risch	$\frac{b\,d^2x}{f^3}+\frac{\frac{f\left(3c^2a\,f^3+2acde\,f^2-5a\,d^2e^2f+b\,c^2e\,f^2-10bcd\,e^2f+9b\,d^2e^3\right)x^3}{8e^2}+\frac{\left(5c^2a\,f^3-2acde\,f^2-3a\,d^2e^2f-b\,c^2e\,f^2-6bcd\,e^2f+7b\,d^2e^3\right)x}{8e}}{f^3(f\,x^2+e)^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b*d^2/f^3*x+1/f^3*((1/8*f*(3*a*c^2*f^3+2*a*c*d*e*f^2-5*a*d^2*e^2*f+b*c^2*e*
f^2-10*b*c*d*e^2*f+9*b*d^2*e^3)/e^2*x^3+1/8*(5*a*c^2*f^3-2*a*c*d*e*f^2-3*a*
d^2*e^2*f-b*c^2*e*f^2-6*b*c*d*e^2*f+7*b*d^2*e^3)/e*x)/(f*x^2+e)^2+1/8*(3*a*
c^2*f^3+2*a*c*d*e*f^2+3*a*d^2*e^2*f+b*c^2*e*f^2+6*b*c*d*e^2*f-15*b*d^2*e^3)
/e^2/(f*e)^(1/2)*arctan(f*x/(f*e)^(1/2)))
```

Maxima [A]
time = 0.50, size = 233, normalized size = 1.13

$$\frac{bd^2x}{f^3}+\frac{(3\,ac^2f^4+9\,bd^2fe^3+(bc^2e+2\,acde)f^3-5\,(2\,bcde^2+ad^2e^2)f^2)x^3+(5\,ac^2f^3e+7\,bd^2e^4-(bc^2e^2+2\,acde^2)f^2-3\,(2\,bcde^3+ad^2e^3)f)x}{8\,(f^5x^4e^2+2\,f^4x^2e^3+f^3e^4)}+\frac{(3\,ac^2f^3-15\,bd^2e^3+(bc^2e+2\,acde)f^2+3\,(2\,bcde^2+ad^2e^2)f)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{5}{2}\right)}}{8\,f^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] b*d^2*x/f^3 + 1/8*((3*a*c^2*f^4 + 9*b*d^2*f*e^3 + (b*c^2*e + 2*a*c*d*e)*f^3
 - 5*(2*b*c*d*e^2 + a*d^2*e^2)*f^2)*x^3 + (5*a*c^2*f^3*e + 7*b*d^2*e^4 - (b
*c^2*e^2 + 2*a*c*d*e^2)*f^2 - 3*(2*b*c*d*e^3 + a*d^2*e^3)*f)*x)/(f^5*x^4*e^
2 + 2*f^4*x^2*e^3 + f^3*e^4) + 1/8*(3*a*c^2*f^3 - 15*b*d^2*e^3 + (b*c^2*e +
 2*a*c*d*e)*f^2 + 3*(2*b*c*d*e^2 + a*d^2*e^2)*f)*arctan(sqrt(f)*x*e^(-1/2))
*e^(-5/2)/f^(7/2)
```

Fricas [A]
time = 2.48, size = 776, normalized size = 3.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*c^2*f^5*x^3*e + 30*b*d^2*f*x*e^5 + (3*a*c^2*f^5*x^4 - 15*b*d^2*e
^5 - 3*(10*b*d^2*f*x^2 - (2*b*c*d + a*d^2)*f)*e^4 - (15*b*d^2*f^2*x^4 - 6*(
2*b*c*d + a*d^2)*f^2*x^2 - (b*c^2 + 2*a*c*d)*f^2)*e^3 + (3*(2*b*c*d + a*d^2
)*f^3*x^4 + 3*a*c^2*f^3 + 2*(b*c^2 + 2*a*c*d)*f^3*x^2)*e^2 + (6*a*c^2*f^4*x
^2 + (b*c^2 + 2*a*c*d)*f^4*x^4)*e)*sqrt(-f*e)*log((f*x^2 + 2*sqrt(-f*e)*x -
 e)/(f*x^2 + e)) + 2*(25*b*d^2*f^2*x^3 - 3*(2*b*c*d + a*d^2)*f^2*x)*e^4 + 2
*(8*b*d^2*f^3*x^5 - 5*(2*b*c*d + a*d^2)*f^3*x^3 - (b*c^2 + 2*a*c*d)*f^3*x)*
e^3 + 2*(5*a*c^2*f^4*x + (b*c^2 + 2*a*c*d)*f^4*x^3)*e^2)/(f^6*x^4*e^3 + 2*f
^5*x^2*e^4 + f^4*e^5), 1/8*(3*a*c^2*f^5*x^3*e + 15*b*d^2*f*x*e^5 + (3*a*c^2
```

```
*f^5*x^4 - 15*b*d^2*e^5 - 3*(10*b*d^2*f*x^2 - (2*b*c*d + a*d^2)*f)*e^4 - (1
5*b*d^2*f^2*x^4 - 6*(2*b*c*d + a*d^2)*f^2*x^2 - (b*c^2 + 2*a*c*d)*f^2)*e^3
+ (3*(2*b*c*d + a*d^2)*f^3*x^4 + 3*a*c^2*f^3 + 2*(b*c^2 + 2*a*c*d)*f^3*x^2)
*e^2 + (6*a*c^2*f^4*x^2 + (b*c^2 + 2*a*c*d)*f^4*x^4)*e)*sqrt(f)*arctan(sqrt
(f)*x*e^(-1/2))*e^(1/2) + (25*b*d^2*f^2*x^3 - 3*(2*b*c*d + a*d^2)*f^2*x)*e^
4 + (8*b*d^2*f^3*x^5 - 5*(2*b*c*d + a*d^2)*f^3*x^3 - (b*c^2 + 2*a*c*d)*f^3*
x)*e^3 + (5*a*c^2*f^4*x + (b*c^2 + 2*a*c*d)*f^4*x^3)*e^2)/(f^6*x^4*e^3 + 2*
f^5*x^2*e^4 + f^4*e^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(199) = 398$.
time = 7.02, size = 400, normalized size = 1.93

$$\frac{bd^2x}{f^3} - \frac{\sqrt{-\frac{1}{e^5f^7}}\cdot(3ac^2f^3 + 2acdef^2 + 3ad^2e^2f + bc^2ef^2 + 6bcde^2f - 15bd^2e^3)\log\left(-e^3f^3\sqrt{-\frac{1}{e^5f^7}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{e^5f^7}}\cdot(3ac^2f^3 + 2acdef^2 + 3ad^2e^2f + bc^2ef^2 + 6bcde^2f - 15bd^2e^3)\log\left(e^3f^3\sqrt{-\frac{1}{e^5f^7}} + x\right)}{16} + \frac{x^3\cdot(3ac^2f^4 + 2acdef^3 - 5ad^2e^2f^2 + bc^2ef^3 - 10bcde^2f^2 + 9bd^2e^3f) + x(5ac^2ef^3 - 2acde^2f^2 - 3ad^2e^3f - bc^2e^2f^2 - 6bcde^3f + 7bd^2e^4)}{8e^4f^3 + 16e^3f^4x^2 + 8e^2f^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**3,x)
```

```
[Out] b*d**2*x/f**3 - sqrt(-1/(e**5*f**7))*(3*a*c**2*f**3 + 2*a*c*d*e*f**2 + 3*a*
d**2*e**2*f + b*c**2*e*f**2 + 6*b*c*d*e**2*f - 15*b*d**2*e**3)*log(-e**3*f*
*3*sqrt(-1/(e**5*f**7)) + x)/16 + sqrt(-1/(e**5*f**7))*(3*a*c**2*f**3 + 2*a
*c*d*e*f**2 + 3*a*d**2*e**2*f + b*c**2*e*f**2 + 6*b*c*d*e**2*f - 15*b*d**2*
e**3)*log(e**3*f**3*sqrt(-1/(e**5*f**7)) + x)/16 + (x**3*(3*a*c**2*f**4 + 2
*a*c*d*e*f**3 - 5*a*d**2*e**2*f**2 + b*c**2*e*f**3 - 10*b*c*d*e**2*f**2 + 9
*b*d**2*e**3*f) + x*(5*a*c**2*e*f**3 - 2*a*c*d*e**2*f**2 - 3*a*d**2*e**3*f
- b*c**2*e**2*f**2 - 6*b*c*d*e**3*f + 7*b*d**2*e**4))/(8*e**4*f**3 + 16*e**
3*f**4*x**2 + 8*e**2*f**5*x**4)
```

Giac [A]
time = 0.63, size = 238, normalized size = 1.15

$$\frac{bd^2x}{f^3} + \frac{(3\,ac^2f^3 + bc^2f^2e + 2\,acdf^2e + 6\,bcdfe^2 + 3\,ad^2fe^2 - 15\,bd^2e^3)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{5}{2}\right)}}{8\,f^{\frac{7}{2}}} + \frac{(3\,ac^2f^4x^3 + bc^2f^3x^3e + 2\,acdf^3x^3e - 10\,bcdf^2x^3e^2 - 5\,ad^2f^2x^3e^2 + 9\,bd^2fx^3e^3 + 5\,ac^2f^3xe - bc^2f^2xe^2 - 2\,acdf^2xe^2 - 6\,bcdfxe^3 - 3\,ad^2fxe^3 + 7\,bd^2xe^4)e^{(-2)}}{8\,(fx^2 + e)^2f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] b*d^2*x/f^3 + 1/8*(3*a*c^2*f^3 + b*c^2*f^2*e + 2*a*c*d*f^2*e + 6*b*c*d*f*e^
2 + 3*a*d^2*f*e^2 - 15*b*d^2*e^3)*arctan(sqrt(f)*x*e^(-1/2))*e^(-5/2)/f^(7/
2) + 1/8*(3*a*c^2*f^4*x^3 + b*c^2*f^3*x^3*e + 2*a*c*d*f^3*x^3*e - 10*b*c*d*
f^2*x^3*e^2 - 5*a*d^2*f^2*x^3*e^2 + 9*b*d^2*f*x^3*e^3 + 5*a*c^2*f^3*x*e - b
*c^2*f^2*x*e^2 - 2*a*c*d*f^2*x*e^2 - 6*b*c*d*f*x*e^3 - 3*a*d^2*f*x*e^3 + 7*
b*d^2*x*e^4)*e^(-2)/((f*x^2 + e)^2*f^3)
```

Mupad [B]
time = 0.98, size = 243, normalized size = 1.17

$$\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,\left(b\,c^2\,e\,f^2+3\,a\,c^2\,f^3+6\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2-15\,b\,d^2\,e^3+3\,a\,d^2\,e^2\,f\right)}{8\,e^{5/2}\,f^{7/2}}-\frac{\frac{x\,\left(b\,c^2\,e\,f^2-5\,a\,c^2\,f^3+6\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2-7\,b\,d^2\,e^3+3\,a\,d^2\,e^2\,f\right)}{8\,e}-\frac{x^3\,\left(b\,c^2\,e\,f^3+3\,a\,c^2\,f^4-10\,b\,c\,d\,e^2\,f^2+2\,a\,c\,d\,e\,f^3+9\,b\,d^2\,e^3\,f-5\,a\,d^2\,e^2\,f^2\right)}{8\,e^2}}{e^2\,f^3+2\,e\,f^4\,x^2+f^5\,x^4}+\frac{b\,d^2\,x}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^3,x)
```

```
[Out] (atan((f^(1/2)*x)/e^(1/2))*(3*a*c^2*f^3 - 15*b*d^2*e^3 + 3*a*d^2*e^2*f + b*
c^2*e*f^2 + 2*a*c*d*e*f^2 + 6*b*c*d*e^2*f))/(8*e^(5/2)*f^(7/2)) - ((x*(3*a*
d^2*e^2*f - 7*b*d^2*e^3 - 5*a*c^2*f^3 + b*c^2*e*f^2 + 2*a*c*d*e*f^2 + 6*b*c
*d*e^2*f))/(8*e) - (x^3*(3*a*c^2*f^4 - 5*a*d^2*e^2*f^2 + b*c^2*e*f^3 + 9*b*
d^2*e^3*f - 10*b*c*d*e^2*f^2 + 2*a*c*d*e*f^3))/(8*e^2))/(e^2*f^3 + f^5*x^4
+ 2*e*f^4*x^2) + (b*d^2*x)/f^3
```

3.15 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4} dx$

Optimal. Leaf size=240

$$-\frac{(be-af)x(c+dx^2)^2}{6ef(e+fx^2)^3}-\frac{(de(5be+af)-cf(be+5af))x(c+dx^2)}{24e^2f^2(e+fx^2)^2}-\frac{(af(3d^2e^2+4cdef-15c^2f^2)+be(15d^2 \cdots}{48e^3f^3(e+fx^2) \cdots}$$

```
[Out] -1/6*(-a*f+b*e)*x*(d*x^2+c)^2/e/f/(f*x^2+e)^3-1/24*(d*e*(a*f+5*b*e)-c*f*(5*
a*f+b*e))*x*(d*x^2+c)/e^2/f^2/(f*x^2+e)^2-1/48*(a*f*(-15*c^2*f^2+4*c*d*e*f+
3*d^2*e^2)+b*e*(-3*c^2*f^2-4*c*d*e*f+15*d^2*e^2))*x/e^3/f^3/(f*x^2+e)+1/16*
(b*e*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)+a*f*(5*c^2*f^2+2*c*d*e*f+d^2*e^2))*arcta
n(x*f^(1/2)/e^(1/2))/e^(7/2)/f^(7/2)
```

Rubi [A]
time = 0.18, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {540, 393, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(5c^2f^2+2cdef+d^2e^2)+be(c^2f^2+2cdef+5d^2e^2))}{16e^{7/2}f^{7/2}}-\frac{x(af(-15c^2f^2+4cdef+3d^2e^2)+be(-3c^2f^2-4cdef+15d^2e^2))}{48e^3f^3(e+fx^2)}-\frac{x(c+dx^2)(de(af+5be)-cf(5af+be))}{24e^2f^2(e+fx^2)^2}-\frac{x(c+dx^2)^2(be-af)}{6ef(e+fx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```

```
[Out] -1/6*((b*e - a*f)*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)^3) - ((d*e*(5*b*e + a*f
) - c*f*(b*e + 5*a*f))*x*(c + d*x^2))/(24*e^2*f^2*(e + f*x^2)^2) - ((a*f*(3
*d^2*e^2 + 4*c*d*e*f - 15*c^2*f^2) + b*e*(15*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^
2))*x)/(48*e^3*f^3*(e + f*x^2)) + ((b*e*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) +
 a*f*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^
(7/2)*f^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx = -\frac{(be-af)x(c+dx^2)^2}{6ef(e+fx^2)^3} - \frac{\int \frac{(c+dx^2)(-c(be+5af)-d(5be+af)x^2)}{(e+fx^2)^3}\,dx}{6ef}$$

$$= -\frac{(be-af)x(c+dx^2)^2}{6ef(e+fx^2)^3} - \frac{(de(5be+af)-cf(be+5af))x(c+dx^2)}{24e^2f^2(e+fx^2)^2} + \frac{\int \frac{c(de(5\ldots}{\ldots}}{\ldots}$$

$$= -\frac{(be-af)x(c+dx^2)^2}{6ef(e+fx^2)^3} - \frac{(de(5be+af)-cf(be+5af))x(c+dx^2)}{24e^2f^2(e+fx^2)^2} - \frac{(af(3a\ldots}{\ldots}$$

$$= -\frac{(be-af)x(c+dx^2)^2}{6ef(e+fx^2)^3} - \frac{(de(5be+af)-cf(be+5af))x(c+dx^2)}{24e^2f^2(e+fx^2)^2} - \frac{(af(3a\ldots}{\ldots}$$

Mathematica [A]
time = 0.11, size = 242, normalized size = 1.01

$$-\frac{(be-af)(de-cf)^2x}{6ef^3(e+fx^2)^3} + \frac{(de-cf)(be(13de-cf)-af(7de+5cf))x}{24e^2f^3(e+fx^2)^2} + \frac{(be(-11d^2e^2+2cdef+c^2f^2)+af(d^2e^2+2cdef+5c^2f^2))\,x}{16e^3f^3(e+fx^2)} + \frac{(be(5d^2e^2+2cdef+c^2f^2)+af(d^2e^2+2cdef+5c^2f^2))\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{16e^{7/2}f^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```

```
[Out] -1/6*((b*e - a*f)*(d*e - c*f)^2*x)/(e*f^3*(e + f*x^2)^3) + ((d*e - c*f)*(b*
e*(13*d*e - c*f) - a*f*(7*d*e + 5*c*f))*x)/(24*e^2*f^3*(e + f*x^2)^2) + ((b
*e*(-11*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + a*f*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f
^2))*x)/(16*e^3*f^3*(e + f*x^2)) + ((b*e*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)
+ a*f*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e
^(7/2)*f^(7/2))
```

Maple [A]
time = 0.16, size = 289, normalized size = 1.20

method	result
default	$\frac{\frac{\left(5c^2 a\, f^3+2acde\, f^2+a\, d^2 e^2 f+b\, c^2 e\, f^2+2bcd\, e^2 f-11b\, d^2 e^3\right)x^5}{16e^3 f}+\frac{\left(5c^2 a\, f^3+2acde\, f^2-a\, d^2 e^2 f+b\, c^2 e\, f^2-2bcd\, e^2 f-5b\, d^2 e^3\right)x^3}{6e^2 f^2}+\frac{\left(11c^2 a\, f^3-2acde\right.}{}}{\left(f\, x^2+e\right)^3}$
risch	$\frac{\frac{\left(5c^2 a\, f^3+2acde\, f^2+a\, d^2 e^2 f+b\, c^2 e\, f^2+2bcd\, e^2 f-11b\, d^2 e^3\right)x^5}{16e^3 f}+\frac{\left(5c^2 a\, f^3+2acde\, f^2-a\, d^2 e^2 f+b\, c^2 e\, f^2-2bcd\, e^2 f-5b\, d^2 e^3\right)x^3}{6e^2 f^2}+\frac{\left(11c^2 a\, f^3-2acde\right.}{}}{\left(f\, x^2+e\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(5*a*c^2*f^3+2*a*c*d*e*f^2+a*d^2*e^2*f+b*c^2*e*f^2+2*b*c*d*e^2*f-11*b
*d^2*e^3)/e^3/f*x^5+1/6*(5*a*c^2*f^3+2*a*c*d*e*f^2-a*d^2*e^2*f+b*c^2*e*f^2-
2*b*c*d*e^2*f-5*b*d^2*e^3)/e^2/f^2*x^3+1/16*(11*a*c^2*f^3-2*a*c*d*e*f^2-a*d
^2*e^2*f-b*c^2*e*f^2-2*b*c*d*e^2*f-5*b*d^2*e^3)/f^3/e*x)/(f*x^2+e)^3+1/16*(
5*a*c^2*f^3+2*a*c*d*e*f^2+a*d^2*e^2*f+b*c^2*e*f^2+2*b*c*d*e^2*f+5*b*d^2*e^3
)/e^3/f^3/(f*e)^(1/2)*arctan(f*x/(f*e)^(1/2))
```

<u>Maxima</u> [A]
time = 0.51, size = 301, normalized size = 1.25

$$\frac{3\,(5\,ac^2f^5-11\,bd^2f^2e^3+(bc^2e+2\,acde)f^4+(2\,bcde^2+ad^2e^2)f^3)x^5+8\,(5\,ac^2f^4e-5\,bd^2fe^4+(bc^2e^2+2\,acde^2)f^3-(2\,bcde^3+ad^2e^3)f^2)x^3+3\,(11\,ac^2f^3e^2-5\,bd^2e^5-(bc^2e^3+2\,acde^3)f^2-(2\,bcde^4+ad^2e^4)f)x}{48\,(f^6x^6e^3+3\,f^5x^4e^4+3\,f^4x^2e^5+f^3e^6)}+\frac{(5\,ac^2f^3+5\,bd^2e^3+(bc^2e+2\,acde)f^2+(2\,bcde^2+ad^2e^2)f)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{7}{2}\right)}}{16\,f^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(5*a*c^2*f^5 - 11*b*d^2*f^2*e^3 + (b*c^2*e + 2*a*c*d*e)*f^4 + (2*b*
c*d*e^2 + a*d^2*e^2)*f^3)*x^5 + 8*(5*a*c^2*f^4*e - 5*b*d^2*f*e^4 + (b*c^2*e
^2 + 2*a*c*d*e^2)*f^3 - (2*b*c*d*e^3 + a*d^2*e^3)*f^2)*x^3 + 3*(11*a*c^2*f^
3*e^2 - 5*b*d^2*e^5 - (b*c^2*e^3 + 2*a*c*d*e^3)*f^2 - (2*b*c*d*e^4 + a*d^2*
e^4)*f)*x)/(f^6*x^6*e^3 + 3*f^5*x^4*e^4 + 3*f^4*x^2*e^5 + f^3*e^6) + 1/16*(
5*a*c^2*f^3 + 5*b*d^2*e^3 + (b*c^2*e + 2*a*c*d*e)*f^2 + (2*b*c*d*e^2 + a*d^
2*e^2)*f)*arctan(sqrt(f)*x*e^(-1/2))*e^(-7/2)/f^(7/2)
```

<u>Fricas</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(228) = 456$.
time = 1.22, size = 1033, normalized size = 4.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(30*a*c^2*f^6*x^5*e - 30*b*d^2*f*x*e^6 - 3*(5*a*c^2*f^6*x^6 + 5*b*d^2
*e^6 + (15*b*d^2*f*x^2 + (2*b*c*d + a*d^2)*f)*e^5 + (15*b*d^2*f^2*x^4 + 3*(
```

```
2*b*c*d + a*d^2)*f^2*x^2 + (b*c^2 + 2*a*c*d)*f^2)*e^4 + (5*b*d^2*f^3*x^6 +
3*(2*b*c*d + a*d^2)*f^3*x^4 + 5*a*c^2*f^3 + 3*(b*c^2 + 2*a*c*d)*f^3*x^2)*e^
3 + ((2*b*c*d + a*d^2)*f^4*x^6 + 15*a*c^2*f^4*x^2 + 3*(b*c^2 + 2*a*c*d)*f^4
*x^4)*e^2 + (15*a*c^2*f^5*x^4 + (b*c^2 + 2*a*c*d)*f^5*x^6)*e)*sqrt(-f*e)*lo
g((f*x^2 - 2*sqrt(-f*e)*x - e)/(f*x^2 + e)) - 2*(40*b*d^2*f^2*x^3 + 3*(2*b*
c*d + a*d^2)*f^2*x)*e^5 - 2*(33*b*d^2*f^3*x^5 + 8*(2*b*c*d + a*d^2)*f^3*x^3
 + 3*(b*c^2 + 2*a*c*d)*f^3*x)*e^4 + 2*(3*(2*b*c*d + a*d^2)*f^4*x^5 + 33*a*c
^2*f^4*x + 8*(b*c^2 + 2*a*c*d)*f^4*x^3)*e^3 + 2*(40*a*c^2*f^5*x^3 + 3*(b*c^
2 + 2*a*c*d)*f^5*x^5)*e^2)/(f^7*x^6*e^4 + 3*f^6*x^4*e^5 + 3*f^5*x^2*e^6 + f
^4*e^7), 1/48*(15*a*c^2*f^6*x^5*e - 15*b*d^2*f*x*e^6 + 3*(5*a*c^2*f^6*x^6 +
 5*b*d^2*e^6 + (15*b*d^2*f*x^2 + (2*b*c*d + a*d^2)*f)*e^5 + (15*b*d^2*f^2*x
^4 + 3*(2*b*c*d + a*d^2)*f^2*x^2 + (b*c^2 + 2*a*c*d)*f^2)*e^4 + (5*b*d^2*f^
3*x^6 + 3*(2*b*c*d + a*d^2)*f^3*x^4 + 5*a*c^2*f^3 + 3*(b*c^2 + 2*a*c*d)*f^3
*x^2)*e^3 + ((2*b*c*d + a*d^2)*f^4*x^6 + 15*a*c^2*f^4*x^2 + 3*(b*c^2 + 2*a*
c*d)*f^4*x^4)*e^2 + (15*a*c^2*f^5*x^4 + (b*c^2 + 2*a*c*d)*f^5*x^6)*e)*sqrt(
f)*arctan(sqrt(f)*x*e^(-1/2))*e^(1/2) - (40*b*d^2*f^2*x^3 + 3*(2*b*c*d + a*
d^2)*f^2*x)*e^5 - (33*b*d^2*f^3*x^5 + 8*(2*b*c*d + a*d^2)*f^3*x^3 + 3*(b*c^
2 + 2*a*c*d)*f^3*x)*e^4 + (3*(2*b*c*d + a*d^2)*f^4*x^5 + 33*a*c^2*f^4*x + 8
*(b*c^2 + 2*a*c*d)*f^4*x^3)*e^3 + (40*a*c^2*f^5*x^3 + 3*(b*c^2 + 2*a*c*d)*f
^5*x^5)*e^2)/(f^7*x^6*e^4 + 3*f^6*x^4*e^5 + 3*f^5*x^2*e^6 + f^4*e^7)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.63, size = 311, normalized size = 1.30

$$\frac{\left(5\,ac^2f^3 + bc^2f^2e + 2\,acdf^2e + 2\,bcdfe^2 + ad^2fe^2 + 5\,bd^2e^3\right)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{7}{2}\right)}}{16\,f^{\frac{7}{2}}} + \frac{\left(15\,ac^2f^5x^5 + 3\,bc^2f^4x^5e + 6\,acdf^4x^5e + 6\,bcdf^3x^5e^2 + 3\,ad^2f^3x^5e^2 - 33\,bd^2f^2x^5e^3 + 40\,ac^2f^4x^3e + 8\,bc^2f^3x^3e^2 + 16\,acdf^3x^3e^2 - 16\,bcdf^2x^3e^3 - 8\,ad^2f^2x^3e^3 - 40\,bd^2fx^3e^4 + 33\,ac^2f^3xe^2 - 3\,bc^2f^2xe^3 - 6\,acdf^2xe^3 - 6\,bcdfxe^4 - 3\,ad^2fxe^4 - 15\,bd^2xe^5\right)e^{(-3)}}{48\,\left(fx^2 + e\right)^3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="giac")
```

```
[Out] 1/16*(5*a*c^2*f^3 + b*c^2*f^2*e + 2*a*c*d*f^2*e + 2*b*c*d*f*e^2 + a*d^2*f*e
^2 + 5*b*d^2*e^3)*arctan(sqrt(f)*x*e^(-1/2))*e^(-7/2)/f^(7/2) + 1/48*(15*a*
c^2*f^5*x^5 + 3*b*c^2*f^4*x^5*e + 6*a*c*d*f^4*x^5*e + 6*b*c*d*f^3*x^5*e^2 +
 3*a*d^2*f^3*x^5*e^2 - 33*b*d^2*f^2*x^5*e^3 + 40*a*c^2*f^4*x^3*e + 8*b*c^2*
f^3*x^3*e^2 + 16*a*c*d*f^3*x^3*e^2 - 16*b*c*d*f^2*x^3*e^3 - 8*a*d^2*f^2*x^3
*e^3 - 40*b*d^2*f*x^3*e^4 + 33*a*c^2*f^3*x*e^2 - 3*b*c^2*f^2*x*e^3 - 6*a*c*
```

```
d*f^2*x*e^3 - 6*b*c*d*f*x*e^4 - 3*a*d^2*f*x*e^4 - 15*b*d^2*x*e^5)*e^(-3)/((
f*x^2 + e)^3*f^3)
```

Mupad [B]
time = 0.98, size = 303, normalized size = 1.26

$$\frac{\frac{x^3\,\left(b\,c^2\,e\,f^2+5\,a\,c^2\,f^3-2\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2-5\,b\,d^2\,e^3-a\,d^2\,e^2\,f\right)}{6\,e^2\,f^2}-\frac{x\,\left(b\,c^2\,e\,f^2-11\,a\,c^2\,f^3+2\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2+5\,b\,d^2\,e^3+a\,d^2\,e^2\,f\right)}{16\,e\,f^3}+\frac{x^5\,\left(b\,c^2\,e\,f^2+5\,a\,c^2\,f^3+2\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2-11\,b\,d^2\,e^3+a\,d^2\,e^2\,f\right)}{16\,e^3\,f}}{e^3+3\,e^2\,f\,x^2+3\,e\,f^2\,x^4+f^3\,x^6}+\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,\left(b\,c^2\,e\,f^2+5\,a\,c^2\,f^3+2\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2+5\,b\,d^2\,e^3+a\,d^2\,e^2\,f\right)}{16\,e^{7/2}\,f^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^4,x)
```

```
[Out] ((x^3*(5*a*c^2*f^3 - 5*b*d^2*e^3 - a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^
2 - 2*b*c*d*e^2*f))/(6*e^2*f^2) - (x*(5*b*d^2*e^3 - 11*a*c^2*f^3 + a*d^2*e^
2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^2 + 2*b*c*d*e^2*f))/(16*e*f^3) + (x^5*(5*a*
c^2*f^3 - 11*b*d^2*e^3 + a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^2 + 2*b*c*
d*e^2*f))/(16*e^3*f))/(e^3 + f^3*x^6 + 3*e^2*f*x^2 + 3*e*f^2*x^4) + (atan((
f^(1/2)*x)/e^(1/2))*(5*a*c^2*f^3 + 5*b*d^2*e^3 + a*d^2*e^2*f + b*c^2*e*f^2
+ 2*a*c*d*e*f^2 + 2*b*c*d*e^2*f))/(16*e^(7/2)*f^(7/2))
```

3.16 $\int (a+bx^2) (c+dx^2)^3 (e+fx^2)^3 \, dx$

Optimal. Leaf size=310

$$ac^3e^3x+\frac{1}{3}c^2e^2(bce+3a(de+cf))x^3+\frac{3}{5}ce\left(bce(de+cf)+a\left(d^2e^2+3cdef+c^2f^2\right)\right)x^5+\frac{1}{7}\left(3bce\left(d^2e^2+3cdef\right.\right.$$

```
[Out] a*c^3*e^3*x+1/3*c^2*e^2*(b*c*e+3*a*(c*f+d*e))*x^3+3/5*c*e*(b*c*e*(c*f+d*e)+
a*(c^2*f^2+3*c*d*e*f+d^2*e^2))*x^5+1/7*(3*b*c*e*(c^2*f^2+3*c*d*e*f+d^2*e^2)
+a*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*x^7+1/9*(3*a*d*f*(c^2*f^2
+3*c*d*e*f+d^2*e^2)+b*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*x^9+3/
11*d*f*(a*d*f*(c*f+d*e)+b*(c^2*f^2+3*c*d*e*f+d^2*e^2))*x^11+1/13*d^2*f^2*(a
*d*f+3*b*(c*f+d*e))*x^13+1/15*b*d^3*f^3*x^15
```

Rubi [A]
time = 0.21, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {535}

$$\frac{3}{11}dfx^{11}\left(adf(cf+de)+b\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{3}{5}cex^5\left(a\left(c^2f^2+3cdef+d^2e^2\right)+bce(cf+de)\right)+\frac{1}{3}c^2e^2x^3(3a(cf+de)+bce)+\frac{1}{9}x^9\left(3adf\left(c^2f^2+3cdef+d^2e^2\right)+b\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)\right)+\frac{1}{7}x^7\left(a\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)+3bce\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{1}{13}d^2f^2x^{13}(adf+3b(cf+de))+ac^3e^3x+\frac{1}{15}bd^3f^3x^{15}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3,x]
```

```
[Out] a*c^3*e^3*x + (c^2*e^2*(b*c*e + 3*a*(d*e + c*f))*x^3)/3 + (3*c*e*(b*c*e*(d*
e + c*f) + a*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^5)/5 + ((3*b*c*e*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2) + a*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^
3))*x^7)/7 + ((3*a*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + b*(d^3*e^3 + 9*c*d
^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^9)/9 + (3*d*f*(a*d*f*(d*e + c*f) + b
*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^11)/11 + (d^2*f^2*(a*d*f + 3*b*(d*e + c
*f))*x^13)/13 + (b*d^3*f^3*x^15)/15
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\int (a+bx^2)\left(c+dx^2\right)^3\left(e+fx^2\right)^3 dx = \int \left(ac^3e^3+c^2e^2(bce+3a(de+cf))x^2+3ce\left(bce(de+cf)+a\left(d^2e\right.\right.\right.$$

$$= ac^3e^3x+\frac{1}{3}c^2e^2(bce+3a(de+cf))x^3+\frac{3}{5}ce\left(bce(de+cf)+a\left(d^2e\right.\right.$$

Mathematica [A]
time = 0.09, size = 310, normalized size = 1.00

$$ac^3e^3x+\frac{1}{3}c^2e^2(bce+3a(de+cf))x^3+\frac{3}{5}ce\left(bce(de+cf)+a\left(d^2e^2+3cdef+c^2f^2\right)\right)x^5+\frac{1}{7}\left(3bce\left(d^2e^2+3cdef+c^2f^2\right)+a\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^7+\frac{1}{9}\left(3adf\left(d^2e^2+3cdef+c^2f^2\right)+b\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^9+\frac{3}{11}df\left(adf(de+cf)+b\left(d^2e^2+3cdef+c^2f^2\right)\right)x^{11}+\frac{1}{13}d^2f^2(adf+3b(de+cf))x^{13}+\frac{1}{15}bd^3f^3x^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3,x]
```

```
[Out] a*c^3*e^3*x + (c^2*e^2*(b*c*e + 3*a*(d*e + c*f))*x^3)/3 + (3*c*e*(b*c*e*(d*
e + c*f) + a*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^5)/5 + ((3*b*c*e*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2) + a*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^
3))*x^7)/7 + ((3*a*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + b*(d^3*e^3 + 9*c*d
^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^9)/9 + (3*d*f*(a*d*f*(d*e + c*f) + b
*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^11)/11 + (d^2*f^2*(a*d*f + 3*b*(d*e + c
*f))*x^13)/13 + (b*d^3*f^3*x^15)/15
```

Maple [A]
time = 0.17, size = 339, normalized size = 1.09

method	result
default	$\frac{b d^3 f^3 x^{15}}{15}+\frac{((a d^3+3bc d^2) f^3+3b d^3 e f^2) x^{13}}{13}+\frac{((3ac d^2+3b c^2 d) f^3+3(a d^3+3bc d^2) e f^2+3b d^3 e^2 f) x^{11}}{11}+\frac{((3a c^2 d+b c^3) f^3+3($
norman	$a c^3 e^3 x+\left(c^3 a e^2 f+a c^2 d e^3+\frac{1}{3} b c^3 e^3\right) x^3+\left(\frac{3}{5} c^3 a e f^2+\frac{9}{5} a c^2 d e^2 f+\frac{3}{5} ac d^2 e^3+\frac{3}{5} b c^3 e^2 f+\frac{3}{5} b c^2$
gosper	$\frac{9}{5} x^5 a c^2 d e^2 f+\frac{9}{7} x^7 a c^2 de f^2+\frac{9}{7} x^7 ac d^2 e^2 f+\frac{9}{7} x^7 b c^2 d e^2 f+x^9 ac d^2 e f^2+x^9 b c^2 de f^2+x^9 bc d^2 e^2$
risch	$\frac{9}{5} x^5 a c^2 d e^2 f+\frac{9}{7} x^7 a c^2 de f^2+\frac{9}{7} x^7 ac d^2 e^2 f+\frac{9}{7} x^7 b c^2 d e^2 f+x^9 ac d^2 e f^2+x^9 b c^2 de f^2+x^9 bc d^2 e^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*b*d^3*f^3*x^15+1/13*((a*d^3+3*b*c*d^2)*f^3+3*b*d^3*e*f^2)*x^13+1/11*((
3*a*c*d^2+3*b*c^2*d)*f^3+3*(a*d^3+3*b*c*d^2)*e*f^2+3*b*d^3*e^2*f)*x^11+1/9*
((3*a*c^2*d+b*c^3)*f^3+3*(3*a*c*d^2+3*b*c^2*d)*e*f^2+3*(a*d^3+3*b*c*d^2)*e^
2*f+b*d^3*e^3)*x^9+1/7*(c^3*a*f^3+3*(3*a*c^2*d+b*c^3)*e*f^2+3*(3*a*c*d^2+3*
b*c^2*d)*e^2*f+(a*d^3+3*b*c*d^2)*e^3)*x^7+1/5*(3*c^3*a*e*f^2+3*(3*a*c^2*d+b
*c^3)*e^2*f+(3*a*c*d^2+3*b*c^2*d)*e^3)*x^5+1/3*(3*c^3*a*e^2*f+(3*a*c^2*d+b*
c^3)*e^3)*x^3+a*c^3*e^3*x
```

Maxima [A]
time = 0.28, size = 333, normalized size = 1.07

$$\frac{1}{15}bd^3f^3x^{15}+\frac{1}{13}\left(3bd^3f^2e+\left(3bcd^2+ad^3\right)f^3\right)x^{13}+\frac{3}{11}\left(bd^3fe^2+\left(bc^2d+acd^2\right)f^3+\left(3bcd^2e+ad^3e\right)f^2\right)x^{11}+\frac{1}{9}\left(bd^3e^3+\left(bc^3+3ac^2d\right)f^3+9\left(bc^2de+acd^2e\right)f^2+3\left(3bcd^2e^2+ad^3e^2\right)f\right)x^9+\frac{1}{7}\left(ac^3f^3+3bcd^2e^3+ad^3e^3+3\left(bc^3e+3ac^2de\right)f^2+9\left(bc^2de^2+acd^2e^2\right)f\right)x^7+\frac{3}{5}\left(ac^3f^2e+bc^2de^3+acd^2e^3+\left(bc^3e^2+3ac^2de^2\right)f\right)x^5+ac^3xe^3+\frac{1}{3}\left(3ac^3fe^2+bc^3e^3+3ac^2de^3\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b*d^3*f^3*x^15 + 1/13*(3*b*d^3*f^2*e + (3*b*c*d^2 + a*d^3)*f^3)*x^13 +
 3/11*(b*d^3*f*e^2 + (b*c^2*d + a*c*d^2)*f^3 + (3*b*c*d^2*e + a*d^3*e)*f^2)
*x^11 + 1/9*(b*d^3*e^3 + (b*c^3 + 3*a*c^2*d)*f^3 + 9*(b*c^2*d*e + a*c*d^2*e
)*f^2 + 3*(3*b*c*d^2*e^2 + a*d^3*e^2)*f)*x^9 + 1/7*(a*c^3*f^3 + 3*b*c*d^2*e
^3 + a*d^3*e^3 + 3*(b*c^3*e + 3*a*c^2*d*e)*f^2 + 9*(b*c^2*d*e^2 + a*c*d^2*e
^2)*f)*x^7 + 3/5*(a*c^3*f^2*e + b*c^2*d*e^3 + a*c*d^2*e^3 + (b*c^3*e^2 + 3*
a*c^2*d*e^2)*f)*x^5 + a*c^3*x*e^3 + 1/3*(3*a*c^3*f*e^2 + b*c^3*e^3 + 3*a*c^
2*d*e^3)*x^3
```

Fricas [**A**]
time = 0.81, size = 337, normalized size = 1.09

$$\frac{1}{15}bd^3f^3x^{15}+\frac{1}{13}(3bcd^2+ad^3)f^3x^{13}+\frac{3}{11}(bc^2d+acd^2)f^3x^{11}+\frac{1}{7}ac^3f^3x^7+\frac{1}{9}(bc^3+3ac^2d)f^3x^9+\frac{1}{315}(35bd^3x^9+45(3bcd^2+ad^3)x^7+189(bc^2d+acd^2)x^5+315ac^3x+105(bc^3+3ac^2d)x^3)e^3+\frac{1}{1155}(315bd^3fx^{11}+385(3bcd^2+ad^3)fx^9+1485(bc^2d+acd^2)fx^7+1155ac^3fx^3+693(bc^3+3ac^2d)fx^5)e^2+\frac{1}{5005}(1155bd^3f^2x^{13}+1365(3bcd^2+ad^3)f^2x^{11}+5005(bc^2d+acd^2)f^2x^9+3003ac^3f^2x^5+2145(bc^3+3ac^2d)f^2x^7)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] 1/15*b*d^3*f^3*x^15 + 1/13*(3*b*c*d^2 + a*d^3)*f^3*x^13 + 3/11*(b*c^2*d + a
*c*d^2)*f^3*x^11 + 1/7*a*c^3*f^3*x^7 + 1/9*(b*c^3 + 3*a*c^2*d)*f^3*x^9 + 1/
315*(35*b*d^3*x^9 + 45*(3*b*c*d^2 + a*d^3)*x^7 + 189*(b*c^2*d + a*c*d^2)*x^
5 + 315*a*c^3*x + 105*(b*c^3 + 3*a*c^2*d)*x^3)*e^3 + 1/1155*(315*b*d^3*f*x^
11 + 385*(3*b*c*d^2 + a*d^3)*f*x^9 + 1485*(b*c^2*d + a*c*d^2)*f*x^7 + 1155*
a*c^3*f*x^3 + 693*(b*c^3 + 3*a*c^2*d)*f*x^5)*e^2 + 1/5005*(1155*b*d^3*f^2*x
^13 + 1365*(3*b*c*d^2 + a*d^3)*f^2*x^11 + 5005*(b*c^2*d + a*c*d^2)*f^2*x^9
+ 3003*a*c^3*f^2*x^5 + 2145*(b*c^3 + 3*a*c^2*d)*f^2*x^7)*e
```

Sympy [**A**]
time = 0.03, size = 423, normalized size = 1.36

$$ac^3e^3x+\frac{bd^3f^3x^{15}}{15}+x^{13}\left(\frac{ad^3f^3}{13}+\frac{3bcd^2f^3}{13}+\frac{3bd^3ef^2}{13}\right)+x^{11}\cdot\left(\frac{3acd^2f^3}{11}+\frac{3ad^3ef^2}{11}+\frac{3bc^2df^3}{11}+\frac{9bcd^2ef^2}{11}+\frac{3bd^3e^2f}{11}\right)+x^9\left(\frac{ac^2df^3}{3}+acd^2ef^2+\frac{ad^3e^2f}{3}+\frac{bc^3f^3}{9}+bc^2def^2+bcd^2e^2f+\frac{bd^3e^3}{9}\right)+x^7\left(\frac{ac^3f^3}{7}+\frac{9ac^2def^2}{7}+\frac{9acd^2e^2f}{7}+\frac{ad^3e^3}{7}+\frac{3bc^3ef^2}{7}+\frac{9bc^2de^2f}{7}+\frac{3bcd^2e^3}{7}\right)+x^5\cdot\left(\frac{3ac^3ef^2}{5}+\frac{9ac^2de^2f}{5}+\frac{3acd^2e^3}{5}+\frac{3bc^3e^2f}{5}+\frac{3bc^2de^3}{5}\right)+x^3\left(ac^3e^2f+ac^2de^3+\frac{bc^3e^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e)**3,x)
```

```
[Out] a*c**3*e**3*x + b*d**3*f**3*x**15/15 + x**13*(a*d**3*f**3/13 + 3*b*c*d**2*f
**3/13 + 3*b*d**3*e*f**2/13) + x**11*(3*a*c*d**2*f**3/11 + 3*a*d**3*e*f**2/
11 + 3*b*c**2*d*f**3/11 + 9*b*c*d**2*e*f**2/11 + 3*b*d**3*e**2*f/11) + x**9
*(a*c**2*d*f**3/3 + a*c*d**2*e*f**2 + a*d**3*e**2*f/3 + b*c**3*f**3/9 + b*c
**2*d*e*f**2 + b*c*d**2*e**2*f + b*d**3*e**3/9) + x**7*(a*c**3*f**3/7 + 9*a
*c**2*d*e*f**2/7 + 9*a*c*d**2*e**2*f/7 + a*d**3*e**3/7 + 3*b*c**3*e*f**2/7
+ 9*b*c**2*d*e**2*f/7 + 3*b*c*d**2*e**3/7) + x**5*(3*a*c**3*e*f**2/5 + 9*a*
c**2*d*e**2*f/5 + 3*a*c*d**2*e**3/5 + 3*b*c**3*e**2*f/5 + 3*b*c**2*d*e**3/5
) + x**3*(a*c**3*e**2*f + a*c**2*d*e**3 + b*c**3*e**3/3)
```

Giac [**A**]
time = 0.55, size = 401, normalized size = 1.29

$$\frac{1}{15}bd^3f^3x^{15}+\frac{3}{13}bcd^2f^3x^{13}+\frac{1}{13}ad^3f^3x^{13}+\frac{3}{13}bd^3f^2x^{13}e+\frac{3}{11}bc^2df^3x^{11}+\frac{3}{11}acd^2f^3x^{11}+\frac{9}{11}bcd^2f^2x^{11}e+\frac{3}{11}ad^3f^2x^{11}e+\frac{3}{11}bd^3fx^{11}e^2+\frac{1}{9}bc^3f^3x^9+\frac{1}{3}ac^2df^3x^9+bc^2df^2x^9e+acd^2f^2x^9e+bcd^2fx^9e^2+\frac{1}{3}ad^3fx^9e^2+\frac{1}{7}ac^3f^3x^7+\frac{1}{9}bd^3x^9e^3+\frac{3}{7}bc^3f^2x^7e+\frac{9}{7}ac^2df^2x^7e+\frac{9}{7}bc^2dfx^7e^2+\frac{9}{7}acd^2fx^7e^2+\frac{3}{7}bcd^2x^7e^3+\frac{1}{7}ad^3x^7e^3+\frac{3}{5}ac^3f^2x^5e+\frac{3}{5}bc^3fx^5e^2+\frac{9}{5}ac^2dfx^5e^2+\frac{3}{5}bc^2dx^5e^3+\frac{3}{5}acd^2x^5e^3+ac^3fx^3e^2+\frac{1}{3}bc^3x^3e^3+ac^2dx^3e^3+ac^3xe^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] 1/15*b*d^3*f^3*x^15 + 3/13*b*c*d^2*f^3*x^13 + 1/13*a*d^3*f^3*x^13 + 3/13*b*
d^3*f^2*x^13*e + 3/11*b*c^2*d*f^3*x^11 + 3/11*a*c*d^2*f^3*x^11 + 9/11*b*c*d
^2*f^2*x^11*e + 3/11*a*d^3*f^2*x^11*e + 3/11*b*d^3*f*x^11*e^2 + 1/9*b*c^3*f
^3*x^9 + 1/3*a*c^2*d*f^3*x^9 + b*c^2*d*f^2*x^9*e + a*c*d^2*f^2*x^9*e + b*c*
d^2*f*x^9*e^2 + 1/3*a*d^3*f*x^9*e^2 + 1/7*a*c^3*f^3*x^7 + 1/9*b*d^3*x^9*e^3
 + 3/7*b*c^3*f^2*x^7*e + 9/7*a*c^2*d*f^2*x^7*e + 9/7*b*c^2*d*f*x^7*e^2 + 9/
7*a*c*d^2*f*x^7*e^2 + 3/7*b*c*d^2*x^7*e^3 + 1/7*a*d^3*x^7*e^3 + 3/5*a*c^3*f
^2*x^5*e + 3/5*b*c^3*f*x^5*e^2 + 9/5*a*c^2*d*f*x^5*e^2 + 3/5*b*c^2*d*x^5*e^
3 + 3/5*a*c*d^2*x^5*e^3 + a*c^3*f*x^3*e^2 + 1/3*b*c^3*x^3*e^3 + a*c^2*d*x^3
*e^3 + a*c^3*x*e^3
```

Mupad [B]
time = 0.12, size = 335, normalized size = 1.08

$$x^5\left(\frac{3bc^3e^2f}{5}+\frac{3ac^3ef^2}{5}+\frac{3bc^2de^3}{5}+\frac{9ac^2de^2f}{5}+\frac{3acd^2e^3}{5}\right)+x^{11}\left(\frac{3bc^2df^3}{11}+\frac{9bcd^2ef^2}{11}+\frac{3acd^2f^3}{11}+\frac{3bd^3e^2f}{11}+\frac{3ad^3ef^2}{11}\right)+x^7\left(\frac{3bc^3ef^2}{7}+\frac{ac^3f^3}{7}+\frac{9bc^2de^2f}{7}+\frac{9ac^2def^2}{7}+\frac{3bcd^2e^3}{7}+\frac{9acd^2e^2f}{7}+\frac{ad^3e^3}{7}\right)+x^9\left(\frac{bc^3f^3}{9}+bc^2def^2+\frac{ac^2df^3}{3}+bcd^2e^2f+acd^2ef^2+\frac{bd^3e^3}{9}+\frac{ad^3e^2f}{3}\right)+\frac{bd^3f^3x^{15}}{15}+\frac{c^2e^2x^3(3acf+3ade+bce)}{3}+\frac{d^2f^2x^{13}(adf+3bcf+3bde)}{13}+ac^3e^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3,x)
```

```
[Out] x^5*((3*a*c*d^2*e^3)/5 + (3*b*c^2*d*e^3)/5 + (3*a*c^3*e*f^2)/5 + (3*b*c^3*e
^2*f)/5 + (9*a*c^2*d*e^2*f)/5) + x^11*((3*a*c*d^2*f^3)/11 + (3*b*c^2*d*f^3)
/11 + (3*a*d^3*e*f^2)/11 + (3*b*d^3*e^2*f)/11 + (9*b*c*d^2*e*f^2)/11) + x^7
*((a*c^3*f^3)/7 + (a*d^3*e^3)/7 + (3*b*c*d^2*e^3)/7 + (3*b*c^3*e*f^2)/7 + (
9*a*c*d^2*e^2*f)/7 + (9*a*c^2*d*e*f^2)/7 + (9*b*c^2*d*e^2*f)/7) + x^9*((b*c
^3*f^3)/9 + (b*d^3*e^3)/9 + (a*c^2*d*f^3)/3 + (a*d^3*e^2*f)/3 + a*c*d^2*e*f
^2 + b*c*d^2*e^2*f + b*c^2*d*e*f^2) + (b*d^3*f^3*x^15)/15 + (c^2*e^2*x^3*(3
*a*c*f + 3*a*d*e + b*c*e))/3 + (d^2*f^2*x^13*(a*d*f + 3*b*c*f + 3*b*d*e))/1
3 + a*c^3*e^3*x
```

3.17 $\int (a+bx^2)(c+dx^2)^3(e+fx^2)^2 \, dx$

Optimal. Leaf size=226

$$ac^3e^2x+\frac{1}{3}c^2e(bce+3ade+2acf)x^3+\frac{1}{5}c\left(bce(3de+2cf)+a\left(3d^2e^2+6cdef+c^2f^2\right)\right)x^5+\frac{1}{7}\left(bc\left(3d^2e^2+6cdef\right.\right.$$

```
[Out] a*c^3*e^2*x+1/3*c^2*e*(2*a*c*f+3*a*d*e+b*c*e)*x^3+1/5*c*(b*c*e*(2*c*f+3*d*e
)+a*(c^2*f^2+6*c*d*e*f+3*d^2*e^2))*x^5+1/7*(b*c*(c^2*f^2+6*c*d*e*f+3*d^2*e^
2)+a*d*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x^7+1/9*d*(a*d*f*(3*c*f+2*d*e)+b*(3*c
^2*f^2+6*c*d*e*f+d^2*e^2))*x^9+1/11*d^2*f*(a*d*f+3*b*c*f+2*b*d*e)*x^11+1/13
*b*d^3*f^2*x^13
```

Rubi [A]
time = 0.15, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{535\}$

$$\frac{1}{9}dx^9\left(adf(3cf+2de)+b\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{7}x^7\left(ad\left(3c^2f^2+6cdef+d^2e^2\right)+bc\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{5}cx^5\left(a\left(c^2f^2+6cdef+3d^2e^2\right)+bce(2cf+3de)\right)+\frac{1}{3}c^2ex^3(2acf+3ade+bce)+\frac{1}{11}d^2fx^{11}(adf+3bcf+2bde)+ac^3e^2x+\frac{1}{13}bd^3f^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^2,x]
```

```
[Out] a*c^3*e^2*x + (c^2*e*(b*c*e + 3*a*d*e + 2*a*c*f)*x^3)/3 + (c*(b*c*e*(3*d*e
+ 2*c*f) + a*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^5)/5 + ((b*c*(3*d^2*e^2 +
 6*c*d*e*f + c^2*f^2) + a*d*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 + (d*
(a*d*f*(2*d*e + 3*c*f) + b*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^9)/9 + (d^2
*f*(2*b*d*e + 3*b*c*f + a*d*f)*x^11)/11 + (b*d^3*f^2*x^13)/13
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)^2 \, dx &= \int \left(ac^3e^2+c^2e(bce+3ade+2acf)x^2+c\left(bce(3de+2cf)+a\left(3d^2\right.\right.\right.\\ &= ac^3e^2x+\frac{1}{3}c^2e(bce+3ade+2acf)x^3+\frac{1}{5}c\left(bce(3de+2cf)+a\left(3d\right.\right.\end{aligned}$$

Mathematica [A]
time = 0.06, size = 226, normalized size = 1.00

$$ac^3e^2x+\frac{1}{3}c^2e(bce+3ade+2acf)x^3+\frac{1}{5}c\left(bce(3de+2cf)+a\left(3d^2e^2+6cdef+c^2f^2\right)\right)x^5+\frac{1}{7}\left(bc\left(3d^2e^2+6cdef+c^2f^2\right)+ad\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^7+\frac{1}{9}d\left(adf(2de+3cf)+b\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^9+\frac{1}{11}d^2f(2bde+3bcf+adf)x^{11}+\frac{1}{13}bd^3f^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^2,x]
```

```
[Out] a*c^3*e^2*x + (c^2*e*(b*c*e + 3*a*d*e + 2*a*c*f)*x^3)/3 + (c*(b*c*e*(3*d*e
 + 2*c*f) + a*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^5)/5 + ((b*c*(3*d^2*e^2 +
 6*c*d*e*f + c^2*f^2) + a*d*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 + (d*
(a*d*f*(2*d*e + 3*c*f) + b*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^9)/9 + (d^2
*f*(2*b*d*e + 3*b*c*f + a*d*f)*x^11)/11 + (b*d^3*f^2*x^13)/13
```

Maple [A]
time = 0.17, size = 244, normalized size = 1.08

method	result
default	$\frac{b\,d^3f^2x^{13}}{13}+\frac{\left(\left(a\,d^3+3bc\,d^2\right)f^2+2b\,d^3fe\right)x^{11}}{11}+\frac{\left(\left(3ac\,d^2+3b\,c^2d\right)f^2+2\left(a\,d^3+3bc\,d^2\right)fe+b\,d^3e^2\right)x^9}{9}+\frac{\left(\left(3a\,c^2d+b\,c^3\right)f^2+2\left(3ac\,d^2\right.\right.}{}$
norman	$\frac{b\,d^3f^2x^{13}}{13}+\left(\frac{1}{11}a\,d^3f^2+\frac{3}{11}bc\,d^2f^2+\frac{2}{11}b\,d^3fe\right)x^{11}+\left(\frac{1}{3}ac\,d^2f^2+\frac{2}{9}a\,d^3ef+\frac{1}{3}b\,c^2d\,f^2+\frac{2}{3}bc\,d^2ef+\right.$
gosper	$\frac{1}{13}b\,d^3f^2x^{13}+\frac{1}{11}x^{11}a\,d^3f^2+\frac{3}{11}x^{11}bc\,d^2f^2+\frac{2}{11}x^{11}b\,d^3fe+\frac{1}{3}x^9ac\,d^2f^2+\frac{2}{9}x^9a\,d^3ef+\frac{1}{3}x^9b\,c^2d\,f^2-$
risch	$\frac{1}{13}b\,d^3f^2x^{13}+\frac{1}{11}x^{11}a\,d^3f^2+\frac{3}{11}x^{11}bc\,d^2f^2+\frac{2}{11}x^{11}b\,d^3fe+\frac{1}{3}x^9ac\,d^2f^2+\frac{2}{9}x^9a\,d^3ef+\frac{1}{3}x^9b\,c^2d\,f^2-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b*d^3*f^2*x^13+1/11*((a*d^3+3*b*c*d^2)*f^2+2*b*d^3*f*e)*x^11+1/9*((3*a
*c*d^2+3*b*c^2*d)*f^2+2*(a*d^3+3*b*c*d^2)*f*e+b*d^3*e^2)*x^9+1/7*((3*a*c^2*
d+b*c^3)*f^2+2*(3*a*c*d^2+3*b*c^2*d)*f*e+(a*d^3+3*b*c*d^2)*e^2)*x^7+1/5*(c^
3*a*f^2+2*(3*a*c^2*d+b*c^3)*f*e+(3*a*c*d^2+3*b*c^2*d)*e^2)*x^5+1/3*(2*c^3*a
*f*e+(3*a*c^2*d+b*c^3)*e^2)*x^3+a*c^3*e^2*x
```

Maxima [A]
time = 0.28, size = 246, normalized size = 1.09

$$\frac{1}{13}bd^3f^2x^{13}+\frac{1}{11}\left(2bd^3fe+\left(3bcd^2+ad^3\right)f^2\right)x^{11}+\frac{1}{9}\left(bd^3e^2+3\left(bc^2d+acd^2\right)f^2+2\left(3bcd^2e+ad^3e\right)f\right)x^9+\frac{1}{7}\left(3bcd^2e^2+ad^3e^2+\left(bc^3+3ac^2d\right)f^2+6\left(bc^2de+acd^2e\right)f\right)x^7+\frac{1}{5}\left(ac^3f^2+3bc^2de^2+3acd^2e^2+2\left(bc^3e+3ac^2de\right)f\right)x^5+ac^3xe^2+\frac{1}{3}\left(2ac^3fe+bc^3e^2+3ac^2de^2\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^3*f^2*x^13 + 1/11*(2*b*d^3*f*e + (3*b*c*d^2 + a*d^3)*f^2)*x^11 + 1
/9*(b*d^3*e^2 + 3*(b*c^2*d + a*c*d^2)*f^2 + 2*(3*b*c*d^2*e + a*d^3*e)*f)*x^
9 + 1/7*(3*b*c*d^2*e^2 + a*d^3*e^2 + (b*c^3 + 3*a*c^2*d)*f^2 + 6*(b*c^2*d*e
```

```
 + a*c*d^2*e)*f)*x^7 + 1/5*(a*c^3*f^2 + 3*b*c^2*d*e^2 + 3*a*c*d^2*e^2 + 2*(
b*c^3*e + 3*a*c^2*d*e)*f)*x^5 + a*c^3*x*e^2 + 1/3*(2*a*c^3*f*e + b*c^3*e^2
+ 3*a*c^2*d*e^2)*x^3
```

Fricas [A]
time = 1.06, size = 245, normalized size = 1.08

$$\frac{1}{13}\,bd^3f^2x^{13}+\frac{1}{11}\,\left(3\,bcd^2+ad^3\right)f^2x^{11}+\frac{1}{3}\,\left(bc^2d+acd^2\right)f^2x^9+\frac{1}{5}\,ac^3f^2x^5+\frac{1}{7}\,\left(bc^3+3\,ac^2d\right)f^2x^7+\frac{1}{315}\,\left(35\,bd^3x^9+45\,\left(3\,bcd^2+ad^3\right)x^7+189\,\left(bc^2d+acd^2\right)x^5+315\,ac^3x+105\,\left(bc^3+3\,ac^2d\right)x^3\right)e^2+\frac{2}{3465}\,\left(315\,bd^3fx^{11}+385\,\left(3\,bcd^2+ad^3\right)fx^9+1485\,\left(bc^2d+acd^2\right)fx^7+1155\,ac^3fx^3+693\,\left(bc^3+3\,ac^2d\right)fx^5\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x, algorithm="fricas")
```

```
[Out] 1/13*b*d^3*f^2*x^13 + 1/11*(3*b*c*d^2 + a*d^3)*f^2*x^11 + 1/3*(b*c^2*d + a*
c*d^2)*f^2*x^9 + 1/5*a*c^3*f^2*x^5 + 1/7*(b*c^3 + 3*a*c^2*d)*f^2*x^7 + 1/31
5*(35*b*d^3*x^9 + 45*(3*b*c*d^2 + a*d^3)*x^7 + 189*(b*c^2*d + a*c*d^2)*x^5
+ 315*a*c^3*x + 105*(b*c^3 + 3*a*c^2*d)*x^3)*e^2 + 2/3465*(315*b*d^3*f*x^11
 + 385*(3*b*c*d^2 + a*d^3)*f*x^9 + 1485*(b*c^2*d + a*c*d^2)*f*x^7 + 1155*a*
c^3*f*x^3 + 693*(b*c^3 + 3*a*c^2*d)*f*x^5)*e
```

Sympy [A]
time = 0.03, size = 304, normalized size = 1.35

$$ac^3e^2x+\frac{bd^3f^2x^{13}}{13}+x^{11}\left(\frac{ad^3f^2}{11}+\frac{3bcd^2f^2}{11}+\frac{2bd^3ef}{11}\right)+x^9\left(\frac{acd^2f^2}{3}+\frac{2ad^3ef}{9}+\frac{bc^2df^2}{3}+\frac{2bcd^2ef}{3}+\frac{bd^3e^2}{9}\right)+x^7\cdot\left(\frac{3ac^2df^2}{7}+\frac{6acd^2ef}{7}+\frac{ad^3e^2}{7}+\frac{bc^3f^2}{7}+\frac{6bc^2def}{7}+\frac{3bcd^2e^2}{7}\right)+x^5\left(\frac{ac^3f^2}{5}+\frac{6ac^2def}{5}+\frac{3acd^2e^2}{5}+\frac{2bc^3ef}{5}+\frac{3bc^2de^2}{5}\right)+x^3\cdot\left(\frac{2ac^3ef}{3}+ac^2de^2+\frac{bc^3e^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e)**2,x)
```

```
[Out] a*c**3*e**2*x + b*d**3*f**2*x**13/13 + x**11*(a*d**3*f**2/11 + 3*b*c*d**2*f
**2/11 + 2*b*d**3*e*f/11) + x**9*(a*c*d**2*f**2/3 + 2*a*d**3*e*f/9 + b*c**2
*d*f**2/3 + 2*b*c*d**2*e*f/3 + b*d**3*e**2/9) + x**7*(3*a*c**2*d*f**2/7 + 6
*a*c*d**2*e*f/7 + a*d**3*e**2/7 + b*c**3*f**2/7 + 6*b*c**2*d*e*f/7 + 3*b*c*
d**2*e**2/7) + x**5*(a*c**3*f**2/5 + 6*a*c**2*d*e*f/5 + 3*a*c*d**2*e**2/5 +
 2*b*c**3*e*f/5 + 3*b*c**2*d*e**2/5) + x**3*(2*a*c**3*e*f/3 + a*c**2*d*e**2
 + b*c**3*e**2/3)
```

Giac [A]
time = 0.58, size = 289, normalized size = 1.28

$$\frac{1}{13}\,bd^3f^2x^{13}+\frac{3}{11}\,bcd^2f^2x^{11}+\frac{1}{11}\,ad^3f^2x^{11}+\frac{2}{11}\,bd^3fx^{11}e+\frac{1}{3}\,bc^2df^2x^9+\frac{1}{3}\,acd^2f^2x^9+\frac{2}{3}\,bcd^2fx^9e+\frac{2}{9}\,ad^3fx^9e+\frac{1}{9}\,bd^3x^9e^2+\frac{1}{7}\,bc^3f^2x^7+\frac{3}{7}\,ac^2df^2x^7+\frac{6}{7}\,bc^2dfx^7e+\frac{6}{7}\,acd^2fx^7e+\frac{3}{7}\,bcd^2x^7e^2+\frac{1}{7}\,ad^3x^7e^2+\frac{1}{5}\,ac^3f^2x^5+\frac{2}{5}\,bc^3fx^5e+\frac{6}{5}\,ac^2dfx^5e+\frac{3}{5}\,bc^2dx^5e^2+\frac{3}{5}\,acd^2x^5e^2+\frac{2}{3}\,ac^3fx^3e+\frac{1}{3}\,bc^3x^3e^2+ac^2dx^3e^2+ac^3xe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] 1/13*b*d^3*f^2*x^13 + 3/11*b*c*d^2*f^2*x^11 + 1/11*a*d^3*f^2*x^11 + 2/11*b*
d^3*f*x^11*e + 1/3*b*c^2*d*f^2*x^9 + 1/3*a*c*d^2*f^2*x^9 + 2/3*b*c*d^2*f*x^
9*e + 2/9*a*d^3*f*x^9*e + 1/9*b*d^3*x^9*e^2 + 1/7*b*c^3*f^2*x^7 + 3/7*a*c^2
```

```
*d*f^2*x^7 + 6/7*b*c^2*d*f*x^7*e + 6/7*a*c*d^2*f*x^7*e + 3/7*b*c*d^2*x^7*e^
2 + 1/7*a*d^3*x^7*e^2 + 1/5*a*c^3*f^2*x^5 + 2/5*b*c^3*f*x^5*e + 6/5*a*c^2*d
*f*x^5*e + 3/5*b*c^2*d*x^5*e^2 + 3/5*a*c*d^2*x^5*e^2 + 2/3*a*c^3*f*x^3*e +
1/3*b*c^3*x^3*e^2 + a*c^2*d*x^3*e^2 + a*c^3*x*e^2
```

Mupad [B]
time = 0.85, size = 233, normalized size = 1.03

$$x^5\left(\frac{2bc^3ef}{5}+\frac{ac^3f^2}{5}+\frac{3bc^2de^2}{5}+\frac{6ac^2def}{5}+\frac{3acd^2e^2}{5}\right)+x^9\left(\frac{bc^2df^2}{3}+\frac{2bcd^2ef}{3}+\frac{acd^2f^2}{3}+\frac{bd^3e^2}{9}+\frac{2ad^3ef}{9}\right)+x^7\left(\frac{bc^3f^2}{7}+\frac{6bc^2def}{7}+\frac{3ac^2df^2}{7}+\frac{3bcd^2e^2}{7}+\frac{6acd^2ef}{7}+\frac{ad^3e^2}{7}\right)+\frac{bd^3f^2x^{13}}{13}+\frac{c^2ex^3\left(2acf+3ade+bce\right)}{3}+\frac{d^2fx^{11}\left(adf+3bcf+2bde\right)}{11}+ac^3e^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^2,x)
```

```
[Out] x^5*((a*c^3*f^2)/5 + (2*b*c^3*e*f)/5 + (3*a*c*d^2*e^2)/5 + (3*b*c^2*d*e^2)/
5 + (6*a*c^2*d*e*f)/5) + x^9*((b*d^3*e^2)/9 + (2*a*d^3*e*f)/9 + (a*c*d^2*f^
2)/3 + (b*c^2*d*f^2)/3 + (2*b*c*d^2*e*f)/3) + x^7*((a*d^3*e^2)/7 + (b*c^3*f
^2)/7 + (3*a*c^2*d*f^2)/7 + (3*b*c*d^2*e^2)/7 + (6*a*c*d^2*e*f)/7 + (6*b*c^
2*d*e*f)/7) + (b*d^3*f^2*x^13)/13 + (c^2*e*x^3*(2*a*c*f + 3*a*d*e + b*c*e))
/3 + (d^2*f*x^11*(a*d*f + 3*b*c*f + 2*b*d*e))/11 + a*c^3*e^2*x
```

3.18 $\int (a+bx^2)(c+dx^2)^3(e+fx^2)\,dx$

Optimal. Leaf size=130

$$ac^3ex+\frac{1}{3}c^2(bce+3ade+acf)x^3+\frac{1}{5}c(3ad(de+cf)+bc(3de+cf))x^5+\frac{1}{7}d(3bc(de+cf)+ad(de+3cf))x^7+\frac{1}{9}d^2(ba$$

```
[Out] a*c^3*e*x+1/3*c^2*(a*c*f+3*a*d*e+b*c*e)*x^3+1/5*c*(3*a*d*(c*f+d*e)+b*c*(c*f
+3*d*e))*x^5+1/7*d*(3*b*c*(c*f+d*e)+a*d*(3*c*f+d*e))*x^7+1/9*d^2*(a*d*f+3*b
*c*f+b*d*e)*x^9+1/11*b*d^3*f*x^11
```

Rubi [A]
time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {535}

$$\frac{1}{3}c^2x^3(acf+3ade+bce)+\frac{1}{9}d^2x^9(adf+3bcf+bde)+\frac{1}{7}dx^7(ad(3cf+de)+3bc(cf+de))+\frac{1}{5}cx^5(3ad(cf+de)+bc(cf+3de))+ac^3ex+\frac{1}{11}bd^3fx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2),x]
```

```
[Out] a*c^3*e*x + (c^2*(b*c*e + 3*a*d*e + a*c*f)*x^3)/3 + (c*(3*a*d*(d*e + c*f) +
 b*c*(3*d*e + c*f))*x^5)/5 + (d*(3*b*c*(d*e + c*f) + a*d*(d*e + 3*c*f))*x^7
)/7 + (d^2*(b*d*e + 3*b*c*f + a*d*f)*x^9)/9 + (b*d^3*f*x^11)/11
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)(c+dx^2)^3(e+fx^2)\,dx &= \int \left(ac^3e+c^2(bce+3ade+acf)x^2+c(3ad(de+cf)+bc(3de+cf\right.\\ &= ac^3ex+\frac{1}{3}c^2(bce+3ade+acf)x^3+\frac{1}{5}c(3ad(de+cf)+bc(3de+c\end{aligned}$$

Mathematica [A]
time = 0.04, size = 130, normalized size = 1.00

$$ac^3ex+\frac{1}{3}c^2(bce+3ade+acf)x^3+\frac{1}{5}c(3ad(de+cf)+bc(3de+cf))x^5+\frac{1}{7}d(3bc(de+cf)+ad(de+3cf))x^7+\frac{1}{9}d^2(bde+3bcf+adf)x^9+\frac{1}{11}bd^3fx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2),x]
```

```
[Out] a*c^3*e*x + (c^2*(b*c*e + 3*a*d*e + a*c*f)*x^3)/3 + (c*(3*a*d*(d*e + c*f) +
 b*c*(3*d*e + c*f))*x^5)/5 + (d*(3*b*c*(d*e + c*f) + a*d*(d*e + 3*c*f))*x^7
)/7 + (d^2*(b*d*e + 3*b*c*f + a*d*f)*x^9)/9 + (b*d^3*f*x^11)/11
```

Maple [A]
time = 0.15, size = 149, normalized size = 1.15

method	result
norman	$\frac{b\,d^{3} f\,x^{11}}{11}+\left(\frac{1}{9} a\,d^{3} f+\frac{1}{3} b c\,d^{2} f+\frac{1}{9} b\,d^{3} e\right) x^{9}+\left(\frac{3}{7} a c\,d^{2} f+\frac{1}{7} a\,d^{3} e+\frac{3}{7} b\,c^{2} d f+\frac{3}{7} b c\,d^{2} e\right) x^{7}+\left(\frac{3}{5} a\,c^{2} d f\right.$
default	$\frac{b\,d^{3} f\,x^{11}}{11}+\frac{\left(\left(a\,d^{3}+3 b c\,d^{2}\right) f+b\,d^{3} e\right) x^{9}}{9}+\frac{\left(\left(3 a c\,d^{2}+3 b\,c^{2} d\right) f+\left(a\,d^{3}+3 b c\,d^{2}\right) e\right) x^{7}}{7}+\frac{\left(\left(3 a\,c^{2} d+b\,c^{3}\right) f+\left(3 a c\,d^{2}+3 b\,c^{2} d\right) e\right) x^{5}}{5}+$
gosper	$\frac{1}{11} b\,d^{3} f\,x^{11}+\frac{1}{9} x^{9} a\,d^{3} f+\frac{1}{3} x^{9} b c\,d^{2} f+\frac{1}{9} x^{9} b\,d^{3} e+\frac{3}{7} x^{7} a c\,d^{2} f+\frac{1}{7} x^{7} a\,d^{3} e+\frac{3}{7} x^{7} b\,c^{2} d f+\frac{3}{7} x^{7} b c\,d^{2} e+$
risch	$\frac{1}{11} b\,d^{3} f\,x^{11}+\frac{1}{9} x^{9} a\,d^{3} f+\frac{1}{3} x^{9} b c\,d^{2} f+\frac{1}{9} x^{9} b\,d^{3} e+\frac{3}{7} x^{7} a c\,d^{2} f+\frac{1}{7} x^{7} a\,d^{3} e+\frac{3}{7} x^{7} b\,c^{2} d f+\frac{3}{7} x^{7} b c\,d^{2} e+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b*d^3*f*x^11+1/9*((a*d^3+3*b*c*d^2)*f+b*d^3*e)*x^9+1/7*((3*a*c*d^2+3*b
*c^2*d)*f+(a*d^3+3*b*c*d^2)*e)*x^7+1/5*((3*a*c^2*d+b*c^3)*f+(3*a*c*d^2+3*b*
c^2*d)*e)*x^5+1/3*(c^3*a*f+(3*a*c^2*d+b*c^3)*e)*x^3+a*c^3*e*x
```

Maxima [A]
time = 0.27, size = 152, normalized size = 1.17

$$\frac{1}{11}\,bd^3fx^{11}+\frac{1}{9}\left(bd^3e+\left(3\,bcd^2+ad^3\right)f\right)x^9+\frac{1}{7}\left(3\,bcd^2e+ad^3e+3\left(bc^2d+acd^2\right)f\right)x^7+\frac{1}{5}\left(3\,bc^2de+3\,acd^2e+\left(bc^3+3\,ac^2d\right)f\right)x^5+ac^3xe+\frac{1}{3}\left(ac^3f+bc^3e+3\,ac^2de\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x, algorithm="maxima")
```

```
[Out] 1/11*b*d^3*f*x^11 + 1/9*(b*d^3*e + (3*b*c*d^2 + a*d^3)*f)*x^9 + 1/7*(3*b*c*
d^2*e + a*d^3*e + 3*(b*c^2*d + a*c*d^2)*f)*x^7 + 1/5*(3*b*c^2*d*e + 3*a*c*d
^2*e + (b*c^3 + 3*a*c^2*d)*f)*x^5 + a*c^3*x*e + 1/3*(a*c^3*f + b*c^3*e + 3*
a*c^2*d*e)*x^3
```

Fricas [A]
time = 1.13, size = 153, normalized size = 1.18

$$\frac{1}{11}\,bd^3fx^{11}+\frac{1}{9}\left(3\,bcd^2+ad^3\right)fx^9+\frac{3}{7}\left(bc^2d+acd^2\right)fx^7+\frac{1}{3}\,ac^3fx^3+\frac{1}{5}\left(bc^3+3\,ac^2d\right)fx^5+\frac{1}{315}\left(35\,bd^3x^9+45\left(3\,bcd^2+ad^3\right)x^7+189\left(bc^2d+acd^2\right)x^5+315\,ac^3x+105\left(bc^3+3\,ac^2d\right)x^3\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x, algorithm="fricas")
```

```
[Out] 1/11*b*d^3*f*x^11 + 1/9*(3*b*c*d^2 + a*d^3)*f*x^9 + 3/7*(b*c^2*d + a*c*d^2)
*f*x^7 + 1/3*a*c^3*f*x^3 + 1/5*(b*c^3 + 3*a*c^2*d)*f*x^5 + 1/315*(35*b*d^3*
x^9 + 45*(3*b*c*d^2 + a*d^3)*x^7 + 189*(b*c^2*d + a*c*d^2)*x^5 + 315*a*c^3*
x + 105*(b*c^3 + 3*a*c^2*d)*x^3)*e
```

Sympy [A]
time = 0.02, size = 173, normalized size = 1.33

$$ac^3ex + \frac{bd^3fx^{11}}{11} + x^9\left(\frac{ad^3f}{9} + \frac{bcd^2f}{3} + \frac{bd^3e}{9}\right) + x^7 \cdot \left(\frac{3acd^2f}{7} + \frac{ad^3e}{7} + \frac{3bc^2df}{7} + \frac{3bcd^2e}{7}\right) + x^5 \cdot \left(\frac{3ac^2df}{5} + \frac{3acd^2e}{5} + \frac{bc^3f}{5} + \frac{3bc^2de}{5}\right) + x^3\left(\frac{ac^3f}{3} + ac^2de + \frac{bc^3e}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e),x)
```

```
[Out] a*c**3*e*x + b*d**3*f*x**11/11 + x**9*(a*d**3*f/9 + b*c*d**2*f/3 + b*d**3*e
/9) + x**7*(3*a*c*d**2*f/7 + a*d**3*e/7 + 3*b*c**2*d*f/7 + 3*b*c*d**2*e/7)
+ x**5*(3*a*c**2*d*f/5 + 3*a*c*d**2*e/5 + b*c**3*f/5 + 3*b*c**2*d*e/5) + x*
*3*(a*c**3*f/3 + a*c**2*d*e + b*c**3*e/3)
```

Giac [A]
time = 0.77, size = 173, normalized size = 1.33

$$\frac{1}{11}bd^3fx^{11} + \frac{1}{3}bcd^2fx^9 + \frac{1}{9}ad^3fx^9 + \frac{1}{9}bd^3x^9e + \frac{3}{7}bc^2dfx^7 + \frac{3}{7}acd^2fx^7 + \frac{3}{7}bcd^2x^7e + \frac{1}{7}ad^3x^7e + \frac{1}{5}bc^3fx^5 + \frac{3}{5}ac^2dfx^5 + \frac{3}{5}bc^2dx^5e + \frac{3}{5}acd^2x^5e + \frac{1}{3}ac^3fx^3 + \frac{1}{3}bc^3x^3e + ac^2dx^3e + ac^3xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x, algorithm="giac")
```

```
[Out] 1/11*b*d^3*f*x^11 + 1/3*b*c*d^2*f*x^9 + 1/9*a*d^3*f*x^9 + 1/9*b*d^3*x^9*e +
 3/7*b*c^2*d*f*x^7 + 3/7*a*c*d^2*f*x^7 + 3/7*b*c*d^2*x^7*e + 1/7*a*d^3*x^7*
e + 1/5*b*c^3*f*x^5 + 3/5*a*c^2*d*f*x^5 + 3/5*b*c^2*d*x^5*e + 3/5*a*c*d^2*x
^5*e + 1/3*a*c^3*f*x^3 + 1/3*b*c^3*x^3*e + a*c^2*d*x^3*e + a*c^3*x*e
```

Mupad [B]
time = 0.81, size = 143, normalized size = 1.10

$$x^5\left(\frac{b\,c^3\,f}{5} + \frac{3\,a\,c\,d^2\,e}{5} + \frac{3\,a\,c^2\,d\,f}{5} + \frac{3\,b\,c^2\,d\,e}{5}\right) + x^7\left(\frac{a\,d^3\,e}{7} + \frac{3\,a\,c\,d^2\,f}{7} + \frac{3\,b\,c\,d^2\,e}{7} + \frac{3\,b\,c^2\,d\,f}{7}\right) + x^3\left(\frac{a\,c^3\,f}{3} + \frac{b\,c^3\,e}{3} + a\,c^2\,d\,e\right) + x^9\left(\frac{a\,d^3\,f}{9} + \frac{b\,d^3\,e}{9} + \frac{b\,c\,d^2\,f}{3}\right) + a\,c^3\,e\,x + \frac{b\,d^3\,f\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)^3*(e + f*x^2),x)
```

```
[Out] x^5*((b*c^3*f)/5 + (3*a*c*d^2*e)/5 + (3*a*c^2*d*f)/5 + (3*b*c^2*d*e)/5) + x
^7*((a*d^3*e)/7 + (3*a*c*d^2*f)/7 + (3*b*c*d^2*e)/7 + (3*b*c^2*d*f)/7) + x^
3*((a*c^3*f)/3 + (b*c^3*e)/3 + a*c^2*d*e) + x^9*((a*d^3*f)/9 + (b*d^3*e)/9
+ (b*c*d^2*f)/3) + a*c^3*e*x + (b*d^3*f*x^11)/11
```

3.19 $\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx$

Optimal. Leaf size=227

$$\frac{(7adf(15d^2e^2-40cdef+33c^2f^2)-b(105d^3e^3-280cd^2e^2f+231c^2def^2-48c^3f^3))\,x}{105f^4}-\frac{(7adf(5de-9cf)-$$

```
[Out] 1/105*(7*a*d*f*(33*c^2*f^2-40*c*d*e*f+15*d^2*e^2)-b*(-48*c^3*f^3+231*c^2*d*
e*f^2-280*c*d^2*e^2*f+105*d^3*e^3))*x/f^4-1/105*(7*a*d*f*(-9*c*f+5*d*e)-b*(
24*c^2*f^2-63*c*d*e*f+35*d^2*e^2))*x*(d*x^2+c)/f^3-1/35*(-7*a*d*f-6*b*c*f+7
*b*d*e)*x*(d*x^2+c)^2/f^2+1/7*b*x*(d*x^2+c)^3/f+(-a*f+b*e)*(-c*f+d*e)^3*arc
tan(x*f^(1/2)/e^(1/2))/f^(9/2)/e^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {542, 396, 211}

$$\frac{(be-af)\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(de-cf)^3}{\sqrt{e}\,f^{9/2}}-\frac{x(c+dx^2)\,(7adf(5de-9cf)-b(24c^2f^2-63cdef+35d^2e^2))}{105f^3}+\frac{x(7adf(33c^2f^2-40cdef+15d^2e^2)-b(-48c^3f^3+231c^2def^2-280cd^2e^2f+105d^3e^3))}{105f^4}-\frac{x(c+dx^2)^2\,(-7adf-6bcf+7bde)}{35f^2}+\frac{bx(c+dx^2)^3}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2),x]
```

```
[Out] ((7*a*d*f*(15*d^2*e^2 - 40*c*d*e*f + 33*c^2*f^2) - b*(105*d^3*e^3 - 280*c*d
^2*e^2*f + 231*c^2*d*e*f^2 - 48*c^3*f^3))*x)/(105*f^4) - ((7*a*d*f*(5*d*e -
 9*c*f) - b*(35*d^2*e^2 - 63*c*d*e*f + 24*c^2*f^2))*x*(c + d*x^2))/(105*f^3
) - ((7*b*d*e - 6*b*c*f - 7*a*d*f)*x*(c + d*x^2)^2)/(35*f^2) + (b*x*(c + d*
x^2)^3)/(7*f) + ((b*e - a*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sq
rt[e]*f^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)\,(c+dx^2)^3}{e+fx^2}\,dx = \frac{bx(c+dx^2)^3}{7f} + \frac{\int \frac{(c+dx^2)^2(-c(be-7af)+(-7bde+6bcf+7adf)x^2)}{e+fx^2}\,dx}{7f}$$

$$= -\frac{(7bde-6bcf-7adf)x(c+dx^2)^2}{35f^2} + \frac{bx(c+dx^2)^3}{7f} + \frac{\int \frac{(c+dx^2)\,(c(be(7de-11cf)-7a}{}}{}$$

$$= -\frac{(7adf(5de-9cf)-b(35d^2e^2-63cdef+24c^2f^2))\,x(c+dx^2)}{105f^3} - \frac{(7bde-6bc}{}$$

$$= \frac{(7adf(15d^2e^2-40cdef+33c^2f^2)-b(105d^3e^3-280cd^2e^2f+231c^2def^2-48}{105f^4}$$

$$= \frac{(7adf(15d^2e^2-40cdef+33c^2f^2)-b(105d^3e^3-280cd^2e^2f+231c^2def^2-48}{105f^4}$$

Mathematica [A]
time = 0.07, size = 179, normalized size = 0.79

$$\frac{(-b(de-cf)^3+adf(d^2e^2-3cdef+3c^2f^2))\,x}{f^4} + \frac{d(adf(-de+3cf)+b(d^2e^2-3cdef+3c^2f^2))\,x^3}{3f^3} + \frac{d^2(-bde+3bcf+adf)x^5}{5f^2} + \frac{bd^3x^7}{7f} + \frac{(be-af)(de-cf)^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}\,f^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2),x]
```

```
[Out] ((-(b*(d*e - c*f)^3) + a*d*f*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*x)/f^4 + (d
*(a*d*f*(-(d*e) + 3*c*f) + b*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*x^3)/(3*f^3
) + (d^2*(-(b*d*e) + 3*b*c*f + a*d*f)*x^5)/(5*f^2) + (b*d^3*x^7)/(7*f) + ((
b*e - a*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(9/2))
```

Maple [A]
time = 0.14, size = 300, normalized size = 1.32

method	result

default	$\frac{\frac{1}{7}b\,d^3x^7f^3+\frac{1}{5}a\,d^3f^3x^5+\frac{3}{5}bc\,d^2f^3x^5-\frac{1}{5}b\,d^3e\,f^2x^5+ac\,d^2f^3x^3-\frac{1}{3}a\,d^3e\,f^2x^3+b\,c^2d\,f^3x^3-bc\,d^2e\,f^2x^3+\frac{1}{3}b\,d^3e^2f\,x^3+3a\,c^2d\,f^3x-3ac\,d}{f^4}$
risch	$\frac{\ln\left(fx+\sqrt{-fe}\right)a\,d^3e^3}{2f^3\sqrt{-fe}}+\frac{\ln\left(fx+\sqrt{-fe}\right)b\,c^3e}{2f\sqrt{-fe}}-\frac{\ln\left(fx+\sqrt{-fe}\right)b\,d^3e^4}{2f^4\sqrt{-fe}}-\frac{\ln\left(-fx+\sqrt{-fe}\right)a\,d^3e^3}{2f^3\sqrt{-fe}}-\frac{\ln\left(-fx+\right.}{2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^4*(1/7*b*d^3*x^7*f^3+1/5*a*d^3*f^3*x^5+3/5*b*c*d^2*f^3*x^5-1/5*b*d^3*e*
f^2*x^5+a*c*d^2*f^3*x^3-1/3*a*d^3*e*f^2*x^3+b*c^2*d*f^3*x^3-b*c*d^2*e*f^2*x
^3+1/3*b*d^3*e^2*f*x^3+3*a*c^2*d*f^3*x-3*a*c*d^2*e*f^2*x+a*d^3*e^2*f*x+b*c^
3*f^3*x-3*b*c^2*d*e*f^2*x+3*b*c*d^2*e^2*f*x-b*d^3*e^3*x)+(a*c^3*f^4-3*a*c^2
*d*e*f^3+3*a*c*d^2*e^2*f^2-a*d^3*e^3*f-b*c^3*e*f^3+3*b*c^2*d*e^2*f^2-3*b*c*
d^2*e^3*f+b*d^3*e^4)/f^4/(f*e)^(1/2)*arctan(f*x/(f*e)^(1/2))
```

Maxima [A]

time = 0.50, size = 272, normalized size = 1.20

$$\frac{\left(ac^3f^4+bd^3e^4-\left(bc^3e+3\,ac^2de\right)f^3+3\left(bc^2de^2+acd^2e^2\right)f^2-\left(3\,bcd^2e^3+ad^3e^3\right)f\right)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}\right)}}{f^{\frac{9}{2}}}+\frac{15\,bd^3f^3x^7-21\left(bd^3f^2e-\left(3\,bcd^2+ad^3\right)f^3\right)x^5+35\left(bd^3fe^2+3\left(bc^2d+acd^2\right)f^3-\left(3\,bcd^2e+ad^3e\right)f^2\right)x^3-105\left(bd^3e^3-\left(bc^3+3\,ac^2d\right)f^3+3\left(bc^2de+acd^2e\right)f^2-\left(3\,bcd^2e^2+ad^3e^2\right)f\right)x}{105\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="maxima")
```

```
[Out] (a*c^3*f^4 + b*d^3*e^4 - (b*c^3*e + 3*a*c^2*d*e)*f^3 + 3*(b*c^2*d*e^2 + a*c
*d^2*e^2)*f^2 - (3*b*c*d^2*e^3 + a*d^3*e^3)*f)*arctan(sqrt(f)*x*e^(-1/2))*e
^(-1/2)/f^(9/2) + 1/105*(15*b*d^3*f^3*x^7 - 21*(b*d^3*f^2*e - (3*b*c*d^2 +
a*d^3)*f^3)*x^5 + 35*(b*d^3*f*e^2 + 3*(b*c^2*d + a*c*d^2)*f^3 - (3*b*c*d^2*
e + a*d^3*e)*f^2)*x^3 - 105*(b*d^3*e^3 - (b*c^3 + 3*a*c^2*d)*f^3 + 3*(b*c^2
*d*e + a*c*d^2*e)*f^2 - (3*b*c*d^2*e^2 + a*d^3*e^2)*f)*x)/f^4
```

Fricas [A]

time = 1.87, size = 581, normalized size = 2.56

$$\left[-\frac{\left(210\,bd^3fxe^4+105\left(ac^3f^4+bd^3e^4-\left(bc^3+3\,ac^2d\right)f^3e+3\left(bc^2d+acd^2\right)f^2e^2-\left(3\,bcd^2+ad^3\right)fe^3\right)\sqrt{-fe}\,\log\left(\frac{fx^2-2\sqrt{-fe}\,x-e}{fx^2+e}\right)-70\left(bd^3f^2x^3+3\left(3\,bcd^2+ad^3\right)f^2x\right)e^3+14\left(3\,bd^3f^3x^5+5\left(3\,bcd^2+ad^3\right)f^3x^3+45\left(bc^2d+acd^2\right)f^3x\right)e^2-6\left(5\,bd^3f^4x^7+7\left(3\,bcd^2+ad^3\right)f^4x^5+35\left(bc^2d+acd^2\right)f^4x^3+35\left(bc^3+3\,ac^2d\right)f^4x\right)e\right)e^{(-1)}}{210\,f^5},\ -\frac{\left(105\,bd^3fxe^4-105\left(ac^3f^4+bd^3e^4-\left(bc^3+3\,ac^2d\right)f^3e+3\left(bc^2d+acd^2\right)f^2e^2-\left(3\,bcd^2+ad^3\right)fe^3\right)\sqrt{f}\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\frac{1}{2}}-35\left(bd^3f^2x^3+3\left(3\,bcd^2+ad^3\right)f^2x\right)e^3+7\left(3\,bd^3f^3x^5+5\left(3\,bcd^2+ad^3\right)f^3x^3+45\left(bc^2d+acd^2\right)f^3x\right)e^2-3\left(5\,bd^3f^4x^7+7\left(3\,bcd^2+ad^3\right)f^4x^5+35\left(bc^2d+acd^2\right)f^4x^3+35\left(bc^3+3\,ac^2d\right)f^4x\right)e\right)e^{(-1)}}{105\,f^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="fricas")
```

```
[Out] [-1/210*(210*b*d^3*f*x*e^4 + 105*(a*c^3*f^4 + b*d^3*e^4 - (b*c^3 + 3*a*c^2*
d)*f^3*e + 3*(b*c^2*d + a*c*d^2)*f^2*e^2 - (3*b*c*d^2 + a*d^3)*f*e^3)*sqrt(
-f*e)*log((f*x^2 - 2*sqrt(-f*e)*x - e)/(f*x^2 + e)) - 70*(b*d^3*f^2*x^3 + 3
*(3*b*c*d^2 + a*d^3)*f^2*x)*e^3 + 14*(3*b*d^3*f^3*x^5 + 5*(3*b*c*d^2 + a*d^
```

```
3)*f^3*x^3 + 45*(b*c^2*d + a*c*d^2)*f^3*x)*e^2 - 6*(5*b*d^3*f^4*x^7 + 7*(3*
b*c*d^2 + a*d^3)*f^4*x^5 + 35*(b*c^2*d + a*c*d^2)*f^4*x^3 + 35*(b*c^3 + 3*a
*c^2*d)*f^4*x)*e)*e^(-1)/f^5, -1/105*(105*b*d^3*f*x*e^4 - 105*(a*c^3*f^4 +
b*d^3*e^4 - (b*c^3 + 3*a*c^2*d)*f^3*e + 3*(b*c^2*d + a*c*d^2)*f^2*e^2 - (3*
b*c*d^2 + a*d^3)*f*e^3)*sqrt(f)*arctan(sqrt(f)*x*e^(-1/2))*e^(1/2) - 35*(b*
d^3*f^2*x^3 + 3*(3*b*c*d^2 + a*d^3)*f^2*x)*e^3 + 7*(3*b*d^3*f^3*x^5 + 5*(3*
b*c*d^2 + a*d^3)*f^3*x^3 + 45*(b*c^2*d + a*c*d^2)*f^3*x)*e^2 - 3*(5*b*d^3*f
^4*x^7 + 7*(3*b*c*d^2 + a*d^3)*f^4*x^5 + 35*(b*c^2*d + a*c*d^2)*f^4*x^3 + 3
5*(b*c^3 + 3*a*c^2*d)*f^4*x)*e)*e^(-1)/f^5]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(228) = 456$.
time = 0.92, size = 508, normalized size = 2.24

$$\frac{bd^3x^7}{7f} + x^5\left(\frac{ad^3}{5f} + \frac{3bcd^2}{5f} - \frac{bd^3e}{5f^2}\right) + x^3\left(\frac{acd^2}{f} - \frac{ad^3e}{3f^2} + \frac{bc^2d}{f} - \frac{bcd^2e}{f^2} + \frac{bd^3e^2}{3f^3}\right) + x\left(\frac{3ac^2d}{f} - \frac{3acd^2e}{f^2} + \frac{ad^3e^2}{f^3} + \frac{bc^3}{f} - \frac{3bc^2de}{f^2} + \frac{3bcd^2e^2}{f^3} - \frac{bd^3e^3}{f^4}\right) - \frac{\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3\log\left(-\frac{ef^4\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3}{ac^3f^4-3ac^2def^3+3acd^2e^2f^2-ad^3e^3f-bc^3ef^3+3bc^2de^2f^2-3bcd^2e^3f+bd^3e^4}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3\log\left(\frac{ef^4\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3}{ac^3f^4-3ac^2def^3+3acd^2e^2f^2-ad^3e^3f-bc^3ef^3+3bc^2de^2f^2-3bcd^2e^3f+bd^3e^4}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e),x)
```

```
[Out] b*d**3*x**7/(7*f) + x**5*(a*d**3/(5*f) + 3*b*c*d**2/(5*f) - b*d**3*e/(5*f**
2)) + x**3*(a*c*d**2/f - a*d**3*e/(3*f**2) + b*c**2*d/f - b*c*d**2*e/f**2 +
 b*d**3*e**2/(3*f**3)) + x*(3*a*c**2*d/f - 3*a*c*d**2*e/f**2 + a*d**3*e**2/
f**3 + b*c**3/f - 3*b*c**2*d*e/f**2 + 3*b*c*d**2*e**2/f**3 - b*d**3*e**3/f*
*4) - sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f - d*e)**3*log(-e*f**4*sqrt(-1/(e*f
**9))*(a*f - b*e)*(c*f - d*e)**3/(a*c**3*f**4 - 3*a*c**2*d*e*f**3 + 3*a*c*d
**2*e**2*f**2 - a*d**3*e**3*f - b*c**3*e*f**3 + 3*b*c**2*d*e**2*f**2 - 3*b*
c*d**2*e**3*f + b*d**3*e**4) + x)/2 + sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f -
d*e)**3*log(e*f**4*sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f - d*e)**3/(a*c**3*f**
4 - 3*a*c**2*d*e*f**3 + 3*a*c*d**2*e**2*f**2 - a*d**3*e**3*f - b*c**3*e*f**
3 + 3*b*c**2*d*e**2*f**2 - 3*b*c*d**2*e**3*f + b*d**3*e**4) + x)/2
```

Giac [A]
time = 1.00, size = 307, normalized size = 1.35

$$\frac{(ac^3f^4 - bc^3f^3e - 3\,ac^2df^3e + 3\,bc^2df^2e^2 + 3\,acd^2f^2e^2 - 3\,bcd^2fe^3 - ad^3fe^3 + bd^3e^4)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}\right)}}{f^{\frac{9}{2}}} + \frac{15\,bd^3f^6x^7 + 63\,bcd^2f^6x^5 + 21\,ad^3f^6x^5 - 21\,bd^3f^5x^5e + 105\,bc^2df^6x^3 + 105\,acd^2f^6x^3 - 105\,bcd^2f^5x^3e - 35\,ad^3f^5x^3e + 35\,bd^3f^4x^3e^2 + 105\,bc^3f^6x + 315\,ac^2df^6x - 315\,bc^2df^5xe - 315\,acd^2f^5xe + 315\,bcd^2f^4xe^2 + 105\,ad^3f^4xe^2 - 105\,bd^3f^3xe^3}{105\,f^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="giac")
```

```
[Out] (a*c^3*f^4 - b*c^3*f^3*e - 3*a*c^2*d*f^3*e + 3*b*c^2*d*f^2*e^2 + 3*a*c*d^2*
f^2*e^2 - 3*b*c*d^2*f*e^3 - a*d^3*f*e^3 + b*d^3*e^4)*arctan(sqrt(f)*x*e^(-1
/2))*e^(-1/2)/f^(9/2) + 1/105*(15*b*d^3*f^6*x^7 + 63*b*c*d^2*f^6*x^5 + 21*a
*d^3*f^6*x^5 - 21*b*d^3*f^5*x^5*e + 105*b*c^2*d*f^6*x^3 + 105*a*c*d^2*f^6*x
^3 - 105*b*c*d^2*f^5*x^3*e - 35*a*d^3*f^5*x^3*e + 35*b*d^3*f^4*x^3*e^2 + 10
5*b*c^3*f^6*x + 315*a*c^2*d*f^6*x - 315*b*c^2*d*f^5*x*e - 315*a*c*d^2*f^5*x
*e + 315*b*c*d^2*f^4*x*e^2 + 105*a*d^3*f^4*x*e^2 - 105*b*d^3*f^3*x*e^3)/f^7
```

Mupad [B]

time = 0.85, size = 312, normalized size = 1.37

$$x\left(\frac{b\,c^3+3\,a\,d\,c^2}{f}+\frac{e\,\left(\frac{e\,\left(\frac{a\,d^3+3\,b\,c\,d^2}{f}-\frac{b\,d^3\,e}{f^2}\right)}{f}-\frac{3\,c\,d\,\left(a\,d+b\,c\right)}{f}\right)}{f}\right)+x^5\,\left(\frac{a\,d^3+3\,b\,c\,d^2}{5\,f}-\frac{b\,d^3\,e}{5\,f^2}\right)-x^3\,\left(\frac{e\,\left(\frac{a\,d^3+3\,b\,c\,d^2}{f}-\frac{b\,d^3\,e}{f^2}\right)}{3\,f}-\frac{c\,d\,\left(a\,d+b\,c\right)}{f}\right)+\frac{b\,d^3\,x^7}{7\,f}+\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,\left(a\,f-b\,e\right)\,{\left(c\,f-d\,e\right)}^3}{\sqrt{e}\,\left(-b\,c^3\,e\,f^3+a\,c^3\,f^4+3\,b\,c^2\,d\,e^2\,f^2-3\,a\,c^2\,d\,e\,f^3-3\,b\,c\,d^2\,e^3\,f+3\,a\,c\,d^2\,e^2\,f^2+b\,d^3\,e^4-a\,d^3\,e^3\,f\right)}\right)\,\left(a\,f-b\,e\right)\,{\left(c\,f-d\,e\right)}^3}{\sqrt{e}\,f^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2),x)
```

```
[Out] x*((b*c^3 + 3*a*c^2*d)/f + (e*((e*((a*d^3 + 3*b*c*d^2)/f - (b*d^3*e)/f^2))/
f - (3*c*d*(a*d + b*c))/f))/f) + x^5*((a*d^3 + 3*b*c*d^2)/(5*f) - (b*d^3*e)
/(5*f^2)) - x^3*((e*((a*d^3 + 3*b*c*d^2)/f - (b*d^3*e)/f^2))/(3*f) - (c*d*(
a*d + b*c))/f) + (b*d^3*x^7)/(7*f) + (atan((f^(1/2)*x*(a*f - b*e)*(c*f - d*
e)^3)/(e^(1/2)*(a*c^3*f^4 + b*d^3*e^4 - a*d^3*e^3*f - b*c^3*e*f^3 - 3*a*c^2
*d*e*f^3 - 3*b*c*d^2*e^3*f + 3*a*c*d^2*e^2*f^2 + 3*b*c^2*d*e^2*f^2)))*(a*f
- b*e)*(c*f - d*e)^3)/(e^(1/2)*f^(9/2))
```

3.20 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx$

Optimal. Leaf size=242

$$-\frac{d(5af(15d^2e^2-22cdef+3c^2f^2)-be(105d^2e^2-190cdef+81c^2f^2))\,x}{30ef^4}-\frac{d(be(35de-33cf)-5af(5de-}{30ef^3}$$

```
[Out] -1/30*d*(5*a*f*(3*c^2*f^2-22*c*d*e*f+15*d^2*e^2)-b*e*(81*c^2*f^2-190*c*d*e*
f+105*d^2*e^2))*x/e/f^4-1/30*d*(b*e*(-33*c*f+35*d*e)-5*a*f*(-3*c*f+5*d*e))*
x*(d*x^2+c)/e/f^3+1/10*d*(-5*a*f+7*b*e)*x*(d*x^2+c)^2/e/f^2-1/2*(-a*f+b*e)*
x*(d*x^2+c)^3/e/f/(f*x^2+e)-1/2*(-c*f+d*e)^2*(b*e*(-c*f+7*d*e)-a*f*(c*f+5*d
*e))*arctan(x*f^(1/2)/e^(1/2))/e^(3/2)/f^(9/2)
```

Rubi [A]
time = 0.28, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {540, 542, 396, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(de-cf)^2(be(7de-cf)-af(cf+5de))}{2e^{3/2}f^{9/2}}-\frac{dx(5af(3c^2f^2-22cdef+15d^2e^2)-be(81c^2f^2-190cdef+105d^2e^2))}{30ef^4}-\frac{dx(c+dx^2)(be(35de-33cf)-5af(5de-3cf))}{30ef^3}+\frac{dx(c+dx^2)^2(7be-5af)}{10ef^2}-\frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

```
[Out] -1/30*(d*(5*a*f*(15*d^2*e^2 - 22*c*d*e*f + 3*c^2*f^2) - b*e*(105*d^2*e^2 -
190*c*d*e*f + 81*c^2*f^2))*x)/(e*f^4) - (d*(b*e*(35*d*e - 33*c*f) - 5*a*f*(
5*d*e - 3*c*f))*x*(c + d*x^2))/(30*e*f^3) + (d*(7*b*e - 5*a*f)*x*(c + d*x^2
)^2)/(10*e*f^2) - ((b*e - a*f)*x*(c + d*x^2)^3)/(2*e*f*(e + f*x^2)) - ((d*e
 - c*f)^2*(b*e*(7*d*e - c*f) - a*f*(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e
]])/(2*e^(3/2)*f^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx = -\frac{(be-af)x(c+dx^2)^3}{2ef(e+fx^2)} - \frac{\int \frac{(c+dx^2)^2(-c(be+af)-d(7be-5af)x^2)}{e+fx^2}\,dx}{2ef}$$

$$= \frac{d(7be-5af)x(c+dx^2)^2}{10ef^2} - \frac{(be-af)x(c+dx^2)^3}{2ef(e+fx^2)} - \frac{\int \frac{(c+dx^2)(c(be(7de-5cf)-5af(de\cdots}{\cdots}}{\cdots}$$

$$= -\frac{d(be(35de-33cf)-5af(5de-3cf))x(c+dx^2)}{30ef^3} + \frac{d(7be-5af)x(c+dx^2)^2}{10ef^2}\cdots$$

$$= -\frac{d(5af(15d^2e^2-22cdef+3c^2f^2)-be(105d^2e^2-190cdef+81c^2f^2))x}{30ef^4} - \frac{d(b\cdots}{\cdots}$$

$$= -\frac{d(5af(15d^2e^2-22cdef+3c^2f^2)-be(105d^2e^2-190cdef+81c^2f^2))x}{30ef^4} - \frac{d(b\cdots}{\cdots}$$

Mathematica [A]
time = 0.09, size = 176, normalized size = 0.73

$$\frac{d(3b(de-cf)^2+adf(-2de+3cf))x}{f^4} + \frac{d^2(-2bde+3bcf+adf)x^3}{3f^3} + \frac{bd^3x^5}{5f^2} + \frac{(be-af)(de-cf)^3x}{2ef^4(e+fx^2)} - \frac{(de-cf)^2(be(7de-cf)-af(5de+cf))\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{2e^{3/2}f^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

```
[Out] (d*(3*b*(d*e - c*f)^2 + a*d*f*(-2*d*e + 3*c*f))*x)/f^4 + (d^2*(-2*b*d*e + 3
*b*c*f + a*d*f)*x^3)/(3*f^3) + (b*d^3*x^5)/(5*f^2) + ((b*e - a*f)*(d*e - c*
f)^3*x)/(2*e*f^4*(e + f*x^2)) - ((d*e - c*f)^2*(b*e*(7*d*e - c*f) - a*f*(5*
d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(9/2))
```

Maple [A]
time = 0.17, size = 308, normalized size = 1.27

method	result
default	$\frac{d\left(\frac{1}{5} b\,d^{2} x^{5} f^{2}+\frac{1}{3} a\,d^{2} f^{2} x^{3}+b c d\,f^{2} x^{3}-\frac{2}{3} b\,d^{2} e f\,x^{3}+3 a c d\,f^{2} x-2 a\,d^{2} e f x+3 b\,c^{2} f^{2} x-6 b c d e f x+3 b\,d^{2} e^{2} x\right)}{f^{4}}+\frac{\left(a\,c^{3} f^{4}-3 a\,c^{2} d e\,f^{3}+3 a c\,d^{2}\right.}{}$
risch	$-\frac{3 \ln\left(f x+\sqrt{-f e}\right) a\,c^{2} d}{4 f \sqrt{-f e}}-\frac{5 e^{2} \ln\left(f x+\sqrt{-f e}\right) a\,d^{3}}{4 f^{3} \sqrt{-f e}}+\frac{7 e^{3} \ln\left(f x+\sqrt{-f e}\right) b\,d^{3}}{4 f^{4} \sqrt{-f e}}+\frac{d^{3} b\,x^{5}}{5 f^{2}}+\frac{d^{3} a\,x^{3}}{3 f^{2}}+\frac{3 d^{2} a c x}{f^{2}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d/f^4*(1/5*b*d^2*x^5*f^2+1/3*a*d^2*f^2*x^3+b*c*d*f^2*x^3-2/3*b*d^2*e*f*x^3+
3*a*c*d*f^2*x-2*a*d^2*e*f*x+3*b*c^2*f^2*x-6*b*c*d*e*f*x+3*b*d^2*e^2*x)+1/f^
4*(1/2*(a*c^3*f^4-3*a*c^2*d*e*f^3+3*a*c*d^2*e^2*f^2-a*d^3*e^3*f-b*c^3*e*f^3
+3*b*c^2*d*e^2*f^2-3*b*c*d^2*e^3*f+b*d^3*e^4)/e*x/(f*x^2+e)+1/2*(a*c^3*f^4+
3*a*c^2*d*e*f^3-9*a*c*d^2*e^2*f^2+5*a*d^3*e^3*f+b*c^3*e*f^3-9*b*c^2*d*e^2*f
^2+15*b*c*d^2*e^3*f-7*b*d^3*e^4)/e/(f*e)^(1/2)*arctan(f*x/(f*e)^(1/2)))
```

Maxima [A]
time = 0.49, size = 298, normalized size = 1.23

$$\frac{(ac^3f^4 + bd^3e^4 - (bc^3e + 3\,ac^2de)f^3 + 3\,(bc^2de^2 + acd^2e^2)f^2 - (3\,bcd^2e^3 + ad^3e^3)f)x}{2\,(f^5x^2e + f^4e^2)} + \frac{(ac^3f^4 - 7\,bd^3e^4 + (bc^3e + 3\,ac^2de)f^3 - 9\,(bc^2de^2 + acd^2e^2)f^2 + 5\,(3\,bcd^2e^3 + ad^3e^3)f)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{3}{2}\right)}}{2\,f^{\frac{9}{2}}} + \frac{3\,bd^3f^2x^5 - 5\,(2\,bd^3fe - (3\,bcd^2 + ad^3)f^2)x^3 + 15\,(3\,bd^3e^2 + 3\,(bc^2d + acd^2)f^2 - 2\,(3\,bcd^2e + ad^3e)f)x}{15\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*c^3*f^4 + b*d^3*e^4 - (b*c^3*e + 3*a*c^2*d*e)*f^3 + 3*(b*c^2*d*e^2 +
 a*c*d^2*e^2)*f^2 - (3*b*c*d^2*e^3 + a*d^3*e^3)*f)*x/(f^5*x^2*e + f^4*e^2)
+ 1/2*(a*c^3*f^4 - 7*b*d^3*e^4 + (b*c^3*e + 3*a*c^2*d*e)*f^3 - 9*(b*c^2*d*e
^2 + a*c*d^2*e^2)*f^2 + 5*(3*b*c*d^2*e^3 + a*d^3*e^3)*f)*arctan(sqrt(f)*x*e
^(-1/2))*e^(-3/2)/f^(9/2) + 1/15*(3*b*d^3*f^2*x^5 - 5*(2*b*d^3*f*e - (3*b*c
*d^2 + a*d^3)*f^2)*x^3 + 15*(3*b*d^3*e^2 + 3*(b*c^2*d + a*c*d^2)*f^2 - 2*(3
*b*c*d^2*e + a*d^3*e)*f)*x)/f^4
```

Fricas [A]
time = 1.18, size = 824, normalized size = 3.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(30*a*c^3*f^5*x*e + 210*b*d^3*f*x*e^5 + 15*(a*c^3*f^5*x^2 - 7*b*d^3*e
^5 - (7*b*d^3*f*x^2 - 5*(3*b*c*d^2 + a*d^3)*f)*e^4 + (5*(3*b*c*d^2 + a*d^3)
*f^2*x^2 - 9*(b*c^2*d + a*c*d^2)*f^2)*e^3 - (9*(b*c^2*d + a*c*d^2)*f^3*x^2
- (b*c^3 + 3*a*c^2*d)*f^3)*e^2 + (a*c^3*f^4 + (b*c^3 + 3*a*c^2*d)*f^4*x^2)*
e)*sqrt(-f*e)*log((f*x^2 + 2*sqrt(-f*e)*x - e)/(f*x^2 + e)) + 10*(14*b*d^3*
f^2*x^3 - 15*(3*b*c*d^2 + a*d^3)*f^2*x)*e^4 - 2*(14*b*d^3*f^3*x^5 + 50*(3*b
*c*d^2 + a*d^3)*f^3*x^3 - 135*(b*c^2*d + a*c*d^2)*f^3*x)*e^3 + 2*(6*b*d^3*f
^4*x^7 + 10*(3*b*c*d^2 + a*d^3)*f^4*x^5 + 90*(b*c^2*d + a*c*d^2)*f^4*x^3 -
15*(b*c^3 + 3*a*c^2*d)*f^4*x)*e^2)/(f^6*x^2*e^2 + f^5*e^3), 1/30*(15*a*c^3*
f^5*x*e + 105*b*d^3*f*x*e^5 + 15*(a*c^3*f^5*x^2 - 7*b*d^3*e^5 - (7*b*d^3*f*
x^2 - 5*(3*b*c*d^2 + a*d^3)*f)*e^4 + (5*(3*b*c*d^2 + a*d^3)*f^2*x^2 - 9*(b*
c^2*d + a*c*d^2)*f^2)*e^3 - (9*(b*c^2*d + a*c*d^2)*f^3*x^2 - (b*c^3 + 3*a*c
^2*d)*f^3)*e^2 + (a*c^3*f^4 + (b*c^3 + 3*a*c^2*d)*f^4*x^2)*e)*sqrt(f)*arcta
n(sqrt(f)*x*e^(-1/2))*e^(1/2) + 5*(14*b*d^3*f^2*x^3 - 15*(3*b*c*d^2 + a*d^3
)*f^2*x)*e^4 - (14*b*d^3*f^3*x^5 + 50*(3*b*c*d^2 + a*d^3)*f^3*x^3 - 135*(b*
c^2*d + a*c*d^2)*f^3*x)*e^3 + (6*b*d^3*f^4*x^7 + 10*(3*b*c*d^2 + a*d^3)*f^4
*x^5 + 90*(b*c^2*d + a*c*d^2)*f^4*x^3 - 15*(b*c^3 + 3*a*c^2*d)*f^4*x)*e^2)/
(f^6*x^2*e^2 + f^5*e^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(231) = 462$.
time = 2.65, size = 661, normalized size = 2.73

$$\frac{bd^3x^5}{5f^2} + x^3\left(\frac{ad^3}{3f^2} + \frac{bcd^2}{f^2} - \frac{2bd^3e}{3f^3}\right) + x\left(\frac{3acd^2}{f^2} - \frac{2ad^3e}{f^3} + \frac{3bc^2d}{f^2} - \frac{6bcd^2e}{f^3} + \frac{3bd^3e^2}{f^4}\right) + \frac{x(ac^3f^4 - 3ac^2def^3 + 3acd^2e^2f^2 - ad^3e^3f - bc^3ef^3 + 3bc^2de^2f^2 - 3bcd^2e^3f + bd^3e^4)}{2e^2f^4 + 2ef^5x^2} - \frac{\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2(acf^2+5adef+bcef-7bde^2)\log\left(-\frac{e^2f^4\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2(acf^2+5adef+bcef-7bde^2)}{ac^3f^4+3ac^2def^3-9acd^2e^2f^2+5ad^3e^3f+bc^3ef^3-9bc^2de^2f^2+15bcd^2e^3f-7bd^3e^4}+x\right)}{4} + \frac{\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2(acf^2+5adef+bcef-7bde^2)\log\left(\frac{e^2f^4\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2(acf^2+5adef+bcef-7bde^2)}{ac^3f^4+3ac^2def^3-9acd^2e^2f^2+5ad^3e^3f+bc^3ef^3-9bc^2de^2f^2+15bcd^2e^3f-7bd^3e^4}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**2,x)
```

```
[Out] b*d**3*x**5/(5*f**2) + x**3*(a*d**3/(3*f**2) + b*c*d**2/f**2 - 2*b*d**3*e/(
3*f**3)) + x*(3*a*c*d**2/f**2 - 2*a*d**3*e/f**3 + 3*b*c**2*d/f**2 - 6*b*c*d
**2*e/f**3 + 3*b*d**3*e**2/f**4) + x*(a*c**3*f**4 - 3*a*c**2*d*e*f**3 + 3*a
*c*d**2*e**2*f**2 - a*d**3*e**3*f - b*c**3*e*f**3 + 3*b*c**2*d*e**2*f**2 -
3*b*c*d**2*e**3*f + b*d**3*e**4)/(2*e**2*f**4 + 2*e*f**5*x**2) - sqrt(-1/(e
**3*f**9))*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f + b*c*e*f - 7*b*d*e**2)*log
(-e**2*f**4*sqrt(-1/(e**3*f**9))*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f + b*c
*e*f - 7*b*d*e**2)/(a*c**3*f**4 + 3*a*c**2*d*e*f**3 - 9*a*c*d**2*e**2*f**2
+ 5*a*d**3*e**3*f + b*c**3*e*f**3 - 9*b*c**2*d*e**2*f**2 + 15*b*c*d**2*e**3
*f - 7*b*d**3*e**4) + x)/4 + sqrt(-1/(e**3*f**9))*(c*f - d*e)**2*(a*c*f**2
+ 5*a*d*e*f + b*c*e*f - 7*b*d*e**2)*log(e**2*f**4*sqrt(-1/(e**3*f**9))*(c*f
 - d*e)**2*(a*c*f**2 + 5*a*d*e*f + b*c*e*f - 7*b*d*e**2)/(a*c**3*f**4 + 3*a
*c**2*d*e*f**3 - 9*a*c*d**2*e**2*f**2 + 5*a*d**3*e**3*f + b*c**3*e*f**3 - 9
*b*c**2*d*e**2*f**2 + 15*b*c*d**2*e**3*f - 7*b*d**3*e**4) + x)/4
```

Giac [A]
time = 1.25, size = 321, normalized size = 1.33

$$\frac{(ac^3f^4+bc^3f^3e+3\,ac^2df^3e-9\,bc^2df^2e^2-9\,acd^2f^2e^2+15\,bcd^2fe^3+5\,ad^3fe^3-7\,bd^3e^4)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{3}{2}\right)}}{2\,f^{\frac{9}{2}}}+\frac{(ac^3f^4x-bc^3f^3xe-3\,ac^2df^3xe+3\,bc^2df^2xe^2+3\,acd^2f^2xe^2-3\,bcd^2fxe^3-ad^3fxe^3+bd^3xe^4)e^{(-1)}}{2\,(fx^2+e)f^4}+\frac{3\,bd^3f^8x^5+15\,bcd^2f^8x^3+5\,ad^3f^8x^3-10\,bd^3f^7x^3e+45\,bc^2df^8x+45\,acd^2f^8x-90\,bcd^2f^7xe-30\,ad^3f^7xe+45\,bd^3f^6xe^2}{15\,f^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*c^3*f^4 + b*c^3*f^3*e + 3*a*c^2*d*f^3*e - 9*b*c^2*d*f^2*e^2 - 9*a*c*
d^2*f^2*e^2 + 15*b*c*d^2*f*e^3 + 5*a*d^3*f*e^3 - 7*b*d^3*e^4)*arctan(sqrt(f
)*x*e^(-1/2))*e^(-3/2)/f^(9/2) + 1/2*(a*c^3*f^4*x - b*c^3*f^3*x*e - 3*a*c^2
*d*f^3*x*e + 3*b*c^2*d*f^2*x*e^2 + 3*a*c*d^2*f^2*x*e^2 - 3*b*c*d^2*f*x*e^3
- a*d^3*f*x*e^3 + b*d^3*x*e^4)*e^(-1)/((f*x^2 + e)*f^4) + 1/15*(3*b*d^3*f^8
*x^5 + 15*b*c*d^2*f^8*x^3 + 5*a*d^3*f^8*x^3 - 10*b*d^3*f^7*x^3*e + 45*b*c^2
*d*f^8*x + 45*a*c*d^2*f^8*x - 90*b*c*d^2*f^7*x*e - 30*a*d^3*f^7*x*e + 45*b*
d^3*f^6*x*e^2)/f^10
```

Mupad [B]
time = 0.98, size = 389, normalized size = 1.61

$$x^3\left(\frac{a\,d^3+3\,b\,c\,d^2}{3\,f^2}-\frac{2\,b\,d^3\,e}{3\,f^3}\right)-x\left(\frac{2\,e\left(\frac{a\,d^3+3\,b\,c\,d^2}{f^2}-\frac{2\,b\,d^3\,e}{f^3}\right)}{f}+\frac{b\,d^3\,e^2}{f^4}-\frac{3\,c\,d\,(a\,d+b\,c)}{f^2}\right)+\frac{b\,d^3\,x^5}{5\,f^2}+\frac{x\,(-b\,c^3\,e\,f^3+a\,c^3\,f^4+3\,b\,c^2\,d\,e^2\,f^2-3\,a\,c^2\,d\,e\,f^3-3\,b\,c\,d^2\,e^3\,f+3\,a\,c\,d^2\,e^2\,f^2+b\,d^3\,e^4-a\,d^3\,e^3\,f)}{2\,e\,(f^5\,x^2+e\,f^4)}+\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(c\,f-d\,e)^2\,(a\,c\,f^2-7\,b\,d\,e^2+5\,a\,d\,e\,f+b\,c\,e\,f)}{\sqrt{e}\,(b\,c^3\,e\,f^3+a\,c^3\,f^4-9\,b\,c^2\,d\,e^2\,f^2+3\,a\,c^2\,d\,e\,f^3+15\,b\,c\,d^2\,e^3\,f-9\,a\,c\,d^2\,e^2\,f^2-7\,b\,d^3\,e^4+5\,a\,d^3\,e^3\,f)}\right)\,(c\,f-d\,e)^2\,(a\,c\,f^2-7\,b\,d\,e^2+5\,a\,d\,e\,f+b\,c\,e\,f)}{2\,e^{3/2}\,f^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^2,x)
```

```
[Out] x^3*((a*d^3 + 3*b*c*d^2)/(3*f^2) - (2*b*d^3*e)/(3*f^3)) - x*((2*e*((a*d^3 +
 3*b*c*d^2)/f^2 - (2*b*d^3*e)/f^3))/f + (b*d^3*e^2)/f^4 - (3*c*d*(a*d + b*c
))/f^2) + (b*d^3*x^5)/(5*f^2) + (x*(a*c^3*f^4 + b*d^3*e^4 - a*d^3*e^3*f - b
*c^3*e*f^3 - 3*a*c^2*d*e*f^3 - 3*b*c*d^2*e^3*f + 3*a*c*d^2*e^2*f^2 + 3*b*c^
2*d*e^2*f^2))/(2*e*(e*f^4 + f^5*x^2)) + (atan((f^(1/2)*x*(c*f - d*e)^2*(a*c
*f^2 - 7*b*d*e^2 + 5*a*d*e*f + b*c*e*f))/(e^(1/2)*(a*c^3*f^4 - 7*b*d^3*e^4
+ 5*a*d^3*e^3*f + b*c^3*e*f^3 + 3*a*c^2*d*e*f^3 + 15*b*c*d^2*e^3*f - 9*a*c*
d^2*e^2*f^2 - 9*b*c^2*d*e^2*f^2)))*(c*f - d*e)^2*(a*c*f^2 - 7*b*d*e^2 + 5*a
*d*e*f + b*c*e*f))/(2*e^(3/2)*f^(9/2))
```

3.21 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx$

Optimal. Leaf size=291

$$\frac{d(3af(15d^2e^2-4cdef-3c^2f^2)-be(105d^2e^2-100cdef+3c^2f^2))\,x}{24e^2f^4}+\frac{d(be(35de-3cf)-3af(5de+3cf))x}{24e^2f^3}$$

```
[Out] 1/24*d*(3*a*f*(-3*c^2*f^2-4*c*d*e*f+15*d^2*e^2)-b*e*(3*c^2*f^2-100*c*d*e*f+
105*d^2*e^2))*x/e^2/f^4+1/24*d*(b*e*(-3*c*f+35*d*e)-3*a*f*(3*c*f+5*d*e))*x*
(d*x^2+c)/e^2/f^3-1/4*(-a*f+b*e)*x*(d*x^2+c)^3/e/f/(f*x^2+e)^2-1/8*(b*e*(-c
*f+7*d*e)-3*a*f*(c*f+d*e))*x*(d*x^2+c)^2/e^2/f^2/(f*x^2+e)+1/8*(-c*f+d*e)*(
b*e*(-c^2*f^2-10*c*d*e*f+35*d^2*e^2)-3*a*f*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))*a
rctan(x*f^(1/2)/e^(1/2))/e^(5/2)/f^(9/2)
```

Rubi [A]
time = 0.27, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {540, 542, 396, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(de-cf)\left(be(-c^2f^2-10cdef+35d^2e^2)-3af(c^2f^2+2cdef+5d^2e^2)\right)}{8e^{5/2}f^{9/2}}+\frac{dx(3af(-3c^2f^2-4cdef+15d^2e^2)-be(3c^2f^2-100cdef+105d^2e^2))}{24e^2f^4}+\frac{dx(c+dx^2)(be(35de-3cf)-3af(3cf+5de))}{24e^2f^3}-\frac{x(c+dx^2)^2(be(7de-cf)-3af(cf+de))}{8e^2f^2(e+fx^2)}-\frac{x(c+dx^2)^3(be-af)}{4ef(e+fx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

```
[Out] (d*(3*a*f*(15*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^2) - b*e*(105*d^2*e^2 - 100*c*d
*e*f + 3*c^2*f^2))*x)/(24*e^2*f^4) + (d*(b*e*(35*d*e - 3*c*f) - 3*a*f*(5*d*
e + 3*c*f))*x*(c + d*x^2))/(24*e^2*f^3) - ((b*e - a*f)*x*(c + d*x^2)^3)/(4*
e*f*(e + f*x^2)^2) - ((b*e*(7*d*e - c*f) - 3*a*f*(d*e + c*f))*x*(c + d*x^2)
^2)/(8*e^2*f^2*(e + f*x^2)) + ((d*e - c*f)*(b*e*(35*d^2*e^2 - 10*c*d*e*f -
c^2*f^2) - 3*a*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt
[e]])/(8*e^(5/2)*f^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx = -\frac{(be-af)x(c+dx^2)^3}{4ef(e+fx^2)^2} - \frac{\int \frac{(c+dx^2)^2(-c(be+3af)-d(7be-3af)x^2)}{(e+fx^2)^2}\,dx}{4ef}$$

$$= -\frac{(be-af)x(c+dx^2)^3}{4ef(e+fx^2)^2} - \frac{(be(7de-cf)-3af(de+cf))x(c+dx^2)^2}{8e^2f^2(e+fx^2)} +$$

$$= \frac{d(be(35de-3cf)-3af(5de+3cf))x(c+dx^2)}{24e^2f^3} - \frac{(be-af)x(c+dx^2)^3}{4ef(e+fx^2)^2} -$$

$$= \frac{d(3af(15d^2e^2-4cdef-3c^2f^2)-be(105d^2e^2-100cdef+3c^2f^2))x}{24e^2f^4} +$$

$$= \frac{d(3af(15d^2e^2-4cdef-3c^2f^2)-be(105d^2e^2-100cdef+3c^2f^2))x}{24e^2f^4} +$$

Mathematica [A]
time = 0.11, size = 219, normalized size = 0.75

$$\frac{d^2(-3bde+3bcf+adf)x}{f^4} + \frac{bd^3x^3}{3f^3} + \frac{(be-af)(de-cf)^3x}{4ef^4(e+fx^2)^2} - \frac{(de-cf)^2(be(13de-cf)-3af(3de+cf))x}{8e^2f^4(e+fx^2)} + \frac{(de-cf)(be(35d^2e^2-10cdef-c^2f^2)-3af(5d^2e^2+2cdef+c^2f^2))\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

```
[Out] (d^2*(-3*b*d*e + 3*b*c*f + a*d*f)*x)/f^4 + (b*d^3*x^3)/(3*f^3) + ((b*e - a*
f)*(d*e - c*f)^3*x)/(4*e*f^4*(e + f*x^2)^2) - ((d*e - c*f)^2*(b*e*(13*d*e -
 c*f) - 3*a*f*(3*d*e + c*f))*x)/(8*e^2*f^4*(e + f*x^2)) + ((d*e - c*f)*(b*e
*(35*d^2*e^2 - 10*c*d*e*f - c^2*f^2) - 3*a*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f
^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(9/2))
```

Maple [A]
time = 0.17, size = 344, normalized size = 1.18

method	result
default	$\frac{d^{2}\left(\frac{1}{3} b d \,x^{3} f +a d f x +3 b c f x -3 b d e x \right)}{f^{4}}+\frac{\frac{f \left(3 a \,c^{3} f^{4}+3 a \,c^{2} d e \,f^{3}-15 a c \,d^{2} e^{2} f^{2}+9 a \,d^{3} e^{3} f +b \,c^{3} e \,f^{3}-15 b \,c^{2} d \,e^{2} f^{2}+27 b c \,d^{2} e^{3} f -13 b \,d^{3} e^{4}\right) x^{3}}{8 e^{2}}+\left(5 a \right.}{\left(f \,x^{2}+e \right)^{2}}$
risch	$\frac{d^{3} b \,x^{3}}{3 f^{3}}+\frac{d^{3} a x}{f^{3}}+\frac{3 d^{2} b c x}{f^{3}}-\frac{3 d^{3} b e x}{f^{4}}+\frac{\frac{f \left(3 a \,c^{3} f^{4}+3 a \,c^{2} d e \,f^{3}-15 a c \,d^{2} e^{2} f^{2}+9 a \,d^{3} e^{3} f +b \,c^{3} e \,f^{3}-15 b \,c^{2} d \,e^{2} f^{2}+27 b c \,d^{2} e^{3} f -13 b \,d^{3} e^{4}\right) x^{3}}{8 e^{2}}+}{f^{4} \left(f \,x^{2}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/f^4*(1/3*b*d*x^3*f+a*d*f*x+3*b*c*f*x-3*b*d*e*x)+1/f^4*((1/8*f*(3*a*c^3*
f^4+3*a*c^2*d*e*f^3-15*a*c*d^2*e^2*f^2+9*a*d^3*e^3*f+b*c^3*e*f^3-15*b*c^2*d
*e^2*f^2+27*b*c*d^2*e^3*f-13*b*d^3*e^4)/e^2*x^3+1/8*(5*a*c^3*f^4-3*a*c^2*d*
e*f^3-9*a*c*d^2*e^2*f^2+7*a*d^3*e^3*f-b*c^3*e*f^3-9*b*c^2*d*e^2*f^2+21*b*c*
d^2*e^3*f-11*b*d^3*e^4)/e*x)/(f*x^2+e)^2+1/8*(3*a*c^3*f^4+3*a*c^2*d*e*f^3+9
*a*c*d^2*e^2*f^2-15*a*d^3*e^3*f+b*c^3*e*f^3+9*b*c^2*d*e^2*f^2-45*b*c*d^2*e^
3*f+35*b*d^3*e^4)/e^2/(f*e)^(1/2)*arctan(f*x/(f*e)^(1/2)))
```

Maxima [A]
time = 0.57, size = 345, normalized size = 1.19

$$\frac{(3\,ac^3f^5 - 13\,bd^3fe^4 + (bc^3e + 3\,ac^2de)f^4 - 15\,(bc^2de^2 + acd^2e^2)f^3 + 9\,(3\,bcd^2e^3 + ad^3e^3)f^2)x^3 + (5\,ac^3f^4e - 11\,bd^3e^5 - (bc^3e^2 + 3\,ac^2de^2)f^3 - 9\,(bc^2de^3 + acd^2e^3)f^2 + 7\,(3\,bcd^2e^4 + ad^3e^4)f)x}{8\,(f^6x^4e^2 + 2\,f^5x^2e^3 + f^4e^4)} + \frac{(3\,ac^3f^4 + 35\,bd^3e^4 + (bc^3e + 3\,ac^2de)f^3 + 9\,(bc^2de^2 + acd^2e^2)f^2 - 15\,(3\,bcd^2e^3 + ad^3e^3)f)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{5}{2}\right)}}{8\,f^{\frac{9}{2}}} + \frac{bd^3fx^3 - 3\,(3\,bd^3e - (3\,bcd^2 + ad^3)f)x}{3\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a*c^3*f^5 - 13*b*d^3*f*e^4 + (b*c^3*e + 3*a*c^2*d*e)*f^4 - 15*(b*c^
2*d*e^2 + a*c*d^2*e^2)*f^3 + 9*(3*b*c*d^2*e^3 + a*d^3*e^3)*f^2)*x^3 + (5*a*
c^3*f^4*e - 11*b*d^3*e^5 - (b*c^3*e^2 + 3*a*c^2*d*e^2)*f^3 - 9*(b*c^2*d*e^3
 + a*c*d^2*e^3)*f^2 + 7*(3*b*c*d^2*e^4 + a*d^3*e^4)*f)*x)/(f^6*x^4*e^2 + 2*
f^5*x^2*e^3 + f^4*e^4) + 1/8*(3*a*c^3*f^4 + 35*b*d^3*e^4 + (b*c^3*e + 3*a*c
^2*d*e)*f^3 + 9*(b*c^2*d*e^2 + a*c*d^2*e^2)*f^2 - 15*(3*b*c*d^2*e^3 + a*d^3
*e^3)*f)*arctan(sqrt(f)*x*e^(-1/2))*e^(-5/2)/f^(9/2) + 1/3*(b*d^3*f*x^3 - 3
*(3*b*d^3*e - (3*b*c*d^2 + a*d^3)*f)*x)/f^4
```

Fricas [A]
time = 1.47, size = 1104, normalized size = 3.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(18*a*c^3*f^6*x^3*e - 210*b*d^3*f*x*e^6 - 3*(3*a*c^3*f^6*x^4 + 35*b*d
^3*e^6 + 5*(14*b*d^3*f*x^2 - 3*(3*b*c*d^2 + a*d^3)*f)*e^5 + (35*b*d^3*f^2*x
^4 - 30*(3*b*c*d^2 + a*d^3)*f^2*x^2 + 9*(b*c^2*d + a*c*d^2)*f^2)*e^4 - (15*
(3*b*c*d^2 + a*d^3)*f^3*x^4 - 18*(b*c^2*d + a*c*d^2)*f^3*x^2 - (b*c^3 + 3*a
*c^2*d)*f^3)*e^3 + (9*(b*c^2*d + a*c*d^2)*f^4*x^4 + 3*a*c^3*f^4 + 2*(b*c^3
+ 3*a*c^2*d)*f^4*x^2)*e^2 + (6*a*c^3*f^5*x^2 + (b*c^3 + 3*a*c^2*d)*f^5*x^4)
*e)*sqrt(-f*e)*log((f*x^2 - 2*sqrt(-f*e)*x - e)/(f*x^2 + e)) - 10*(35*b*d^3
*f^2*x^3 - 9*(3*b*c*d^2 + a*d^3)*f^2*x)*e^5 - 2*(56*b*d^3*f^3*x^5 - 75*(3*b
*c*d^2 + a*d^3)*f^3*x^3 + 27*(b*c^2*d + a*c*d^2)*f^3*x)*e^4 + 2*(8*b*d^3*f^
4*x^7 + 24*(3*b*c*d^2 + a*d^3)*f^4*x^5 - 45*(b*c^2*d + a*c*d^2)*f^4*x^3 - 3
*(b*c^3 + 3*a*c^2*d)*f^4*x)*e^3 + 6*(5*a*c^3*f^5*x + (b*c^3 + 3*a*c^2*d)*f^
5*x^3)*e^2)/(f^7*x^4*e^3 + 2*f^6*x^2*e^4 + f^5*e^5), 1/24*(9*a*c^3*f^6*x^3*
e - 105*b*d^3*f*x*e^6 + 3*(3*a*c^3*f^6*x^4 + 35*b*d^3*e^6 + 5*(14*b*d^3*f*x
^2 - 3*(3*b*c*d^2 + a*d^3)*f)*e^5 + (35*b*d^3*f^2*x^4 - 30*(3*b*c*d^2 + a*d
^3)*f^2*x^2 + 9*(b*c^2*d + a*c*d^2)*f^2)*e^4 - (15*(3*b*c*d^2 + a*d^3)*f^3*
x^4 - 18*(b*c^2*d + a*c*d^2)*f^3*x^2 - (b*c^3 + 3*a*c^2*d)*f^3)*e^3 + (9*(b
*c^2*d + a*c*d^2)*f^4*x^4 + 3*a*c^3*f^4 + 2*(b*c^3 + 3*a*c^2*d)*f^4*x^2)*e^
2 + (6*a*c^3*f^5*x^2 + (b*c^3 + 3*a*c^2*d)*f^5*x^4)*e)*sqrt(f)*arctan(sqrt(
f)*x*e^(-1/2))*e^(1/2) - 5*(35*b*d^3*f^2*x^3 - 9*(3*b*c*d^2 + a*d^3)*f^2*x)
*e^5 - (56*b*d^3*f^3*x^5 - 75*(3*b*c*d^2 + a*d^3)*f^3*x^3 + 27*(b*c^2*d + a
*c*d^2)*f^3*x)*e^4 + (8*b*d^3*f^4*x^7 + 24*(3*b*c*d^2 + a*d^3)*f^4*x^5 - 45
*(b*c^2*d + a*c*d^2)*f^4*x^3 - 3*(b*c^3 + 3*a*c^2*d)*f^4*x)*e^3 + 3*(5*a*c^
3*f^5*x + (b*c^3 + 3*a*c^2*d)*f^5*x^3)*e^2)/(f^7*x^4*e^3 + 2*f^6*x^2*e^4 +
f^5*e^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(291) = 582$.
time = 59.62, size = 865, normalized size = 2.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**3,x)
```

```
[Out] b*d**3*x**3/(3*f**3) + x*(a*d**3/f**3 + 3*b*c*d**2/f**3 - 3*b*d**3*e/f**4)
- sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**2 + 15*a*d
**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)*log(-e**3*f*
```

```
*4*sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**2 + 15*a*
d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)/(3*a*c**3*f
**4 + 3*a*c**2*d*e*f**3 + 9*a*c*d**2*e**2*f**2 - 15*a*d**3*e**3*f + b*c**3*
e*f**3 + 9*b*c**2*d*e**2*f**2 - 45*b*c*d**2*e**3*f + 35*b*d**3*e**4) + x)/1
6 + sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**2 + 15*a
*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)*log(e**3*f
**4*sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**2 + 15*a
*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)/(3*a*c**3*
f**4 + 3*a*c**2*d*e*f**3 + 9*a*c*d**2*e**2*f**2 - 15*a*d**3*e**3*f + b*c**3
*e*f**3 + 9*b*c**2*d*e**2*f**2 - 45*b*c*d**2*e**3*f + 35*b*d**3*e**4) + x)/
16 + (x**3*(3*a*c**3*f**5 + 3*a*c**2*d*e*f**4 - 15*a*c*d**2*e**2*f**3 + 9*a
*d**3*e**3*f**2 + b*c**3*e*f**4 - 15*b*c**2*d*e**2*f**3 + 27*b*c*d**2*e**3*
f**2 - 13*b*d**3*e**4*f) + x*(5*a*c**3*e*f**4 - 3*a*c**2*d*e**2*f**3 - 9*a*
c*d**2*e**3*f**2 + 7*a*d**3*e**4*f - b*c**3*e**2*f**3 - 9*b*c**2*d*e**3*f**
2 + 21*b*c*d**2*e**4*f - 11*b*d**3*e**5))/(8*e**4*f**4 + 16*e**3*f**5*x**2
+ 8*e**2*f**6*x**4)
```

Giac [A]
time = 1.68, size = 371, normalized size = 1.27

$$\frac{\left(3\,ac^3f^4+bc^3f^3e+3\,ac^2df^3e+9\,bc^2df^2e^2+9\,acd^2f^2e^2-45\,bcd^2fe^3-15\,ad^3fe^3+35\,bd^3e^4\right)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{5}{2}\right)}}{8\,f^{\frac{9}{2}}}+\frac{\left(3\,ac^3f^5x^3+bc^3f^4x^3e+3\,ac^2df^4x^3e-15\,bc^2df^3x^3e^2-15\,acd^2f^3x^3e^2+27\,bcd^2f^2x^3e^3+9\,ad^3f^2x^3e^3+5\,ac^3f^4xe-13\,bd^3fx^3e^4-bc^3f^3xe^2-3\,ac^2df^3xe^2-9\,bc^2df^2xe^3-9\,acd^2f^2xe^3+21\,bcd^2fxe^4+7\,ad^3fxe^4-11\,bd^3xe^5\right)e^{(-2)}}{8\,(fx^2+e)^2f^4}+\frac{bd^3f^6x^3+9\,bcd^2f^6x+3\,ad^3f^6x-9\,bd^3f^5xe}{3\,f^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*a*c^3*f^4 + b*c^3*f^3*e + 3*a*c^2*d*f^3*e + 9*b*c^2*d*f^2*e^2 + 9*a*
c*d^2*f^2*e^2 - 45*b*c*d^2*f*e^3 - 15*a*d^3*f*e^3 + 35*b*d^3*e^4)*arctan(sq
rt(f)*x*e^(-1/2))*e^(-5/2)/f^(9/2) + 1/8*(3*a*c^3*f^5*x^3 + b*c^3*f^4*x^3*e
 + 3*a*c^2*d*f^4*x^3*e - 15*b*c^2*d*f^3*x^3*e^2 - 15*a*c*d^2*f^3*x^3*e^2 +
27*b*c*d^2*f^2*x^3*e^3 + 9*a*d^3*f^2*x^3*e^3 + 5*a*c^3*f^4*x*e - 13*b*d^3*f
*x^3*e^4 - b*c^3*f^3*x*e^2 - 3*a*c^2*d*f^3*x*e^2 - 9*b*c^2*d*f^2*x*e^3 - 9*
a*c*d^2*f^2*x*e^3 + 21*b*c*d^2*f*x*e^4 + 7*a*d^3*f*x*e^4 - 11*b*d^3*x*e^5)*
e^(-2)/((f*x^2 + e)^2*f^4) + 1/3*(b*d^3*f^6*x^3 + 9*b*c*d^2*f^6*x + 3*a*d^3
*f^6*x - 9*b*d^3*f^5*x*e)/f^9
```

Mupad [B]
time = 0.22, size = 495, normalized size = 1.70

$$\frac{\frac{x^3\,\left(b\,c^3\,e\,f^4+3\,a\,c^3\,f^5-15\,b\,c^2\,d\,e^2\,f^3+3\,a\,c^2\,d\,e\,f^4+27\,b\,c\,d^2\,e^3\,f^2-15\,a\,c\,d^2\,e^2\,f^3-13\,b\,d^3\,e^4\,f+9\,a\,d^3\,e^3\,f^2\right)}{8\,e^2}-\frac{x\,\left(b\,c^3\,e\,f^3-5\,a\,c^3\,f^4+9\,b\,c^2\,d\,e^2\,f^2+3\,a\,c^2\,d\,e\,f^3-21\,b\,c\,d^2\,e^3\,f+9\,a\,c\,d^2\,e^2\,f^2+11\,b\,d^3\,e^4-7\,a\,d^3\,e^3\,f\right)}{8\,e}}{e^2\,f^4+2\,e\,f^5\,x^2+f^6\,x^4}+x\,\left(\frac{a\,d^3+3\,b\,c\,d^2}{f^3}-\frac{3\,b\,d^3\,e}{f^4}\right)+\frac{b\,d^3\,x^3}{3\,f^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,\left(c\,f-d\,e\right)\,\left(b\,c^2\,e\,f^2+3\,a\,c^2\,f^3+10\,b\,c\,d\,e^2\,f+6\,a\,c\,d\,e\,f^2-35\,b\,d^2\,e^3+15\,a\,d^2\,e^2\,f\right)}{\sqrt{e}\,\left(b\,c^3\,e\,f^3+3\,a\,c^3\,f^4+9\,b\,c^2\,d\,e^2\,f^2+3\,a\,c^2\,d\,e\,f^3-45\,b\,c\,d^2\,e^3\,f+9\,a\,c\,d^2\,e^2\,f^2+35\,b\,d^3\,e^4-15\,a\,d^3\,e^3\,f\right)}\right)\,\left(c\,f-d\,e\right)\,\left(b\,c^2\,e\,f^2+3\,a\,c^2\,f^3+10\,b\,c\,d\,e^2\,f+6\,a\,c\,d\,e\,f^2-35\,b\,d^2\,e^3+15\,a\,d^2\,e^2\,f\right)}{8\,e^{5/2}\,f^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^3,x)
```

```
[Out] ((x^3*(3*a*c^3*f^5 + 9*a*d^3*e^3*f^2 + b*c^3*e*f^4 - 13*b*d^3*e^4*f + 3*a*c
^2*d*e*f^4 - 15*a*c*d^2*e^2*f^3 + 27*b*c*d^2*e^3*f^2 - 15*b*c^2*d*e^2*f^3))
/(8*e^2) - (x*(11*b*d^3*e^4 - 5*a*c^3*f^4 - 7*a*d^3*e^3*f + b*c^3*e*f^3 + 3
```

```
*a*c^2*d*e*f^3 - 21*b*c*d^2*e^3*f + 9*a*c*d^2*e^2*f^2 + 9*b*c^2*d*e^2*f^2))
/(8*e))/(e^2*f^4 + f^6*x^4 + 2*e*f^5*x^2) + x*((a*d^3 + 3*b*c*d^2)/f^3 - (3
*b*d^3*e)/f^4) + (b*d^3*x^3)/(3*f^3) + (atan((f^(1/2)*x*(c*f - d*e)*(3*a*c^
2*f^3 - 35*b*d^2*e^3 + 15*a*d^2*e^2*f + b*c^2*e*f^2 + 6*a*c*d*e*f^2 + 10*b*
c*d*e^2*f))/(e^(1/2)*(3*a*c^3*f^4 + 35*b*d^3*e^4 - 15*a*d^3*e^3*f + b*c^3*e
*f^3 + 3*a*c^2*d*e*f^3 - 45*b*c*d^2*e^3*f + 9*a*c*d^2*e^2*f^2 + 9*b*c^2*d*e
^2*f^2)))*(c*f - d*e)*(3*a*c^2*f^3 - 35*b*d^2*e^3 + 15*a*d^2*e^2*f + b*c^2*
e*f^2 + 6*a*c*d*e*f^2 + 10*b*c*d*e^2*f))/(8*e^(5/2)*f^(9/2))
```

3.22 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4} \, dx$

Optimal. Leaf size=348

$$\frac{d(be(105d^2e^2-10cdef-3c^2f^2)-af(15d^2e^2+14cdef+15c^2f^2))\,x}{48e^3f^4}-\frac{(be-af)x(c+dx^2)^3}{6ef\,(e+fx^2)^3}-\frac{(be(7de-cf)}{24}$$

```
[Out] 1/48*d*(b*e*(-3*c^2*f^2-10*c*d*e*f+105*d^2*e^2)-a*f*(15*c^2*f^2+14*c*d*e*f+
15*d^2*e^2))*x/e^3/f^4-1/6*(-a*f+b*e)*x*(d*x^2+c)^3/e/f/(f*x^2+e)^3-1/24*(b
*e*(-c*f+7*d*e)-a*f*(5*c*f+d*e))*x*(d*x^2+c)^2/e^2/f^2/(f*x^2+e)^2-1/48*(b*
e*(-3*c^2*f^2-8*c*d*e*f+35*d^2*e^2)-a*f*(15*c^2*f^2+4*c*d*e*f+5*d^2*e^2))*x
*(d*x^2+c)/e^3/f^3/(f*x^2+e)-1/16*(b*e*(-c^3*f^3-3*c^2*d*e*f^2-15*c*d^2*e^2
*f+35*d^3*e^3)-a*f*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^2*e^2*f+5*d^3*e^3))*arcta
n(x*f^(1/2)/e^(1/2))/e^(7/2)/f^(9/2)
```

Rubi [A]
time = 0.29, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {540, 396, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(be(-c^3f^3-3c^2def^2-15cd^2e^2f+35d^3e^3)-af(5c^3f^3+3c^2def^2+3cd^2e^2f+5d^3e^3))}{16e^{7/2}f^{9/2}}+\frac{dx(be(-3c^2f^2-10cdef+105d^2e^2)-af(15c^2f^2+14cdef+15d^2e^2))}{48e^3f^4}-\frac{x(c+dx^2)(be(-3c^2f^2-8cdef+35d^2e^2)-af(15c^2f^2+4cdef+5d^2e^2))}{48e^3f^3(e+fx^2)}-\frac{x(c+dx^2)^2(be(7de-cf)-af(5cf+de))}{24e^2f^2(e+fx^2)^2}-\frac{x(c+dx^2)^3(be-af)}{6ef(e+fx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

```
[Out] (d*(b*e*(105*d^2*e^2 - 10*c*d*e*f - 3*c^2*f^2) - a*f*(15*d^2*e^2 + 14*c*d*e
*f + 15*c^2*f^2))*x)/(48*e^3*f^4) - ((b*e - a*f)*x*(c + d*x^2)^3)/(6*e*f*(e
 + f*x^2)^3) - ((b*e*(7*d*e - c*f) - a*f*(d*e + 5*c*f))*x*(c + d*x^2)^2)/(2
4*e^2*f^2*(e + f*x^2)^2) - ((b*e*(35*d^2*e^2 - 8*c*d*e*f - 3*c^2*f^2) - a*f
*(5*d^2*e^2 + 4*c*d*e*f + 15*c^2*f^2))*x*(c + d*x^2))/(48*e^3*f^3*(e + f*x^
2)) - ((b*e*(35*d^3*e^3 - 15*c*d^2*e^2*f - 3*c^2*d*e*f^2 - c^3*f^3) - a*f*(
5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f]*x)/
Sqrt[e]])/(16*e^(7/2)*f^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx = -\frac{(be-af)x(c+dx^2)^3}{6ef(e+fx^2)^3} - \frac{\int \frac{(c+dx^2)^2(-c(be+5af)-d(7be-af)x^2)}{(e+fx^2)^3}\,dx}{6ef}$$

$$= -\frac{(be-af)x(c+dx^2)^3}{6ef(e+fx^2)^3} - \frac{(be(7de-cf)-af(de+5cf))x(c+dx^2)^2}{24e^2f^2(e+fx^2)^2} + \frac{\int (c+\ldots}{\ldots}$$

$$= -\frac{(be-af)x(c+dx^2)^3}{6ef(e+fx^2)^3} - \frac{(be(7de-cf)-af(de+5cf))x(c+dx^2)^2}{24e^2f^2(e+fx^2)^2} - \frac{(be(35\ldots}{\ldots}$$

$$= \frac{d(be(105d^2e^2-10cdef-3c^2f^2)-af(15d^2e^2+14cdef+15c^2f^2))x}{48e^3f^4} - \frac{(be-\ldots}{6e\ldots}$$

$$= \frac{d(be(105d^2e^2-10cdef-3c^2f^2)-af(15d^2e^2+14cdef+15c^2f^2))x}{48e^3f^4} - \frac{(be-\ldots}{6e\ldots}$$

Mathematica [A]
time = 0.14, size = 295, normalized size = 0.85

$$\frac{bd^3x}{f^4} + \frac{(be-af)(de-cf)^3x}{6ef^4(e+fx^2)^3} - \frac{(de-cf)^2(be(19de-cf)-af(13de+5cf))x}{24e^2f^4(e+fx^2)^2} + \frac{(de-cf)(be(29d^2e^2-4cdef-c^2f^2)-af(11d^2e^2+8cdef+5c^2f^2))x}{16e^3f^4(e+fx^2)} - \frac{(be(35d^3e^3-15cd^2e^2f-3c^2def^2-c^3f^3)-af(5d^3e^3+3cd^2e^2f+3c^2def^2+5c^3f^3))\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

```
[Out] (b*d^3*x)/f^4 + ((b*e - a*f)*(d*e - c*f)^3*x)/(6*e*f^4*(e + f*x^2)^3) - ((d
*e - c*f)^2*(b*e*(19*d*e - c*f) - a*f*(13*d*e + 5*c*f))*x)/(24*e^2*f^4*(e +
 f*x^2)^2) + ((d*e - c*f)*(b*e*(29*d^2*e^2 - 4*c*d*e*f - c^2*f^2) - a*f*(11
*d^2*e^2 + 8*c*d*e*f + 5*c^2*f^2))*x)/(16*e^3*f^4*(e + f*x^2)) - ((b*e*(35*
d^3*e^3 - 15*c*d^2*e^2*f - 3*c^2*d*e*f^2 - c^3*f^3) - a*f*(5*d^3*e^3 + 3*c*
```

```
d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^
(7/2)*f^(9/2))
```

Maple [A]
time = 0.18, size = 417, normalized size = 1.20

method	result
default	$\frac{b\,d^{3} x}{f^{4}}+\frac{\frac{f^{2} \left(5 a\,c^{3} f^{4}+3 a\,c^{2} d e\,f^{3}+3 a c\,d^{2} e^{2} f^{2}-11 a\,d^{3} e^{3} f+b\,c^{3} e\,f^{3}+3 b\,c^{2} d\,e^{2} f^{2}-33 b c\,d^{2} e^{3} f+29 b\,d^{3} e^{4}\right) x^{5}}{16 e^{3}}+\frac{f \left(5 a\,c^{3} f^{4}+3 a\,c^{2} d e\,f^{3}-3 a c\,d^{2} e^{2} f^{2}-\right.}{}}{}$
risch	$\frac{b\,d^{3} x}{f^{4}}+\frac{\frac{f^{2} \left(5 a\,c^{3} f^{4}+3 a\,c^{2} d e\,f^{3}+3 a c\,d^{2} e^{2} f^{2}-11 a\,d^{3} e^{3} f+b\,c^{3} e\,f^{3}+3 b\,c^{2} d\,e^{2} f^{2}-33 b c\,d^{2} e^{3} f+29 b\,d^{3} e^{4}\right) x^{5}}{16 e^{3}}+\frac{f \left(5 a\,c^{3} f^{4}+3 a\,c^{2} d e\,f^{3}-3 a c\,d^{2} e^{2} f^{2}-\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] b*d^3/f^4*x+1/f^4*((1/16*f^2*(5*a*c^3*f^4+3*a*c^2*d*e*f^3+3*a*c*d^2*e^2*f^2
-11*a*d^3*e^3*f+b*c^3*e*f^3+3*b*c^2*d*e^2*f^2-33*b*c*d^2*e^3*f+29*b*d^3*e^4
)/e^3*x^5+1/6*f*(5*a*c^3*f^4+3*a*c^2*d*e*f^3-3*a*c*d^2*e^2*f^2-5*a*d^3*e^3*
f+b*c^3*e*f^3-3*b*c^2*d*e^2*f^2-15*b*c*d^2*e^3*f+17*b*d^3*e^4)/e^2*x^3+1/16
*(11*a*c^3*f^4-3*a*c^2*d*e*f^3-3*a*c*d^2*e^2*f^2-5*a*d^3*e^3*f-b*c^3*e*f^3-
3*b*c^2*d*e^2*f^2-15*b*c*d^2*e^3*f+19*b*d^3*e^4)/e*x)/(f*x^2+e)^3+1/16*(5*a
*c^3*f^4+3*a*c^2*d*e*f^3+3*a*c*d^2*e^2*f^2+5*a*d^3*e^3*f+b*c^3*e*f^3+3*b*c^
2*d*e^2*f^2+15*b*c*d^2*e^3*f-35*b*d^3*e^4)/e^3/(f*e)^(1/2)*arctan(f*x/(f*e)
^(1/2)))
```

Maxima [A]
time = 0.49, size = 416, normalized size = 1.20

$$\frac{bd^3x}{f^4}+\frac{3\,(5\,ac^3f^6+29\,bd^3f^2e^4+(bc^3e+3\,ac^2de)f^5+3\,(bc^2de^2+acd^2e^2)f^4-11\,(3\,bcd^2e^3+ad^3e^3)f^3)x^5+8\,(5\,ac^3f^5e+17\,bd^3fe^5+(bc^3e^2+3\,ac^2de^2)f^4-3\,(bc^2de^3+acd^2e^3)f^3-5\,(3\,bcd^2e^4+ad^3e^4)f^2)x^3+3\,(11\,ac^3f^4e^2+19\,bd^3e^6-(bc^3e^3+3\,ac^2de^3)f^3-3\,(bc^2de^4+acd^2e^4)f^2-5\,(3\,bcd^2e^5+ad^3e^5)f)x}{48\,(f^7x^6e^3+3\,f^6x^4e^4+3\,f^5x^2e^5+f^4e^6)}+\frac{(5\,ac^3f^4-35\,bd^3e^4+(bc^3e+3\,ac^2de)f^3+3\,(bc^2de^2+acd^2e^2)f^2+5\,(3\,bcd^2e^3+ad^3e^3)f)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{7}{2}\right)}}{16\,f^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="maxima")
```

```
[Out] b*d^3*x/f^4 + 1/48*(3*(5*a*c^3*f^6 + 29*b*d^3*f^2*e^4 + (b*c^3*e + 3*a*c^2*
d*e)*f^5 + 3*(b*c^2*d*e^2 + a*c*d^2*e^2)*f^4 - 11*(3*b*c*d^2*e^3 + a*d^3*e^
3)*f^3)*x^5 + 8*(5*a*c^3*f^5*e + 17*b*d^3*f*e^5 + (b*c^3*e^2 + 3*a*c^2*d*e^
2)*f^4 - 3*(b*c^2*d*e^3 + a*c*d^2*e^3)*f^3 - 5*(3*b*c*d^2*e^4 + a*d^3*e^4)*
f^2)*x^3 + 3*(11*a*c^3*f^4*e^2 + 19*b*d^3*e^6 - (b*c^3*e^3 + 3*a*c^2*d*e^3)
*f^3 - 3*(b*c^2*d*e^4 + a*c*d^2*e^4)*f^2 - 5*(3*b*c*d^2*e^5 + a*d^3*e^5)*f)
*x)/(f^7*x^6*e^3 + 3*f^6*x^4*e^4 + 3*f^5*x^2*e^5 + f^4*e^6) + 1/16*(5*a*c^3
*f^4 - 35*b*d^3*e^4 + (b*c^3*e + 3*a*c^2*d*e)*f^3 + 3*(b*c^2*d*e^2 + a*c*d^
2*e^2)*f^2 + 5*(3*b*c*d^2*e^3 + a*d^3*e^3)*f)*arctan(sqrt(f)*x*e^(-1/2))*e^
(-7/2)/f^(9/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(326) = 652$.
time = 1.45, size = 1424, normalized size = 4.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(30*a*c^3*f^7*x^5*e + 210*b*d^3*f*x*e^7 + 3*(5*a*c^3*f^7*x^6 - 35*b*d
^3*e^7 - 5*(21*b*d^3*f*x^2 - (3*b*c*d^2 + a*d^3)*f)*e^6 - 3*(35*b*d^3*f^2*x
^4 - 5*(3*b*c*d^2 + a*d^3)*f^2*x^2 - (b*c^2*d + a*c*d^2)*f^2)*e^5 - (35*b*d
^3*f^3*x^6 - 15*(3*b*c*d^2 + a*d^3)*f^3*x^4 - 9*(b*c^2*d + a*c*d^2)*f^3*x^2
 - (b*c^3 + 3*a*c^2*d)*f^3)*e^4 + (5*(3*b*c*d^2 + a*d^3)*f^4*x^6 + 9*(b*c^2
*d + a*c*d^2)*f^4*x^4 + 5*a*c^3*f^4 + 3*(b*c^3 + 3*a*c^2*d)*f^4*x^2)*e^3 +
3*((b*c^2*d + a*c*d^2)*f^5*x^6 + 5*a*c^3*f^5*x^2 + (b*c^3 + 3*a*c^2*d)*f^5*
x^4)*e^2 + (15*a*c^3*f^6*x^4 + (b*c^3 + 3*a*c^2*d)*f^6*x^6)*e)*sqrt(-f*e)*l
og((f*x^2 + 2*sqrt(-f*e)*x - e)/(f*x^2 + e)) + 10*(56*b*d^3*f^2*x^3 - 3*(3*
b*c*d^2 + a*d^3)*f^2*x)*e^6 + 2*(231*b*d^3*f^3*x^5 - 40*(3*b*c*d^2 + a*d^3)
*f^3*x^3 - 9*(b*c^2*d + a*c*d^2)*f^3*x)*e^5 + 6*(16*b*d^3*f^4*x^7 - 11*(3*b
*c*d^2 + a*d^3)*f^4*x^5 - 8*(b*c^2*d + a*c*d^2)*f^4*x^3 - (b*c^3 + 3*a*c^2*
d)*f^4*x)*e^4 + 2*(9*(b*c^2*d + a*c*d^2)*f^5*x^5 + 33*a*c^3*f^5*x + 8*(b*c^
3 + 3*a*c^2*d)*f^5*x^3)*e^3 + 2*(40*a*c^3*f^6*x^3 + 3*(b*c^3 + 3*a*c^2*d)*f
^6*x^5)*e^2)/(f^8*x^6*e^4 + 3*f^7*x^4*e^5 + 3*f^6*x^2*e^6 + f^5*e^7), 1/48*
(15*a*c^3*f^7*x^5*e + 105*b*d^3*f*x*e^7 + 3*(5*a*c^3*f^7*x^6 - 35*b*d^3*e^7
 - 5*(21*b*d^3*f*x^2 - (3*b*c*d^2 + a*d^3)*f)*e^6 - 3*(35*b*d^3*f^2*x^4 - 5
*(3*b*c*d^2 + a*d^3)*f^2*x^2 - (b*c^2*d + a*c*d^2)*f^2)*e^5 - (35*b*d^3*f^3
*x^6 - 15*(3*b*c*d^2 + a*d^3)*f^3*x^4 - 9*(b*c^2*d + a*c*d^2)*f^3*x^2 - (b*
c^3 + 3*a*c^2*d)*f^3)*e^4 + (5*(3*b*c*d^2 + a*d^3)*f^4*x^6 + 9*(b*c^2*d + a
*c*d^2)*f^4*x^4 + 5*a*c^3*f^4 + 3*(b*c^3 + 3*a*c^2*d)*f^4*x^2)*e^3 + 3*((b*
c^2*d + a*c*d^2)*f^5*x^6 + 5*a*c^3*f^5*x^2 + (b*c^3 + 3*a*c^2*d)*f^5*x^4)*e
^2 + (15*a*c^3*f^6*x^4 + (b*c^3 + 3*a*c^2*d)*f^6*x^6)*e)*sqrt(f)*arctan(sqr
t(f)*x*e^(-1/2))*e^(1/2) + 5*(56*b*d^3*f^2*x^3 - 3*(3*b*c*d^2 + a*d^3)*f^2*
x)*e^6 + (231*b*d^3*f^3*x^5 - 40*(3*b*c*d^2 + a*d^3)*f^3*x^3 - 9*(b*c^2*d +
 a*c*d^2)*f^3*x)*e^5 + 3*(16*b*d^3*f^4*x^7 - 11*(3*b*c*d^2 + a*d^3)*f^4*x^5
 - 8*(b*c^2*d + a*c*d^2)*f^4*x^3 - (b*c^3 + 3*a*c^2*d)*f^4*x)*e^4 + (9*(b*c
^2*d + a*c*d^2)*f^5*x^5 + 33*a*c^3*f^5*x + 8*(b*c^3 + 3*a*c^2*d)*f^5*x^3)*e
^3 + (40*a*c^3*f^6*x^3 + 3*(b*c^3 + 3*a*c^2*d)*f^6*x^5)*e^2)/(f^8*x^6*e^4 +
 3*f^7*x^4*e^5 + 3*f^6*x^2*e^6 + f^5*e^7)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.26, size = 447, normalized size = 1.28

$$\frac{bd^3x}{f^4}+\frac{\left(5\,ac^3f^4+bc^3f^3e+3\,ac^2df^3e+3\,bc^2df^2e^2+3\,acd^2f^2e^2+15\,bcd^2fe^3+5\,ad^3fe^3-35\,bd^3e^4\right)\arctan\left(\sqrt{f}\,xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{7}{2}\right)}}{16\,f^{\frac{9}{2}}}+\frac{\left(15\,ac^3f^6x^5+3\,bc^3f^5x^5e+9\,ac^2df^5x^5e+9\,bc^2df^4x^5e^2+9\,acd^2f^4x^5e^2-99\,bcd^2f^3x^5e^3-33\,ad^3f^3x^5e^3+40\,ac^3f^5x^3e+87\,bd^3f^2x^5e^4+8\,bc^3f^4x^3e^2+24\,ac^2df^4x^3e^2-24\,bc^2df^3x^3e^3-24\,acd^2f^3x^3e^3-120\,bcd^2f^2x^3e^4-40\,ad^3f^2x^3e^4+33\,ac^3f^4xe^2+136\,bd^3fx^3e^5-3\,bc^3f^3xe^3-9\,ac^2df^3xe^3-9\,bc^2df^2xe^4-9\,acd^2f^2xe^4-45\,bcd^2fxe^5-15\,ad^3fxe^5+57\,bd^3xe^6\right)e^{(-3)}}{48\left(fx^2+e\right)^3f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="giac")
```

```
[Out] b*d^3*x/f^4 + 1/16*(5*a*c^3*f^4 + b*c^3*f^3*e + 3*a*c^2*d*f^3*e + 3*b*c^2*d
*f^2*e^2 + 3*a*c*d^2*f^2*e^2 + 15*b*c*d^2*f*e^3 + 5*a*d^3*f*e^3 - 35*b*d^3*
e^4)*arctan(sqrt(f)*x*e^(-1/2))*e^(-7/2)/f^(9/2) + 1/48*(15*a*c^3*f^6*x^5 +
 3*b*c^3*f^5*x^5*e + 9*a*c^2*d*f^5*x^5*e + 9*b*c^2*d*f^4*x^5*e^2 + 9*a*c*d^
2*f^4*x^5*e^2 - 99*b*c*d^2*f^3*x^5*e^3 - 33*a*d^3*f^3*x^5*e^3 + 40*a*c^3*f^
5*x^3*e + 87*b*d^3*f^2*x^5*e^4 + 8*b*c^3*f^4*x^3*e^2 + 24*a*c^2*d*f^4*x^3*e
^2 - 24*b*c^2*d*f^3*x^3*e^3 - 24*a*c*d^2*f^3*x^3*e^3 - 120*b*c*d^2*f^2*x^3*
e^4 - 40*a*d^3*f^2*x^3*e^4 + 33*a*c^3*f^4*x*e^2 + 136*b*d^3*f*x^3*e^5 - 3*b
*c^3*f^3*x*e^3 - 9*a*c^2*d*f^3*x*e^3 - 9*b*c^2*d*f^2*x*e^4 - 9*a*c*d^2*f^2*
x*e^4 - 45*b*c*d^2*f*x*e^5 - 15*a*d^3*f*x*e^5 + 57*b*d^3*x*e^6)*e^(-3)/((f*
x^2 + e)^3*f^4)
```

Mupad [B]
time = 1.19, size = 444, normalized size = 1.28

$$\frac{\frac{x^3\,\left(b\,c^3\,e\,f^4+5\,a\,c^3\,f^5-3\,b\,c^2\,d\,e^2\,f^3+3\,a\,c^2\,d\,e\,f^4-15\,b\,c\,d^2\,e^3\,f^2-3\,a\,c\,d^2\,e^2\,f^3+17\,b\,d^3\,e^4\,f-5\,a\,d^3\,e^3\,f^2\right)}{6\,e^2}+\frac{x^5\,\left(b\,c^3\,e\,f^5+5\,a\,c^3\,f^6+3\,b\,c^2\,d\,e^2\,f^4+3\,a\,c^2\,d\,e\,f^5-33\,b\,c\,d^2\,e^3\,f^3+3\,a\,c\,d^2\,e^2\,f^4+29\,b\,d^3\,e^4\,f^2-11\,a\,d^3\,e^3\,f^3\right)}{16\,e^3}-\frac{x\,\left(b\,c^3\,e\,f^3-11\,a\,c^3\,f^4+3\,b\,c^2\,d\,e^2\,f^2+3\,a\,c^2\,d\,e\,f^3+15\,b\,c\,d^2\,e^3\,f+3\,a\,c\,d^2\,e^2\,f^2-19\,b\,d^3\,e^4+5\,a\,d^3\,e^3\,f\right)}{16\,e}}{e^3\,f^4+3\,e^2\,f^5\,x^2+3\,e\,f^6\,x^4+f^7\,x^6}+\frac{b\,d^3\,x}{f^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,\left(b\,c^3\,e\,f^3+5\,a\,c^3\,f^4+3\,b\,c^2\,d\,e^2\,f^2+3\,a\,c^2\,d\,e\,f^3+15\,b\,c\,d^2\,e^3\,f+3\,a\,c\,d^2\,e^2\,f^2-35\,b\,d^3\,e^4+5\,a\,d^3\,e^3\,f\right)}{16\,e^{7/2}\,f^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^4,x)
```

```
[Out] ((x^3*(5*a*c^3*f^5 - 5*a*d^3*e^3*f^2 + b*c^3*e*f^4 + 17*b*d^3*e^4*f + 3*a*c
^2*d*e*f^4 - 3*a*c*d^2*e^2*f^3 - 15*b*c*d^2*e^3*f^2 - 3*b*c^2*d*e^2*f^3))/(
6*e^2) + (x^5*(5*a*c^3*f^6 - 11*a*d^3*e^3*f^3 + 29*b*d^3*e^4*f^2 + b*c^3*e*
f^5 + 3*a*c^2*d*e*f^5 + 3*a*c*d^2*e^2*f^4 - 33*b*c*d^2*e^3*f^3 + 3*b*c^2*d*
e^2*f^4))/(16*e^3) - (x*(5*a*d^3*e^3*f - 19*b*d^3*e^4 - 11*a*c^3*f^4 + b*c^
3*e*f^3 + 3*a*c^2*d*e*f^3 + 15*b*c*d^2*e^3*f + 3*a*c*d^2*e^2*f^2 + 3*b*c^2*
d*e^2*f^2))/(16*e))/(e^3*f^4 + f^7*x^6 + 3*e*f^6*x^4 + 3*e^2*f^5*x^2) + (b*
d^3*x)/f^4 + (atan((f^(1/2)*x)/e^(1/2))*(5*a*c^3*f^4 - 35*b*d^3*e^4 + 5*a*d
^3*e^3*f + b*c^3*e*f^3 + 3*a*c^2*d*e*f^3 + 15*b*c*d^2*e^3*f + 3*a*c*d^2*e^2
*f^2 + 3*b*c^2*d*e^2*f^2))/(16*e^(7/2)*f^(9/2))
```

3.23 $\int (a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}\,dx$

Optimal. Leaf size=544

$$-\frac{(7adf(2d^2e^2-7cdef-3c^2f^2)-b(8d^3e^3-19cd^2e^2f+9c^2def^2-6c^3f^3))\,x\sqrt{c+dx^2}}{105d^2f^2\sqrt{e+fx^2}}+\frac{(7adf(de+3cf)\cdots}{}$$

```
[Out] -1/105*(7*a*d*f*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e^2)-b*(-6*c^3*f^3+9*c^2*d*e*f^
2-19*c*d^2*e^2*f+8*d^3*e^3))*x*(d*x^2+c)^(1/2)/d^2/f^2/(f*x^2+e)^(1/2)-1/10
5*e^(3/2)*(7*a*d*f*(-9*c*f+d*e)-b*(-3*c^2*f^2-9*c*d*e*f+4*d^2*e^2))*(1/(1+f
*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1
/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/d/f^(5/2)/(e*(d*x^2+c)/c/(f*x^2+e))^
(1/2)/(f*x^2+e)^(1/2)+1/105*(7*a*d*f*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e^2)-b*(-6
*c^3*f^3+9*c^2*d*e*f^2-19*c*d^2*e^2*f+8*d^3*e^3))*(1/(1+f*x^2/e))^(1/2)*(1+
f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1
/2))*e^(1/2)*(d*x^2+c)^(1/2)/d^2/f^(5/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f
*x^2+e)^(1/2)+1/35*(7*a*d*f-2*b*c*f+b*d*e)*x*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2
)/d/f+1/7*b*x*(d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/d+1/105*(7*a*d*f*(3*c*f+d*e)-
b*(6*c^2*f^2-6*c*d*e*f+4*d^2*e^2))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d/f^2
```

Rubi [A]
time = 0.45, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{542, 545, 429, 506, 422\}$

$$-\frac{e^{3/2}\sqrt{c+dx^2}\,(7adf(de-9cf)-b(-3c^2f^2-9cdef+4d^2e^2))\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105df^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{e}\sqrt{c+dx^2}\,(7adf(-3c^2f^2-7cdef+2d^2e^2)-b(-6c^3f^3+9c^2def^2-19cd^2e^2f+8d^3e^3))\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105d^2f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\,(7adf(3cf+de)-b(6c^2f^2-6cdef+4d^2e^2))}{105df^2}-\frac{x\sqrt{c+dx^2}\,(7adf(-3c^2f^2-7cdef+2d^2e^2)-b(-6c^3f^3+9c^2def^2-19cd^2e^2f+8d^3e^3))}{105d^2f^2\sqrt{e+fx^2}}+\frac{x(c+dx^2)^{3/2}\sqrt{e+fx^2}\,(7adf-2bcf+bde)}{35df}+\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2],x]
```

```
[Out] -1/105*((7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) - b*(8*d^3*e^3 - 19*c*
d^2*e^2*f + 9*c^2*d*e*f^2 - 6*c^3*f^3))*x*Sqrt[c + d*x^2])/(d^2*f^2*Sqrt[e
+ f*x^2]) + ((7*a*d*f*(d*e + 3*c*f) - b*(4*d^2*e^2 - 6*c*d*e*f + 6*c^2*f^2)
)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(105*d*f^2) + ((b*d*e - 2*b*c*f + 7*a*
d*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(35*d*f) + (b*x*(c + d*x^2)^(5/2)
*Sqrt[e + f*x^2])/(7*d) + (Sqrt[e]*(7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*
f^2) - b*(8*d^3*e^3 - 19*c*d^2*e^2*f + 9*c^2*d*e*f^2 - 6*c^3*f^3))*Sqrt[c +
 d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d^2*f
^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (e^(3/2)*(7
*a*d*f*(d*e - 9*c*f) - b*(4*d^2*e^2 - 9*c*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^
2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d*f^(5/2)*
Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx = \frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7d} + \frac{\int \frac{(c+dx^2)^{3/2}(-(bc-7ad)e+(bde-2bcf+7adf)\ldots}{\sqrt{e+fx^2}}}{7d}$$

$$= \frac{(bde-2bcf+7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35df} + \frac{bx(c+dx^2)^{5/2}\ldots}{7d}$$

$$= \frac{(7adf(de+3cf)-b(4d^2e^2-6cdef+6c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+\ldots}}{105df^2}$$

$$= \frac{(7adf(de+3cf)-b(4d^2e^2-6cdef+6c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+\ldots}}{105df^2}$$

$$= -\frac{(7adf(2d^2e^2-7cdef-3c^2f^2)-b(8d^3e^3-19cd^2e^2f+9c^2def\ldots}{105d^2f^2\sqrt{e+fx^2}}$$

$$= -\frac{(7adf(2d^2e^2-7cdef-3c^2f^2)-b(8d^3e^3-19cd^2e^2f+9c^2def\ldots}{105d^2f^2\sqrt{e+fx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.37, size = 373, normalized size = 0.69

$$\frac{\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)(7adf(6cf+d(e+3fx^2))+b(3c^2f^2+3cdf(3e+8fx^2)+d^2(-4e^2+3efx^2+15f^2x^4)))+ie(7adf(2d^2e^2-7cdef-3c^2f^2)+b(-8d^3e^3+19cd^2e^2f-9c^2def^2+6c^3f^3))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ie(-de+cf)(-14adf(de-3cf)+b(8d^2e^2-15cdef+3c^2f^2))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)}{105d\sqrt{\frac{d}{c}}f^3\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2],x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(7*a*d*f*(6*c*f + d*(e + 3*f*x^2)) +
 b*(3*c^2*f^2 + 3*c*d*f*(3*e + 8*f*x^2) + d^2*(-4*e^2 + 3*e*f*x^2 + 15*f^2*
x^4))) + I*e*(7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) + b*(-8*d^3*e^3 +
 19*c*d^2*e^2*f - 9*c^2*d*e*f^2 + 6*c^3*f^3))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
(f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(d*e) + c
*f)*(-14*a*d*f*(d*e - 3*c*f) + b*(8*d^2*e^2 - 15*c*d*e*f + 3*c^2*f^2))*Sqrt
[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)
/(d*e)])/(105*d*Sqrt[d/c]*f^3*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1331 vs. 2(566) = 1132.
time = 0.18, size = 1332, normalized size = 2.45

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{bd\,x^5\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{7}+\frac{\left(a\,d^2f+2bcdf+b\,d^2e-\frac{bd(6cf+6de)}{7}\right)x^3\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{5df}\right.}{}$
risch	$\frac{x\left(15b\,x^4d^2f^2+21a\,d^2f^2x^2+24bcd\,f^2x^2+3b\,d^2ef\,x^2+42acd\,f^2+7a\,d^2ef+3b\,c^2f^2+9bcdef-4b\,d^2e^2\right)\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{105d\,f^2}+$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(-19*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f+9*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*
c^2*d*e^2*f^2+23*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f+21*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3+49*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c
*d^2*e^2*f^2+42*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3-56*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*f^2-18*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*
c^2*d*e^2*f^2+3*(-d/c)^(1/2)*b*c^3*e*f^3*x+51*(-d/c)^(1/2)*b*c*d^2*e*f^3*x^
5+70*(-d/c)^(1/2)*a*c*d^2*e*f^3*x^3+36*(-d/c)^(1/2)*b*c^2*d*e*f^3*x^3+8*(-d
/c)^(1/2)*b*c*d^2*e^2*f^2*x^3+39*(-d/c)^(1/2)*b*c*d^2*f^4*x^7+18*(-d/c)^(1/
2)*b*d^3*e*f^3*x^7+63*(-d/c)^(1/2)*a*c*d^2*f^4*x^5+28*(-d/c)^(1/2)*a*d^3*e*
f^3*x^5+27*(-d/c)^(1/2)*b*c^2*d*f^4*x^5-(-d/c)^(1/2)*b*d^3*e^2*f^2*x^5+14*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*a*d^3*e^3*f+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f^3-14*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^3*f-6*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3
*e*f^3+42*(-d/c)^(1/2)*a*c^2*d*e*f^3*x+7*(-d/c)^(1/2)*a*c*d^2*e^2*f^2*x+9*(
-d/c)^(1/2)*b*c^2*d*e^2*f^2*x-4*(-d/c)^(1/2)*b*c*d^2*e^3*f*x+42*(-d/c)^(1/2
```

```
)*a*c^2*d*f^4*x^3+7*(-d/c)^(1/2)*a*d^3*e^2*f^2*x^3-4*(-d/c)^(1/2)*b*d^3*e^3
*f*x^3+15*(-d/c)^(1/2)*b*d^3*f^4*x^9+21*(-d/c)^(1/2)*a*d^3*f^4*x^7+3*(-d/c)
^(1/2)*b*c^3*f^4*x^3-8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e^4+8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e^4)/d/(d*f*x^4+c*f*
x^2+d*e*x^2+c*e)/f^3/(-d/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx^2\right) \left(c + dx^2\right)^{\frac{3}{2}} \sqrt{e + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(3/2)*(f*x**2+e)**(1/2),x)
[Out] Integral((a + b*x**2)*(c + d*x**2)**(3/2)*sqrt(e + f*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{3/2}\,\sqrt{f\,x^2+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2),x)
[Out] int((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2), x)
```

3.24 $\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$

Optimal. Leaf size=381

$$\frac{(5adf(de+cf)-2b(d^2e^2-cdef+c^2f^2))\,x\sqrt{c+dx^2}}{15d^2f\sqrt{e+fx^2}}+\frac{(bde-2bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15df}+bx(c$$

```
[Out] 1/15*(5*a*d*f*(c*f+d*e)-2*b*(c^2*f^2-c*d*e*f+d^2*e^2))*x*(d*x^2+c)^(1/2)/d^
2/f/(f*x^2+e)^(1/2)-1/15*e^(3/2)*(-10*a*d*f+b*c*f+b*d*e)*(1/(1+f*x^2/e))^(1
/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/
c/f)^(1/2))*(d*x^2+c)^(1/2)/d/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^
2+e)^(1/2)-1/15*(5*a*d*f*(c*f+d*e)-2*b*(c^2*f^2-c*d*e*f+d^2*e^2))*(1/(1+f*x
^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2
),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/d^2/f^(3/2)/(e*(d*x^2+c)/c/(f*
x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/5*b*x*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/d+1/1
5*(5*a*d*f-2*b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d/f
```

Rubi [A]
time = 0.24, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {542, 545, 429, 506, 422}

$$-\frac{\sqrt{e}\sqrt{c+dx^2}\,(5adf(cf+de)-2b(c^2f^2-cdef+d^2e^2))\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15d^2f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{e^{3/2}\sqrt{c+dx^2}\,(-10adf+bcf+bde)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15df^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}\,(5adf(cf+de)-2b(c^2f^2-cdef+d^2e^2))}{15d^2f\sqrt{e+fx^2}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\,(5adf-2bcf+bde)}{15df}+\frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

```
[Out] ((5*a*d*f*(d*e + c*f) - 2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*x*Sqrt[c + d*x^2
])/(15*d^2*f*Sqrt[e + f*x^2]) + ((b*d*e - 2*b*c*f + 5*a*d*f)*x*Sqrt[c + d*x
^2]*Sqrt[e + f*x^2])/(15*d*f) + (b*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*
d) - (Sqrt[e]*(5*a*d*f*(d*e + c*f) - 2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*Sqr
t[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*d
^2*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (e^(3/2
)*(b*d*e + b*c*f - 10*a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(15*d*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2
))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \left(a+bx^2\right)\sqrt{c+dx^2}\,\sqrt{e+fx^2}\,dx = \frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5d} + \frac{\int \frac{\sqrt{c+dx^2}\,(-(bc-5ad)e+(bde-2bcf+5adf)x}{\sqrt{e+fx^2}}}{5d}$$

$$= \frac{(bde-2bcf+5adf)x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{15df} + \frac{bx(c+dx^2)^{3/2}\sqrt{e+}}{5d}$$

$$= \frac{(bde-2bcf+5adf)x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{15df} + \frac{bx(c+dx^2)^{3/2}\sqrt{e+}}{5d}$$

$$= \frac{(5adf(de+cf)-2b(d^2e^2-cdef+c^2f^2))\,x\sqrt{c+dx^2}}{15d^2f\sqrt{e+fx^2}} + \frac{(bde-2b}{}$$

$$= \frac{(5adf(de+cf)-2b(d^2e^2-cdef+c^2f^2))\,x\sqrt{c+dx^2}}{15d^2f\sqrt{e+fx^2}} + \frac{(bde-2b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.70, size = 267, normalized size = 0.70

$$\frac{\sqrt{\frac{d}{c}}\,fx(c+dx^2)(e+fx^2)(bcf+5adf+bd(e+3fx^2))+ie(-5adf(de+cf)+2b(d^2e^2-cdef+c^2f^2))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)\middle|\frac{cf}{de}\right)-ie(-de+cf)(-2bde+bcf+5adf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)\middle|\frac{cf}{de}\right)}{15d\sqrt{\frac{d}{c}}\,f^2\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(b*c*f + 5*a*d*f + b*d*(e + 3*f*x^2)
) + I*e*(-5*a*d*f*(d*e + c*f) + 2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*Sqrt[1 +
 (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*
e)] - I*e*(-(d*e) + c*f)*(-2*b*d*e + b*c*f + 5*a*d*f)*Sqrt[1 + (d*x^2)/c]*S
qrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(15*d*Sq
rt[d/c]*f^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 864 vs. 2(409) = 818.
time = 0.13, size = 865, normalized size = 2.27

method	result

elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(\frac{b x^3 \sqrt{df x^4+cf x^2+de x^2+ce}}{5}+\frac{\left(adf+bcf+bde-\frac{b(4cf+4de)}{5}\right) x \sqrt{df x^4+cf x^2+}}{3df}\right.}{}$
risch	$\frac{x(3bd x^2 f+5adf+bcf+bde)\sqrt{d x^2+c}\,\sqrt{f x^2+e}}{15df}+\frac{\left(-\frac{(5acd f^2+5a d^2 ef-2b c^2 f^2+2bcdef-2b d^2 e^2) e \sqrt{1+\frac{d x^2}{c}}\,\sqrt{1+}}{\sqrt{-\frac{d}{c}}\,\sqrt{}}\right.}{}$
default	$\frac{\sqrt{d x^2+c}\,\sqrt{f x^2+e}\left(3\sqrt{-\frac{d}{c}}\, b d^2 f^3 x^7+5\sqrt{-\frac{d}{c}}\, a d^2 f^3 x^5+4\sqrt{-\frac{d}{c}}\, bcd f^3 x^5+4\sqrt{-\frac{d}{c}}\, b d^2 e f^2 x^5+5\sqrt{-\frac{d}{c}}\, acd\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(3*(-d/c)^(1/2)*b*d^2*f^3*x^7+5*(-d/c)
^(1/2)*a*d^2*f^3*x^5+4*(-d/c)^(1/2)*b*c*d*f^3*x^5+4*(-d/c)^(1/2)*b*d^2*e*f^
2*x^5+5*(-d/c)^(1/2)*a*c*d*f^3*x^3+5*(-d/c)^(1/2)*a*d^2*e*f^2*x^3+(-d/c)^(1
/2)*b*c^2*f^3*x^3+5*(-d/c)^(1/2)*b*c*d*e*f^2*x^3+(-d/c)^(1/2)*b*d^2*e^2*f*x
^3+5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*a*c*d*e*f^2-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*e*f^2-3*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*
c*d*e^2*f+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*b*d^2*e^3+5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f^2+5*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*f-2*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*b*c^2*e*f^2+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^2*e^3+5*(-d/c)^(1/2)*a*c
*d*e*f^2*x+(-d/c)^(1/2)*b*c^2*e*f^2*x+(-d/c)^(1/2)*b*c*d*e^2*f*x)/(d*f*x^4+
c*f*x^2+d*e*x^2+c*e)/d/f^2/(-d/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)\sqrt{c+dx^2}\,\sqrt{e+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2),x)
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2),x)
[Out] int((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2), x)
```

3.25 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=283

$$\frac{(bde-2bcf+3adf)x\sqrt{c+dx^2}}{3d^2\sqrt{e+fx^2}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d}-\frac{\sqrt{e}\,(bde-2bcf+3adf)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\right.\right.}{3d^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] 1/3*(3*a*d*f-2*b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)/d^2/(f*x^2+e)^(1/2)-1/3*(-3*a
*d+b*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)
/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/c/d/f^(1/2)/(
e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(3*a*d*f-2*b*c*f+b*d*e)*
(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^
2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/d^2/f^(1/2)/(e*(d*x^2
+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*b*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1
/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {542, 545, 429, 506, 422}

$$-\frac{\sqrt{e}\sqrt{c+dx^2}\,(3adf-2bcf+bde)E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3d^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{e^{3/2}\sqrt{c+dx^2}\,(bc-3ad)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}\,(3adf-2bcf+bde)}{3d^2\sqrt{e+fx^2}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sqrt[e + f*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] ((b*d*e - 2*b*c*f + 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*d^2*Sqrt[e + f*x^2]) + (
b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*d) - (Sqrt[e]*(b*d*e - 2*b*c*f + 3*
a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(3*d^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
- ((b*c - 3*a*d)*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(3*c*d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]
*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3d} + \frac{\int \frac{-(bc-3ad)e+(bde-2bcf+3adf)x^2}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx}{3d}$$

$$= \frac{bx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3d} - \frac{((bc-3ad)e)\int \frac{1}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx}{3d} + \frac{(bde-}{}$$

$$= \frac{(bde-2bcf+3adf)x\sqrt{c+dx^2}}{3d^2\sqrt{e+fx^2}} + \frac{bx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3d} - \frac{(bc-3ad)e^{3/2}\sqrt{}}{3cd\sqrt{}}$$

$$= \frac{(bde-2bcf+3adf)x\sqrt{c+dx^2}}{3d^2\sqrt{e+fx^2}} + \frac{bx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3d} - \frac{\sqrt{e}\,(bde-2bcf}{3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.53, size = 212, normalized size = 0.75

$$\frac{b\sqrt{\frac{d}{c}}\,fx(c+dx^2)(e+fx^2)+ie(-bde+2bcf-3adf)\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)-ibe(-de+cf)\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)}{3d\sqrt{\frac{d}{c}}\,f\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2) + I*e*(-(b*d*e) + 2*b*c*f - 3*a*d*
f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x]
, (c*f)/(d*e)] - I*b*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/
e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*d*Sqrt[d/c]*f*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.13, size = 394, normalized size = 1.39

method	result
elliptic	$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\left(\frac{bx\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{3d}+\frac{\left(ae-\frac{ceb}{3d}\right)\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left(x\sqrt{}\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de}}\right.}{}$

risch	$\frac{bx\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{3d}+\frac{\left(-\frac{(3adf-2bcf+bde)e\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\left(\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}\right.}{}$
default	$\frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\left(\sqrt{-\frac{d}{c}}\,bd\,f^2x^5+\sqrt{-\frac{d}{c}}\,bc\,f^2x^3+\sqrt{-\frac{d}{c}}\,bdef\,x^3+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,\text{EllipticF}\left(x\sqrt{-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)*((-d/c)^(1/2)*b*d*f^2*x^5+(-d/c)^(1/2)*
b*c*f^2*x^3+(-d/c)^(1/2)*b*d*e*f*x^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e*f-((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d*e^2+3*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a*d*e*f-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*b*c*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d*e^2+(-d/c)^(1/2)*b*c*e*f*x)/(d*f*x^4+c
*f*x^2+d*e*x^2+c*e)/d/(-d/c)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(e + f*x**2)/sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)\,\sqrt{f\,x^2+e}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(1/2), x)
```

3.26 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=271

$$\frac{(2bc-ad)fx\sqrt{c+dx^2}}{cd^2\sqrt{e+fx^2}}-\frac{(bc-ad)x\sqrt{e+fx^2}}{cd\sqrt{c+dx^2}}-\frac{(2bc-ad)\sqrt{e}\,\sqrt{f}\,\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{cd^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}}$$

```
[Out] (-a*d+2*b*c)*f*x*(d*x^2+c)^(1/2)/c/d^2/(f*x^2+e)^(1/2)+b*e^(3/2)*(1/(1+f*x^
2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2)
,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/c/d/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(
1/2)/(f*x^2+e)^(1/2)-(-a*d+2*b*c)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*E
llipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(
1/2)*(d*x^2+c)^(1/2)/c/d^2/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-
(-a*d+b*c)*x*(f*x^2+e)^(1/2)/c/d/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {540, 545, 429, 506, 422}

$$-\frac{\sqrt{e}\,\sqrt{f}\,\sqrt{c+dx^2}\,(2bc-ad)E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{cd^2\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{fx\sqrt{c+dx^2}\,(2bc-ad)}{cd^2\sqrt{e+fx^2}}-\frac{x\sqrt{e+fx^2}\,(bc-ad)}{cd\sqrt{c+dx^2}}+\frac{be^{3/2}\sqrt{c+dx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{cd\sqrt{f}\,\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2),x]
```

```
[Out] ((2*b*c - a*d)*f*x*Sqrt[c + d*x^2])/(c*d^2*Sqrt[e + f*x^2]) - ((b*c - a*d)*
x*Sqrt[e + f*x^2])/(c*d*Sqrt[c + d*x^2]) - ((2*b*c - a*d)*Sqrt[e]*Sqrt[f]*S
qrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*
d^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b*e^(3/2)*Sqr
t[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*d*
Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = -\frac{(bc-ad)x\sqrt{e+fx^2}}{cd\sqrt{c+dx^2}} - \frac{\int \frac{-bce-(2bc-ad)fx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{cd}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{cd\sqrt{c+dx^2}} + \frac{(be)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{d} + \frac{((2bc-ad)f)\int}{}$$

$$= \frac{(2bc-ad)fx\sqrt{c+dx^2}}{cd^2\sqrt{e+fx^2}} - \frac{(bc-ad)x\sqrt{e+fx^2}}{cd\sqrt{c+dx^2}} + \frac{be^{3/2}\sqrt{c+dx^2}\,F\Big(\tan^{-1}}{cd\sqrt{f}\sqrt{\frac{e(c+dx^2}{c(e+fx^2}}}$$

$$= \frac{(2bc-ad)fx\sqrt{c+dx^2}}{cd^2\sqrt{e+fx^2}} - \frac{(bc-ad)x\sqrt{e+fx^2}}{cd\sqrt{c+dx^2}} - \frac{(2bc-ad)\sqrt{e}\sqrt{f}\sqrt{c+}}{cd^2\sqrt{\frac{e(c}{c(e}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.68, size = 192, normalized size = 0.71

$$\frac{-i(2bc-ad)e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)-(bc-ad)\left(\sqrt{\frac{d}{c}}\,x(e+fx^2)-ie\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)\right)}{c^2\left(\frac{d}{c}\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2),x]
```

```
[Out] ((-I)*(2*b*c - a*d)*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*A
rcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (b*c - a*d)*(Sqrt[d/c]*x*(e + f*x^2) -
I*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x
], (c*f)/(d*e)]))/(c^2*(d/c)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.13, size = 328, normalized size = 1.21

method	result
default	$\frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\left(\sqrt{-\frac{d}{c}}\,adf\,x^3-\sqrt{-\frac{d}{c}}\,bcf\,x^3+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ade-\right.}{}$

elliptic	$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\left(\frac{\left(df\,x^2+de\right)(ad-bc)x}{d^2c\sqrt{\left(x^2+\frac{c}{d}\right)(df\,x^2+de)}}+\frac{\left(\frac{adf-bcf+bde}{d^2}-\frac{(ad-bc)(cf-de)}{d^2c}-\frac{e(ad-bc)}{dc}\right)\sqrt{1+\frac{d\,x^2}{c}}}{\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+cf}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)*((-d/c)^(1/2)*a*d*f*x^3-(-d/c)^(1/2)*b*c*f*
x^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d
/e)^(1/2))*a*d*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*b*c*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellip
ticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e+(-d/c)^(1/2)*a*d*
e*x-(-d/c)^(1/2)*b*c*e*x)/d/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/c/(-d/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(e + f*x**2)/(c + d*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)\,\sqrt{f\,x^2+e}}{{\left(d\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(3/2),x)
```

```
[Out] int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(3/2), x)
```

3.27 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=274

$$-\frac{(bc-ad)x\sqrt{e+fx^2}}{3cd\left(c+dx^2\right)^{3/2}}+\frac{(d(bc+2ad)e-c(2bc+ad)f)\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}d^{3/2}(de-cf)\sqrt{c+dx^2}\,\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}+\frac{(bc-ad)e^{3}}{3c}$$

```
[Out] 1/3*(-a*d+b*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*
f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2
)/c^2/d/(-c*f+d*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(-a*
d+b*c)*x*(f*x^2+e)^(1/2)/c/d/(d*x^2+c)^(3/2)+1/3*(d*(2*a*d+b*c)*e-c*(a*d+2*
b*c)*f)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)
/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(3/2)/d^(3/2)/(-c*f
+d*e)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$,
Rules used = {540, 539, 429, 422}

$$\frac{\sqrt{e+fx^2}\,(de(2ad+bc)-cf(ad+2bc))E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}d^{3/2}\sqrt{c+dx^2}\,(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}+\frac{e^{3/2}\sqrt{f}\,\sqrt{c+dx^2}\,(bc-ad)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d\sqrt{e+fx^2}\,(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{x\sqrt{e+fx^2}\,(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2),x]
```

```
[Out] -1/3*((b*c - a*d)*x*Sqrt[e + f*x^2])/(c*d*(c + d*x^2)^(3/2)) + ((d*(b*c + 2
*a*d)*e - c*(2*b*c + a*d)*f)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/S
qrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*d^(3/2)*(d*e - c*f)*Sqrt[c + d*x^2]*S
qrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + ((b*c - a*d)*e^(3/2)*Sqrt[f]*Sqrt[c
 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*d
*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx &= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3cd\,(c+dx^2)^{3/2}} - \frac{\int \frac{-(bc+2ad)e-(2bc+ad)fx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx}{3cd}\\ &= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3cd\,(c+dx^2)^{3/2}} + \frac{((bc-ad)ef)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3cd(de-cf)} + \frac{(d(bc\\ &= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3cd\,(c+dx^2)^{3/2}} + \frac{(d(bc+2ad)e-c(2bc+ad)f)\sqrt{e+fx^2}\,E\left(\tan^{-}\right.}{3c^{3/2}d^{3/2}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+}{e(c+}}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.74, size = 297, normalized size = 1.08

$$\frac{\sqrt{\frac{d}{c}}\,x(e+fx^2)\left(ad(-3cde+2c^2f-2d^2ex^2+cdfx^2)+bc(c^2f-d^2ex^2+2cdfx^2)\right)+ie(ad(-2de+cf)+bc(-de+2cf))\,(c+dx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)-i(bc+2ad)e(-de+cf)\,(c+dx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)}{3c^3\left(\frac{d}{c}\right)^{3/2}(-de+cf)\,(c+dx^2)^{3/2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2),x]

[Out] (Sqrt[d/c]*x*(e + f*x^2)*(a*d*(-3*c*d*e + 2*c^2*f - 2*d^2*e*x^2 + c*d*f*x^2
) + b*c*(c^2*f - d^2*e*x^2 + 2*c*d*f*x^2)) + I*e*(a*d*(-2*d*e + c*f) + b*c*
(-(d*e) + 2*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellip
ticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*(b*c + 2*a*d)*e*(-(d*e) + c*f
)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[S
qrt[d/c]*x], (c*f)/(d*e)])/(3*c^3*(d/c)^(3/2)*(-(d*e) + c*f)*(c + d*x^2)^(3
/2)*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. $2(314) = 628$.
time = 0.14, size = 1237, normalized size = 4.51

method	result
elliptic	$\frac{\sqrt{\left(d\,x^{2}+c\right)\left(f\,x^{2}+e\right)}\left(\frac{\left(a d-b c\right) x \sqrt{d f\,x^{4}+c f\,x^{2}+d e\,x^{2}+c e}}{3 d^{3} c \left(x^{2}+\frac{c}{d}\right)^{2}}+\frac{\left(d f\,x^{2}+d e\right) x \left(a c d f-2 a\,d^{2} e+2 b\,c^{2} f-b c d e\right)}{3 d^{2} c^{2} \left(c f-d e\right) \sqrt{\left(x^{2}+\frac{c}{d}\right)\left(d f\,x^{2}+d e\right)}}+\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/3*(-2*(-d/c)^(1/2)*a*c^2*d*e*f*x+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^2*x^2-2*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e
^2*x^2-(-d/c)^(1/2)*b*c^3*e*f*x+(-d/c)^(1/2)*b*c*d^2*e*f*x^5+2*(-d/c)^(1/2)
*a*c*d^2*e*f*x^3-2*(-d/c)^(1/2)*b*c^2*d*e*f*x^3-2*(-d/c)^(1/2)*b*c^2*d*f^2*
x^5-2*(-d/c)^(1/2)*a*c^2*d*f^2*x^3+(-d/c)^(1/2)*b*c*d^2*e^2*x^3+3*(-d/c)^(1
/2)*a*c*d^2*e^2*x+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^2*x^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^2*x^2-2*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*
c^2*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*a*c^2*d*e*f+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2-((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f+((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b
*c^2*d*e^2-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a*c*d^2*e^2+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f-((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2+2*(-d
/c)^(1/2)*a*d^3*e^2*x^3-(-d/c)^(1/2)*b*c^3*f^2*x^3-(-d/c)^(1/2)*a*c*d^2*f^2
```

```
*x^5+2*(-d/c)^(1/2)*a*d^3*e*f*x^5-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e*f*x^2+2*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d
*e*f*x^2+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*a*c*d^2*e*f*x^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ell
ipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e*f*x^2)/(f*x^2+e)^(1/2)/(-d
/c)^(1/2)/(c*f-d*e)/c^2/d/(d*x^2+c)^(3/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)\sqrt{e + fx^2}}{(c + dx^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(e + f*x**2)/(c + d*x**2)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(5/2),x)
```

```
[Out] int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(5/2), x)
```

3.28 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=385

$$-\frac{(bc-ad)x\sqrt{e+fx^2}}{5cd\,(c+dx^2)^{5/2}}+\frac{(ad(4de-3cf)+bc(de-2cf))x\sqrt{e+fx^2}}{15c^2d(de-cf)\,(c+dx^2)^{3/2}}+\frac{(2bc(d^2e^2-cdef+c^2f^2)+ad(8d^2}{15c^{5/2}d^{3/}}$$

```
[Out] -1/15*e^(3/2)*(2*a*d*(-3*c*f+2*d*e)+b*c*(c*f+d*e))*(1/(1+f*x^2/e))^(1/2)*(1
+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(
1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^3/d/(-c*f+d*e)^2/(e*(d*x^2+c)/c/(f*x^2+e))^
(1/2)/(f*x^2+e)^(1/2)-1/5*(-a*d+b*c)*x*(f*x^2+e)^(1/2)/c/d/(d*x^2+c)^(5/2)+
1/15*(a*d*(-3*c*f+4*d*e)+b*c*(-2*c*f+d*e))*x*(f*x^2+e)^(1/2)/c^2/d/(-c*f+d*
e)/(d*x^2+c)^(3/2)+1/15*(2*b*c*(c^2*f^2-c*d*e*f+d^2*e^2)+a*d*(3*c^2*f^2-13*
c*d*e*f+8*d^2*e^2))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(
1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(5/2)/d
^(3/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {540, 541, 539, 429, 422}

$$-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,(2ad(2de-3cf)+bc(cf+de))F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3d\sqrt{e+fx^2}\,(de-cf)^2\sqrt{\frac{e\,(c+dx^2)}{c\,(e+fx^2)}}}+\frac{\sqrt{e+fx^2}\,(ad(3c^2f^2-13cdef+8d^2e^2)+2bc(c^2f^2-cdef+d^2e^2))\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}d^{3/2}\sqrt{c+dx^2}\,(de-cf)^2\sqrt{\frac{c\,(e+fx^2)}{e\,(c+dx^2)}}}+\frac{x\sqrt{e+fx^2}\,(ad(4de-3cf)+bc(de-2cf))}{15c^2d\,(c+dx^2)^{3/2}\,(de-cf)}-\frac{x\sqrt{e+fx^2}\,(bc-ad)}{5cd\,(c+dx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2),x]
```

```
[Out] -1/5*((b*c - a*d)*x*Sqrt[e + f*x^2])/(c*d*(c + d*x^2)^(5/2)) + ((a*d*(4*d*e
 - 3*c*f) + b*c*(d*e - 2*c*f))*x*Sqrt[e + f*x^2])/(15*c^2*d*(d*e - c*f)*(c
+ d*x^2)^(3/2)) + ((2*b*c*(d^2*e^2 - c*d*e*f + c^2*f^2) + a*d*(8*d^2*e^2 -
13*c*d*e*f + 3*c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (c*f)/(d*e)])/(15*c^(5/2)*d^(3/2)*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sq
rt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (e^(3/2)*Sqrt[f]*(2*a*d*(2*d*e - 3*c
*f) + b*c*(d*e + c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]
], 1 - (d*e)/(c*f)])/(15*c^3*d*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f
*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = -\frac{(bc-ad)x\sqrt{e+fx^2}}{5cd\,(c+dx^2)^{5/2}} - \frac{\int \frac{-(bc+4ad)e-(2bc+3ad)fx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx}{5cd}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5cd\,(c+dx^2)^{5/2}} + \frac{(ad(4de-3cf)+bc(de-2cf))x\sqrt{e+fx^2}}{15c^2d(de-cf)\,(c+dx^2)^{3/2}} + \frac{\int}{}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5cd\,(c+dx^2)^{5/2}} + \frac{(ad(4de-3cf)+bc(de-2cf))x\sqrt{e+fx^2}}{15c^2d(de-cf)\,(c+dx^2)^{3/2}} - \frac{(e}{}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5cd\,(c+dx^2)^{5/2}} + \frac{(ad(4de-3cf)+bc(de-2cf))x\sqrt{e+fx^2}}{15c^2d(de-cf)\,(c+dx^2)^{3/2}} + \frac{(2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.14, size = 379, normalized size = 0.98

$$\frac{-\sqrt{\frac{d}{c}}\,x(e+fx^2)\left(3c^2(bc-ad)(de-cf)^2-c(de-cf)(ad(4de-3cf)+bc(de-2cf))(c+dx^2)-(2bc(d^2e^2-cdef+c^2f^2)+ad(8d^2e^2-13cdef+3c^2f^2))(c+dx^2)^2\right)+ie(c+dx^2)^2\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left((2bc(d^2e^2-cdef+c^2f^2)+ad(8d^2e^2-13cdef+3c^2f^2))\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)-(-de+cf)(bc(-2de+cf)+ad(-8de+9cf))F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)\right)}{15c^4\left(\frac{d}{c}\right)^{3/2}(de-cf)^2\,(c+dx^2)^{5/2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2),x]
```

```
[Out] (-(Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)*(d*e - c*f)^2 - c*(d*e - c*f)
*(a*d*(4*d*e - 3*c*f) + b*c*(d*e - 2*c*f))*(c + d*x^2) - (2*b*c*(d^2*e^2 -
c*d*e*f + c^2*f^2) + a*d*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*(c + d*x^2)^
2)) + I*e*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*((2*b*c*(d^
2*e^2 - c*d*e*f + c^2*f^2) + a*d*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*Elli
pticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (-(d*e) + c*f)*(b*c*(-2*d*e +
c*f) + a*d*(-8*d*e + 9*c*f))*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
))/(15*c^4*(d/c)^(3/2)*(d*e - c*f)^2*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2860 vs. $2(419) = 838$.
time = 0.15, size = 2861, normalized size = 7.43

method	result

elliptic	$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\left(\frac{(ad-bc)x\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{5d^4c\left(x^2+\frac{c}{d}\right)^3}+\frac{\left(3acdf-4a\,d^2e+2b\,c^2f-bcde\right)x\sqrt{df\,x^4+cf\,x^2}}{15d^3c^2(cf-de)\left(x^2+\frac{c}{d}\right)^2}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(26*(-d/c)^(1/2)*a*c^3*d^2*e^2*f*x-(-d/c)^(1/2)*b*c^4*d*e^2*f*x+13*(-
d/c)^(1/2)*a*c*d^4*e*f^2*x^7+2*(-d/c)^(1/2)*b*c^2*d^3*e*f^2*x^7-2*(-d/c)^(1
/2)*b*c*d^4*e^2*f*x^7+30*(-d/c)^(1/2)*a*c^2*d^3*e*f^2*x^5-7*(-d/c)^(1/2)*a*
c*d^4*e^2*f*x^5+5*(-d/c)^(1/2)*b*c^3*d^2*e*f^2*x^5-3*(-d/c)^(1/2)*b*c^2*d^3
*e^2*f*x^5+17*(-d/c)^(1/2)*a*c^3*d^2*e*f^2*x^3+18*(-d/c)^(1/2)*a*c^2*d^3*e^
2*f*x^3-7*(-d/c)^(1/2)*b*c^4*d*e*f^2*x^3+7*(-d/c)^(1/2)*b*c^3*d^2*e^2*f*x^3
-9*(-d/c)^(1/2)*a*c^4*d*e*f^2*x-3*(-d/c)^(1/2)*a*c^2*d^3*f^3*x^7-8*(-d/c)^(
1/2)*a*d^5*e^2*f*x^7-2*(-d/c)^(1/2)*b*c^3*d^2*f^3*x^7-9*(-d/c)^(1/2)*a*c^3*
d^2*f^3*x^5-6*(-d/c)^(1/2)*b*c^4*d*f^3*x^5-2*(-d/c)^(1/2)*b*c*d^4*e^3*x^5-9
*(-d/c)^(1/2)*a*c^4*d*f^3*x^3-20*(-d/c)^(1/2)*a*c*d^4*e^3*x^3-5*(-d/c)^(1/2
)*b*c^2*d^3*e^3*x^3-15*(-d/c)^(1/2)*a*c^2*d^3*e^3*x-(-d/c)^(1/2)*b*c^5*e*f^
2*x+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*b*c^4*d*e^2*f+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipt
icE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^4*d*e*f^2-13*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^3*d^2*e^2*f
-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*b*c^4*d*e^2*f-8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^5*e^3*x^4+8*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^5*e^3*x^4-8*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*a*c^2*d^3*e^3-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*b*c^5*e*f^2-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^3+8*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*
d^3*e^3+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*b*c^5*e*f^2+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^3-8*(-d/c)^(1/2)*a*d^5*e^
3*x^5-(-d/c)^(1/2)*b*c^5*f^3*x^3-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^4*e^3*x^4+2*((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^4*
e^3*x^4-16*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*a*c*d^4*e^3*x^2-4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d^3*e^3*x^2+16*((d*x^2+c)/
```

```
c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*
d^4*e^3*x^2+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*b*c^2*d^3*e^3*x^2-9*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^4*d*e*f^2+17*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c
^3*d^2*e^2*f-9*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*a*c^2*d^3*e*f^2*x^4+17*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^4*e^2*f*x^4-((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*b*c^3*d^2*e*f^2*x^4+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d^3*e^2*f*x^4+3*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^3*e*f^2
*x^4-13*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*a*c*d^4*e^2*f*x^4+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e*f^2*x^4-2*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^
2*d^3*e^2*f*x^4-18*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*a*c^3*d^2*e*f^2*x^2+34*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^3*e^2*f*x^2
-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*b*c^4*d*e*f^2*x^2+6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellip
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^2*f*x^2+6*((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^3*d^2
*e*f^2*x^2-26*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*a*c^2*d^3*e^2*f*x^2+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^4*d*e*f^2*x^2-4*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*b*c^3*d^2*e^2*f*x^2)/(f*x^2+e)^(1/2)/(-d/c)^(1/2)/(c*f-d*e)^2/c^3/d/(d*x^
2+c)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(7/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(e + f*x**2)/(c + d*x**2)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)\,\sqrt{f\,x^2+e}}{{(d\,x^2+c)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(7/2),x)
```

```
[Out] int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(7/2), x)
```

3.29 $\int (a+bx^2)\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$

Optimal. Leaf size=543

$$\frac{(7adf(3d^2e^2+7cdef-2c^2f^2)-b(6d^3e^3-9cd^2e^2f+19c^2def^2-8c^3f^3))\,x\sqrt{c+dx^2}}{105d^3f\sqrt{e+fx^2}}+\frac{(14adf(3de-cf)+$$

```
[Out] 1/7*b*x*(d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/d+1/105*(7*a*d*f*(-2*c^2*f^2+7*c*d*
e*f+3*d^2*e^2)-b*(-8*c^3*f^3+19*c^2*d*e*f^2-9*c*d^2*e^2*f+6*d^3*e^3))*x*(d*
x^2+c)^(1/2)/d^3/f/(f*x^2+e)^(1/2)+1/105*e^(3/2)*(7*a*d*f*(-c*f+9*d*e)-b*(-
4*c^2*f^2+9*c*d*e*f+3*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ell
ipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/
2)/d^2/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/105*(7*a*d
*f*(-2*c^2*f^2+7*c*d*e*f+3*d^2*e^2)-b*(-8*c^3*f^3+19*c^2*d*e*f^2-9*c*d^2*e^
2*f+6*d^3*e^3))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)
/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/d^3/f
^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/35*(7*a*d*f-4*b*c*
f+3*b*d*e)*x*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/d^2+1/105*(14*a*d*f*(-c*f+3*d*
e)+b*(8*c^2*f^2-15*c*d*e*f+3*d^2*e^2))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d^
2/f
```

Rubi [A]
time = 0.43, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {542, 545, 429, 506, 422}

$$\frac{e^{3/2}\sqrt{c+dx^2}\,(7adf(9de-cf)-b(-4c^2f^2+9cdef+3d^2e^2))\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105d^2f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\sqrt{c+dx^2}\,(7adf(-2c^2f^2+7cdef+3d^2e^2)-b(-8c^3f^3+19c^2def^2-9cd^2e^2f+6d^3e^3))\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105d^3f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\,(14adf(3de-cf)+b(8c^2f^2-15cdef+3d^2e^2))}{105d^2f}+\frac{x\sqrt{c+dx^2}\,(7adf(-2c^2f^2+7cdef+3d^2e^2)-b(-8c^3f^3+19c^2def^2-9cd^2e^2f+6d^3e^3))}{105d^3f\sqrt{e+fx^2}}+\frac{x(c+dx^2)^{3/2}\sqrt{e+fx^2}\,(7adf-4bcf+3bde)}{35d^2}+\frac{bx(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

```
[Out] ((7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2) - b*(6*d^3*e^3 - 9*c*d^2*e^2*
f + 19*c^2*d*e*f^2 - 8*c^3*f^3))*x*Sqrt[c + d*x^2])/(105*d^3*f*Sqrt[e + f*x
^2]) + ((14*a*d*f*(3*d*e - c*f) + b*(3*d^2*e^2 - 15*c*d*e*f + 8*c^2*f^2))*x
*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(105*d^2*f) + ((3*b*d*e - 4*b*c*f + 7*a*d
*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(35*d^2) + (b*x*(c + d*x^2)^(3/2)*
(e + f*x^2)^(3/2))/(7*d) - (Sqrt[e]*(7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2
*f^2) - b*(6*d^3*e^3 - 9*c*d^2*e^2*f + 19*c^2*d*e*f^2 - 8*c^3*f^3))*Sqrt[c
+ d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d^3*
f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(
7*a*d*f*(9*d*e - c*f) - b*(3*d^2*e^2 + 9*c*d*e*f - 4*c^2*f^2))*Sqrt[c + d*x
```

```
^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d^2*f^(3/
2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int (a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}\,dx = \frac{bx(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{7d} + \frac{\int \sqrt{c+dx^2}\,\sqrt{e+fx^2}\,(-(bc-7a}{7}$$

$$= \frac{(3bde-4bcf+7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35d^2} + \frac{bx(c+dx^2)^{3/2}}{7d}$$

$$= \frac{(14adf(3de-cf)+b(3d^2e^2-15cdef+8c^2f^2))\,x\sqrt{c+dx^2}\,\sqrt{e}}{105d^2f}$$

$$= \frac{(14adf(3de-cf)+b(3d^2e^2-15cdef+8c^2f^2))\,x\sqrt{c+dx^2}\,\sqrt{e}}{105d^2f}$$

$$= \frac{(7adf(3d^2e^2+7cdef-2c^2f^2)-b(6d^3e^3-9cd^2e^2f+19c^2def^2}{105d^3f\sqrt{e+fx^2}}$$

$$= \frac{(7adf(3d^2e^2+7cdef-2c^2f^2)-b(6d^3e^3-9cd^2e^2f+19c^2def^2}{105d^3f\sqrt{e+fx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.35, size = 372, normalized size = 0.69

$$\frac{-\sqrt{\frac{d}{c}}\,fx(c+dx^2)(e+fx^2)(4bc^2f^2-3bcdf(3e+fx^2)-7adf(6de+cf+3dfx^2)-3bd^2(e^2+8efx^2+5f^2x^4))-ie(7adf(3d^2e^2+7cdef-2c^2f^2)+b(-6d^3e^3+9cd^2e^2f-19c^2def^2+8c^3f^3))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+ie(-de+cf)(-7adf(3de+cf)+b(6d^2e^2-6cdef+4c^2f^2))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)}{105c^2\left(\frac{d}{c}\right)^{5/2}f^2\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

```
[Out] (-(Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(4*b*c^2*f^2 - 3*b*c*d*f*(3*e + f*
x^2) - 7*a*d*f*(6*d*e + c*f + 3*d*f*x^2) - 3*b*d^2*(e^2 + 8*e*f*x^2 + 5*f^2
*x^4))) - I*e*(7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2) + b*(-6*d^3*e^3
+ 9*c*d^2*e^2*f - 19*c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
 (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*e*(-(d*e) +
c*f)*(-7*a*d*f*(3*d*e + c*f) + b*(6*d^2*e^2 - 6*c*d*e*f + 4*c^2*f^2))*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)])/(105*c^2*(d/c)^(5/2)*f^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1330 vs. $2(565) = 1130$.
time = 0.14, size = 1331, normalized size = 2.45

method	result
elliptic	$\dfrac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\left(\dfrac{b f x^5 \sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{7}+\dfrac{\left(a d f^2+b c f^2+2bdef-\frac{bf(6cf+6de)}{7}\right)x^3\sqrt{df\,x^4+c\ldots}}{5df}\right.}{\ldots}$
risch	$\dfrac{x\left(15b\,x^4 d^2 f^2+21a\,d^2 f^2 x^2+3bcd\,f^2 x^2+24b\,d^2 ef\,x^2+7acd\,f^2+42a\,d^2 ef-4b\,c^2 f^2+9bcdef+3b\,d^2 e^2\right)\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{105f\,d^2}-$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(-9*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f+19*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b
*c^2*d*e^2*f^2+12*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f+14*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3-49*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*
c*d^2*e^2*f^2-7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3-14*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*f^2-10*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*
c^2*d*e^2*f^2+4*(-d/c)^(1/2)*b*c^3*e*f^3*x-51*(-d/c)^(1/2)*b*c*d^2*e*f^3*x^
5-70*(-d/c)^(1/2)*a*c*d^2*e*f^3*x^3-8*(-d/c)^(1/2)*b*c^2*d*e*f^3*x^3-36*(-d
/c)^(1/2)*b*c*d^2*e^2*f^2*x^3-18*(-d/c)^(1/2)*b*c*d^2*f^4*x^7-39*(-d/c)^(1/
2)*b*d^3*e*f^3*x^7-28*(-d/c)^(1/2)*a*c*d^2*f^4*x^5-63*(-d/c)^(1/2)*a*d^3*e*
f^3*x^5+(-d/c)^(1/2)*b*c^2*d*f^4*x^5-27*(-d/c)^(1/2)*b*d^3*e^2*f^2*x^5+21*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*a*d^3*e^3*f+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f^3-21*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^3*f-8*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3
*e*f^3-7*(-d/c)^(1/2)*a*c^2*d*e*f^3*x-42*(-d/c)^(1/2)*a*c*d^2*e^2*f^2*x-9*(
-d/c)^(1/2)*b*c^2*d*e^2*f^2*x-3*(-d/c)^(1/2)*b*c*d^2*e^3*f*x-7*(-d/c)^(1/2)
```

```
*a*c^2*d*f^4*x^3-42*(-d/c)^(1/2)*a*d^3*e^2*f^2*x^3-3*(-d/c)^(1/2)*b*d^3*e^3
*f*x^3-15*(-d/c)^(1/2)*b*d^3*f^4*x^9-21*(-d/c)^(1/2)*a*d^3*f^4*x^7+4*(-d/c)
^(1/2)*b*c^3*f^4*x^3-6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e^4+6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e^4)/f^2/(d*f*x^4+c*
f*x^2+d*e*x^2+c*e)/d^2/(-d/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx^2) \sqrt{c + dx^2} (e + fx^2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)*(f*x**2+e)**(3/2),x)
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2),x)
[Out] int((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2), x)
```

3.30 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\, dx$

Optimal. Leaf size=400

$$\frac{(10adf(2de-cf)+b(3d^2e^2-13cdef+8c^2f^2))\,x\sqrt{c+dx^2}}{15d^3\sqrt{e+fx^2}}+\frac{(3bde-4bcf+5adf)x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{15d^2}$$

```
[Out] 1/5*b*x*(f*x^2+e)^(3/2)*(d*x^2+c)^(1/2)/d+1/15*(10*a*d*f*(-c*f+2*d*e)+b*(8*
c^2*f^2-13*c*d*e*f+3*d^2*e^2))*x*(d*x^2+c)^(1/2)/d^3/(f*x^2+e)^(1/2)+1/15*e
^(3/2)*(5*a*d*(-c*f+3*d*e)-b*(-4*c^2*f+6*c*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f
*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/
2))*(d*x^2+c)^(1/2)/c/d^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)
^(1/2)-1/15*(10*a*d*f*(-c*f+2*d*e)+b*(8*c^2*f^2-13*c*d*e*f+3*d^2*e^2))*(1/(
1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)
^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/d^3/f^(1/2)/(e*(d*x^2+c)/
c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/15*(5*a*d*f-4*b*c*f+3*b*d*e)*x*(d*x^2+
c)^(1/2)*(f*x^2+e)^(1/2)/d^2
```

Rubi [A]
time = 0.29, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {542, 545, 429, 506, 422}

$$\frac{e^{3/2}\sqrt{c+dx^2}\,(5ad(3de-cf)-b(6cde-4c^2f))\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15cd^2\sqrt{f}\,\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\,\sqrt{c+dx^2}\,(10adf(2de-cf)+b(8c^2f^2-13cdef+3d^2e^2))\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15d^3\sqrt{f}\,\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}\,(10adf(2de-cf)+b(8c^2f^2-13cdef+3d^2e^2))}{15d^3\sqrt{e+fx^2}}+\frac{x\sqrt{c+dx^2}\,\sqrt{e+fx^2}\,(5adf-4bcf+3bde)}{15d^2}+\frac{bx\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] ((10*a*d*f*(2*d*e - c*f) + b*(3*d^2*e^2 - 13*c*d*e*f + 8*c^2*f^2))*x*Sqrt[c
 + d*x^2])/(15*d^3*Sqrt[e + f*x^2]) + ((3*b*d*e - 4*b*c*f + 5*a*d*f)*x*Sqrt
[c + d*x^2]*Sqrt[e + f*x^2])/(15*d^2) + (b*x*Sqrt[c + d*x^2]*(e + f*x^2)^(3
/2))/(5*d) - (Sqrt[e]*(10*a*d*f*(2*d*e - c*f) + b*(3*d^2*e^2 - 13*c*d*e*f +
 8*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*
e)/(c*f)])/(15*d^3*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f
*x^2]) + (e^(3/2)*(5*a*d*(3*d*e - c*f) - b*(6*c*d*e - 4*c^2*f))*Sqrt[c + d*
x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*c*d^2*Sqr
t[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)\,(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}{5d} + \frac{\int \frac{\sqrt{e+fx^2}\,(-(bc-5ad)e+(3bde-4bcf+5adf)x^2)}{\sqrt{c+dx^2}}\,dx}{5d}$$

$$= \frac{(3bde-4bcf+5adf)x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{15d^2} + \frac{bx\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}{5d} +$$

$$= \frac{(3bde-4bcf+5adf)x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{15d^2} + \frac{bx\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}{5d} -$$

$$= \frac{(10adf(2de-cf)+b(3d^2e^2-13cdef+8c^2f^2))\,x\sqrt{c+dx^2}}{15d^3\sqrt{e+fx^2}} + \frac{(3bde-4bc}{}$$

$$= \frac{(10adf(2de-cf)+b(3d^2e^2-13cdef+8c^2f^2))\,x\sqrt{c+dx^2}}{15d^3\sqrt{e+fx^2}} + \frac{(3bde-4bc}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.27, size = 275, normalized size = 0.69

$$\frac{-\sqrt{\frac{d}{c}}\,fx(c+dx^2)\,(e+fx^2)\,(4bcf-5adf-3bd(2e+fx^2)) - ie(10adf(2de-cf)+b(3d^2e^2-13cdef+8c^2f^2))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right) + ie(-de+cf)(-3bde+4bcf-5adf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)}{15c^2\left(\frac{d}{c}\right)^{5/2} f\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (-(Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(4*b*c*f - 5*a*d*f - 3*b*d*(2*e +
f*x^2))) - I*e*(10*a*d*f*(2*d*e - c*f) + b*(3*d^2*e^2 - 13*c*d*e*f + 8*c^2*
f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)] + I*e*(-(d*e) + c*f)*(-3*b*d*e + 4*b*c*f - 5*a*d*f)*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)])/(15*c^2*(d/c)^(5/2)*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 869 vs. 2(428) = 856.
time = 0.15, size = 870, normalized size = 2.18 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)*(3*(-d/c)^(1/2)*b*d^2*f^3*x^7+5*(-d/c)
^(1/2)*a*d^2*f^3*x^5-(-d/c)^(1/2)*b*c*d*f^3*x^5+9*(-d/c)^(1/2)*b*d^2*e*f^2*
x^5+5*(-d/c)^(1/2)*a*c*d*f^3*x^3+5*(-d/c)^(1/2)*a*d^2*e*f^2*x^3-4*(-d/c)^(1
/2)*b*c^2*f^3*x^3+5*(-d/c)^(1/2)*b*c*d*e*f^2*x^3+6*(-d/c)^(1/2)*b*d^2*e^2*f
*x^3+5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*a*c*d*e*f^2-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipti
cF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*f-4*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*e*f^2+7*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*b*c*d*e^2*f-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*b*d^2*e^3-10*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f^2+20*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*
f+8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d
/e)^(1/2))*b*c^2*e*f^2-13*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f+3*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^2*e^3+5*(-d/c)^(1
/2)*a*c*d*e*f^2*x-4*(-d/c)^(1/2)*b*c^2*e*f^2*x+6*(-d/c)^(1/2)*b*c*d*e^2*f*x
)/d^2/f/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/(-d/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)(e + fx^2)^{\frac{3}{2}}}{\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*(e + f*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)\,{\left(f\,x^2+e\right)}^{3/2}}{\sqrt{d\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

3.31 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=369

$$\frac{f(bc(7de-8cf)-3ad(de-2cf))x\sqrt{c+dx^2}}{3cd^3\sqrt{e+fx^2}}+\frac{(4bc-3ad)fx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3cd^2}-\frac{(bc-ad)x(e+fx^2)^{3/}}{cd\sqrt{c+dx^2}}$$

```
[Out] -(-a*d+b*c)*x*(f*x^2+e)^(3/2)/c/d/(d*x^2+c)^(1/2)+1/3*f*(b*c*(-8*c*f+7*d*e)
-3*a*d*(-2*c*f+d*e))*x*(d*x^2+c)^(1/2)/c/d^3/(f*x^2+e)^(1/2)+1/3*e^(3/2)*(3
*a*d*f-4*b*c*f+3*b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x
*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/c/d^2
/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(b*c*(-8*c*f+7
*d*e)-3*a*d*(-2*c*f+d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE
(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*
x^2+c)^(1/2)/c/d^3/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*(-3*
a*d+4*b*c)*f*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/c/d^2
```

Rubi [A]
time = 0.27, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {540, 542, 545, 429, 506, 422}

$$-\frac{\sqrt{e}\,\sqrt{f}\,\sqrt{c+dx^2}\,(bc(7de-8cf)-3ad(de-2cf))E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd^3\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{c+dx^2}\,(3adf-4bcf+3bde)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd^2\sqrt{f}\,\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{fx\sqrt{c+dx^2}\,(bc(7de-8cf)-3ad(de-2cf))}{3cd^3\sqrt{e+fx^2}}+\frac{fx\sqrt{c+dx^2}\,\sqrt{e+fx^2}\,(4bc-3ad)}{3cd^2}-\frac{x(e+fx^2)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x]
```

```
[Out] (f*(b*c*(7*d*e - 8*c*f) - 3*a*d*(d*e - 2*c*f))*x*Sqrt[c + d*x^2])/(3*c*d^3*
Sqrt[e + f*x^2]) + ((4*b*c - 3*a*d)*f*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3
*c*d^2) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(c*d*Sqrt[c + d*x^2]) - (Sqrt[e
]*Sqrt[f]*(b*c*(7*d*e - 8*c*f) - 3*a*d*(d*e - 2*c*f))*Sqrt[c + d*x^2]*Ellip
ticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c*d^3*Sqrt[(e*(c + d
*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(3*b*d*e - 4*b*c*f + 3*
a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(3*c*d^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]
)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = -\frac{(bc-ad)x(e+fx^2)^{3/2}}{cd\sqrt{c+dx^2}} - \frac{\int \frac{\sqrt{e+fx^2}\,(-bce-(4bc-3ad)fx^2)}{\sqrt{c+dx^2}}\,dx}{cd}$$

$$= \frac{(4bc-3ad)fx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3cd^2} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{cd\sqrt{c+dx^2}} - \frac{\int \frac{-ce(3bde-4b}{\ldots}}{\ldots}$$

$$= \frac{(4bc-3ad)fx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3cd^2} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{(e(3bde-4\ldots}{\ldots}$$

$$= \frac{f(bc(7de-8cf)-3ad(de-2cf))x\sqrt{c+dx^2}}{3cd^3\sqrt{e+fx^2}} + \frac{(4bc-3ad)fx\sqrt{c+dx^2}\,\sqrt{\ldots}}{3cd^2}$$

$$= \frac{f(bc(7de-8cf)-3ad(de-2cf))x\sqrt{c+dx^2}}{3cd^3\sqrt{e+fx^2}} + \frac{(4bc-3ad)fx\sqrt{c+dx^2}\,\sqrt{\ldots}}{3cd^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.06, size = 248, normalized size = 0.67

$$\frac{\sqrt{\frac{d}{c}}\left(\sqrt{\frac{d}{c}}\,x(e+fx^2)\left(3ad(de-cf)+bc(-3de+4cf+dfx^2)\right)+ie\left(3ad(de-2cf)+bc(-7de+8cf)\right)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)\middle|\frac{cf}{de}\right)-i(4bc-3ad)e(-de+cf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)\middle|\frac{cf}{de}\right)\right)}{3d^3\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d/c]*(Sqrt[d/c]*x*(e + f*x^2)*(3*a*d*(d*e - c*f) + b*c*(-3*d*e + 4*c*
f + d*f*x^2)) + I*e*(3*a*d*(d*e - 2*c*f) + b*c*(-7*d*e + 8*c*f))*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)
] - I*(4*b*c - 3*a*d)*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*d^3*Sqrt[c + d*x^2]
*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.17, size = 671, normalized size = 1.82

method	result

elliptic	$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\left(-\frac{(df\,x^2+de)(acdf-a\,d^2e-b\,c^2f+bcde)x}{d^3c\sqrt{\left(x^2+\frac{c}{d}\right)(df\,x^2+de)}}+\frac{bfx\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{3d^2}+\frac{\left(-\frac{acd\,f^2-}{}\right.}{}\right.}{}$
default	$-\frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\left(-\sqrt{-\frac{d}{c}}\,bcd\,f^2x^5+3\sqrt{-\frac{d}{c}}\,acd\,f^2x^3-3\sqrt{-\frac{d}{c}}\,a\,d^2ef\,x^3-4\sqrt{-\frac{d}{c}}\,b\,c^2f^2x^3+2\sqrt{-\frac{d}{c}}\right.}{}$
risch	$\frac{bx\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,f}{3d^2}+\frac{\left(-\frac{(3adf-5bcf+4bde)e\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)*(-(-d/c)^(1/2)*b*c*d*f^2*x^5+3*(-d/c)^
(1/2)*a*c*d*f^2*x^3-3*(-d/c)^(1/2)*a*d^2*e*f*x^3-4*(-d/c)^(1/2)*b*c^2*f^2*x
^3+2*(-d/c)^(1/2)*b*c*d*e*f*x^3+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f-3*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2-4*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*b*c^2*e*f+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2-6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f+3*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2+8*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*b*c^2*e*f-7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2+3*(-d/c)^(1/2)*a*c*d*e*f*x-3*(-d/c)^(1/2)
*a*d^2*e^2*x-4*(-d/c)^(1/2)*b*c^2*e*f*x+3*(-d/c)^(1/2)*b*c*d*e^2*x)/d^2/(d*
f*x^4+c*f*x^2+d*e*x^2+c*e)/(-d/c)^(1/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)\,(e+fx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*(e + f*x**2)**(3/2)/(c + d*x**2)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)\,{\left(f\,x^2+e\right)}^{3/2}}{{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x)
```

```
[Out] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2), x)
```

3.32 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=373

$$-\frac{f(bc(de-8cf)+2ad(de+cf))x\sqrt{c+dx^2}}{3c^2d^3\sqrt{e+fx^2}}+\frac{(bc(de-4cf)+ad(2de+cf))x\sqrt{e+fx^2}}{3c^2d^2\sqrt{c+dx^2}}-\frac{(bc-ad)x(e+}{3cd\,(c+d}$$

```
[Out] -1/3*(-a*d+b*c)*x*(f*x^2+e)^(3/2)/c/d/(d*x^2+c)^(3/2)-1/3*f*(b*c*(-8*c*f+d*
e)+2*a*d*(c*f+d*e))*x*(d*x^2+c)^(1/2)/c^2/d^3/(f*x^2+e)^(1/2)+1/3*(-a*d+4*b
*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(
1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^2/d^2/(
e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*(b*c*(-8*c*f+d*e)+2*a*d*
(c*f+d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1
/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c^
2/d^3/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*(b*c*(-4*c*f+d*e)
+a*d*(c*f+2*d*e))*x*(f*x^2+e)^(1/2)/c^2/d^2/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {540, 545, 429, 506, 422}

$$\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\,(2ad(cf+de)+bc(de-8cf))E\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d^3\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,(4bc-ad)F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d^2\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{fx\sqrt{c+dx^2}\,(2ad(cf+de)+bc(de-8cf))}{3c^2d^3\sqrt{e+fx^2}}+\frac{x\sqrt{e+fx^2}\,(ad(cf+2de)+bc(de-4cf))}{3c^2d^2\sqrt{c+dx^2}}-\frac{x(e+fx^2)^{3/2}(bc-ad)}{3cd\,(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x]
```

```
[Out] -1/3*(f*(b*c*(d*e - 8*c*f) + 2*a*d*(d*e + c*f))*x*Sqrt[c + d*x^2])/(c^2*d^3
*Sqrt[e + f*x^2]) + ((b*c*(d*e - 4*c*f) + a*d*(2*d*e + c*f))*x*Sqrt[e + f*x
^2])/(3*c^2*d^2*Sqrt[c + d*x^2]) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(3*c*d
*(c + d*x^2)^(3/2)) + (Sqrt[e]*Sqrt[f]*(b*c*(d*e - 8*c*f) + 2*a*d*(d*e + c*
f))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)]
)/(3*c^2*d^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((4*b
*c - a*d)*e^(3/2)*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt
[e]], 1 - (d*e)/(c*f)])/(3*c^2*d^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sq
rt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = -\frac{(bc-ad)x(e+fx^2)^{3/2}}{3cd(c+dx^2)^{3/2}} - \frac{\int \frac{\sqrt{e+fx^2}(-(bc+2ad)e-(4bc-ad)fx^2)}{(c+dx^2)^{3/2}}dx}{3cd}$$

$$= \frac{(bc(de-4cf)+ad(2de+cf))x\sqrt{e+fx^2}}{3c^2d^2\sqrt{c+dx^2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{3cd(c+dx^2)^{3/2}} + \frac{\int c \ldots}{\ldots}$$

$$= \frac{(bc(de-4cf)+ad(2de+cf))x\sqrt{e+fx^2}}{3c^2d^2\sqrt{c+dx^2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{3cd(c+dx^2)^{3/2}} + \frac{((4 \ldots}{\ldots}$$

$$= -\frac{f(bc(de-8cf)+2ad(de+cf))x\sqrt{c+dx^2}}{3c^2d^3\sqrt{e+fx^2}} + \frac{(bc(de-4cf)+ad(2de+ \ldots}{3c^2d^2\sqrt{c+d \ldots}}$$

$$= -\frac{f(bc(de-8cf)+2ad(de+cf))x\sqrt{c+dx^2}}{3c^2d^3\sqrt{e+fx^2}} + \frac{(bc(de-4cf)+ad(2de+ \ldots}{3c^2d^2\sqrt{c+d \ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.85, size = 296, normalized size = 0.79

$$\frac{\left(\frac{d}{c}\right)^{3/2}\left(\sqrt{\frac{d}{c}}x(e+fx^2)(bc(-4c^2f+d^2ex^2-5cdfx^2)+ad(c^2f+2d^2ex^2+cd(3e+2fx^2)))-ie(-2ad(de+cf)+bc(-de+8cf))(c+dx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+ie(-ad(2de+cf)+bc(-de+4cf))(c+dx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)}{3d^4(c+dx^2)^{3/2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x]
```

```
[Out] ((d/c)^(3/2)*(Sqrt[d/c]*x*(e + f*x^2)*(b*c*(-4*c^2*f + d^2*e*x^2 - 5*c*d*f*
x^2) + a*d*(c^2*f + 2*d^2*e*x^2 + c*d*(3*e + 2*f*x^2))) - I*e*(-2*a*d*(d*e
+ c*f) + b*c*(-(d*e) + 8*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*
x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*e*(-(a*d*(2*d*e
+ c*f)) + b*c*(-(d*e) + 4*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f
*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*d^4*(c + d*x^2
)^(3/2)*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1224 vs. 2(401) = 802.
time = 0.14, size = 1225, normalized size = 3.28

method	result

elliptic	$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\,\left(-\frac{\left(acdf-a\,d^2e-b\,c^2f+bcde\right)x\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{3d^4c\left(x^2+\frac{c}{d}\right)^2}+\frac{\left(df\,x^2+de\right)\left(2acdf+2a\,d^2e-5b\,c^2\right.}{3d^3c^2\sqrt{\left(x^2+\frac{c}{d}\right)\left(df\,x^2\right.}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-d/c)^(1/2)*a*c^2*d*e*f*x+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^2*x^2-2*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^2*x
^2-4*(-d/c)^(1/2)*b*c^3*e*f*x+(-d/c)^(1/2)*b*c*d^2*e*f*x^5+5*(-d/c)^(1/2)*a
*c*d^2*e*f*x^3-5*(-d/c)^(1/2)*b*c^2*d*e*f*x^3-5*(-d/c)^(1/2)*b*c^2*d*f^2*x^
5+(-d/c)^(1/2)*a*c^2*d*f^2*x^3+(-d/c)^(1/2)*b*c*d^2*e^2*x^3+3*(-d/c)^(1/2)*
a*c*d^2*e^2*x+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*b*c*d^2*e^2*x^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^2*x^2+((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*
e*f-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*a*c^2*d*e*f+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
F(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2-4*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f+((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c
^2*d*e^2-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*a*c*d^2*e^2+8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ell
ipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f-((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2+2*(-d/c
)^(1/2)*a*d^3*e^2*x^3-4*(-d/c)^(1/2)*b*c^3*f^2*x^3+2*(-d/c)^(1/2)*a*c*d^2*f
^2*x^5+2*(-d/c)^(1/2)*a*d^3*e*f*x^5+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e*f*x^2+8*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d
*e*f*x^2-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*a*c*d^2*e*f*x^2-4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e*f*x^2)/(f*x^2+e)^(1/2)
/c^2/(-d/c)^(1/2)/(d*x^2+c)^(3/2)/d^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)\,(e+fx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)*(e + f*x**2)**(3/2)/(c + d*x**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)\,\left(f\,x^2+e\right)^{3/2}}{(d\,x^2+c)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x)
```

```
[Out] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2), x)
```

3.33 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}} \, dx$

Optimal. Leaf size=376

$$\frac{(d(bc+4ad)e-c(4bc+ad)f)x\sqrt{e+fx^2}}{15c^2d^2\left(c+dx^2\right)^{3/2}}-\frac{(bc-ad)x(e+fx^2)^{3/2}}{5cd\left(c+dx^2\right)^{5/2}}+\frac{(bc(2d^2e^2+3cdef-8c^2f^2)+ad(8d^2e}{15c^{5/2}d^{5/2}(d}$$

```
[Out] -1/5*(-a*d+b*c)*x*(f*x^2+e)^(3/2)/c/d/(d*x^2+c)^(5/2)-1/15*e^(3/2)*(b*c*(-4
*c*f+d*e)+a*d*(-c*f+4*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ellipti
cF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)
^(1/2)/c^3/d^2/(-c*f+d*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1
/15*(d*(4*a*d+b*c)*e-c*(a*d+4*b*c)*f)*x*(f*x^2+e)^(1/2)/c^2/d^2/(d*x^2+c)^(
3/2)+1/15*(b*c*(-8*c^2*f^2+3*c*d*e*f+2*d^2*e^2)+a*d*(-2*c^2*f^2-3*c*d*e*f+8
*d^2*e^2))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1
/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(5/2)/d^(5/2)/(-
c*f+d*e)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {540, 539, 429, 422}

$$-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,(ad(4de-cf)+bc(de-4cf))F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3d^2\sqrt{e+fx^2}\,(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{e+fx^2}\,(ad(-2c^2f^2-3cdef+8d^2e^2)+bc(-8c^2f^2+3cdef+2d^2e^2))\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}d^{5/2}\sqrt{c+dx^2}\,(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{x\sqrt{e+fx^2}\,(de(4ad+bc)-cf(ad+4bc))}{15c^2d^2\left(c+dx^2\right)^{3/2}}-\frac{x(e+fx^2)^{3/2}\,(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x]
```

```
[Out] ((d*(b*c + 4*a*d)*e - c*(4*b*c + a*d)*f)*x*Sqrt[e + f*x^2])/(15*c^2*d^2*(c
+ d*x^2)^(3/2)) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(5*c*d*(c + d*x^2)^(5/2
)) + ((b*c*(2*d^2*e^2 + 3*c*d*e*f - 8*c^2*f^2) + a*d*(8*d^2*e^2 - 3*c*d*e*f
 - 2*c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
c*f)/(d*e)])/(15*c^(5/2)*d^(5/2)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f
*x^2))/(e*(c + d*x^2))]) - (e^(3/2)*Sqrt[f]*(b*c*(d*e - 4*c*f) + a*d*(4*d*e
 - c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(
c*f)])/(15*c^3*d^2*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
 + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = -\frac{(bc-ad)x(e+fx^2)^{3/2}}{5cd\,(c+dx^2)^{5/2}} - \frac{\int \frac{\sqrt{e+fx^2}\,(-(bc+4ad)e-(4bc+ad)fx^2)}{(c+dx^2)^{5/2}}\,dx}{5cd}$$

$$= \frac{(d(bc+4ad)e-c(4bc+ad)f)x\sqrt{e+fx^2}}{15c^2d^2\,(c+dx^2)^{3/2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{5cd\,(c+dx^2)^{5/2}} +$$

$$= \frac{(d(bc+4ad)e-c(4bc+ad)f)x\sqrt{e+fx^2}}{15c^2d^2\,(c+dx^2)^{3/2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{5cd\,(c+dx^2)^{5/2}} -$$

$$= \frac{(d(bc+4ad)e-c(4bc+ad)f)x\sqrt{e+fx^2}}{15c^2d^2\,(c+dx^2)^{3/2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{5cd\,(c+dx^2)^{5/2}} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.24, size = 382, normalized size = 1.02

$$\frac{\sqrt{\frac{d}{c}}\left(-\sqrt{\frac{d}{c}}\,x(e+fx^2)\left(3c^2(bc-ad)(de-cf)^2-c(de-cf)(bc(de-7cf)+2ad(2de+cf))\,(c+dx^2)+(ad(-8d^2e^2+3cdef+2c^2f^2)+bc(-2d^2e^2-3cdef+8c^2f^2))\,(c+dx^2)^2\right)-ie(c+dx^2)^2\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left((ad(-8d^2e^2+3cdef+2c^2f^2)+bc(-2d^2e^2-3cdef+8c^2f^2))\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\tfrac{cf}{de}\right)+(de-cf)(ad(8de+cf)+2bc(de+2cf))F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\tfrac{cf}{de}\right)\right)\right)}{15c^2d^3(de-cf)\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d/c]*(-(Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)*(d*e - c*f)^2 - c*
(d*e - c*f)*(b*c*(d*e - 7*c*f) + 2*a*d*(2*d*e + c*f))*(c + d*x^2) + (a*d*(-
8*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2) + b*c*(-2*d^2*e^2 - 3*c*d*e*f + 8*c^2*f^
2))*(c + d*x^2)^2)) - I*e*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2
)/e]*((a*d*(-8*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2) + b*c*(-2*d^2*e^2 - 3*c*d*e
*f + 8*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (d*e - c*
f)*(a*d*(8*d*e + c*f) + 2*b*c*(d*e + 2*c*f))*EllipticF[I*ArcSinh[Sqrt[d/c]*
x], (c*f)/(d*e)])))/(15*c^2*d^3*(d*e - c*f)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^
2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2859 vs. 2(410) = 820.
time = 0.14, size = 2860, normalized size = 7.61

method	result
elliptic	$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\left(-\frac{\left(acdf-a\,d^2e-b\,c^2f+bcde\right)x\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{5d^5c\left(x^2+\frac{c}{d}\right)^3}+\frac{\left(2acdf+4a\,d^2e-7b\,c^2f+bcde\right)x\,\sqrt{}}{15d}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(-11*(-d/c)^(1/2)*a*c^3*d^2*e^2*f*x+(-d/c)^(1/2)*b*c^4*d*e^2*f*x-3*(-
d/c)^(1/2)*a*c*d^4*e*f^2*x^7+3*(-d/c)^(1/2)*b*c^2*d^3*e*f^2*x^7+2*(-d/c)^(1
/2)*b*c*d^4*e^2*f*x^7-10*(-d/c)^(1/2)*a*c^2*d^3*e*f^2*x^5+17*(-d/c)^(1/2)*a
*c*d^4*e^2*f*x^5-10*(-d/c)^(1/2)*b*c^3*d^2*e*f^2*x^5+8*(-d/c)^(1/2)*b*c^2*d
^3*e^2*f*x^5-17*(-d/c)^(1/2)*a*c^3*d^2*e*f^2*x^3+7*(-d/c)^(1/2)*a*c^2*d^3*e
^2*f*x^3-8*(-d/c)^(1/2)*b*c^4*d*e*f^2*x^3-2*(-d/c)^(1/2)*b*c^3*d^2*e^2*f*x^
3-(-d/c)^(1/2)*a*c^4*d*e*f^2*x-2*(-d/c)^(1/2)*a*c^2*d^3*f^3*x^7+8*(-d/c)^(1
/2)*a*d^5*e^2*f*x^7-8*(-d/c)^(1/2)*b*c^3*d^2*f^3*x^7-6*(-d/c)^(1/2)*a*c^3*d
^2*f^3*x^5-9*(-d/c)^(1/2)*b*c^4*d*f^3*x^5+2*(-d/c)^(1/2)*b*c*d^4*e^3*x^5-(-
d/c)^(1/2)*a*c^4*d*f^3*x^3+20*(-d/c)^(1/2)*a*c*d^4*e^3*x^3+5*(-d/c)^(1/2)*b
*c^2*d^3*e^3*x^3+15*(-d/c)^(1/2)*a*c^2*d^3*e^3*x-4*(-d/c)^(1/2)*b*c^5*e*f^2
*x+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/
```

d/e)^(1/2))*b*c^4*d*e^2*f+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipti
cE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^4*d*e*f^2+3*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^3*d^2*e^2*f-3
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*b*c^4*d*e^2*f+8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^5*e^3*x^4-8*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^5*e^3*x^4+8*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*a*c^2*d^3*e^3-4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*b*c^5*e*f^2+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^3-8*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*
d^3*e^3+8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*b*c^5*e*f^2-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^3+8*(-d/c)^(1/2)*a*d^5*e^
3*x^5-4*(-d/c)^(1/2)*b*c^5*f^3*x^3+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^4*e^3*x^4-2*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^
4*e^3*x^4+16*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a*c*d^4*e^3*x^2+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d^3*e^3*x^2-16*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*
c*d^4*e^3*x^2-4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*b*c^2*d^3*e^3*x^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^4*d*e*f^2-7*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^
3*d^2*e^2*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*a*c^2*d^3*e*f^2*x^4-7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^4*e^2*f*x^4-4*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
b*c^3*d^2*e*f^2*x^4+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d^3*e^2*f*x^4+2*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^3*e*f^2*x
^4+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*a*c*d^4*e^2*f*x^4+8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ell
ipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e*f^2*x^4-3*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d
^3*e^2*f*x^4-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*a*c^3*d^2*e*f^2*x^2-14*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^3*e^2*f*x^2-8*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*b*c^4*d*e*f^2*x^2+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^2*f*x^2+4*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^3*d^2*e*f
^2*x^2+6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(

```
c*f/d/e)^(1/2))*a*c^2*d^3*e^2*f*x^2+16*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^4*d*e*f^2*x^2-6*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*
c^3*d^2*e^2*f*x^2)/(f*x^2+e)^(1/2)/c^3/(c*f-d*e)/(-d/c)^(1/2)/(d*x^2+c)^(5/
2)/d^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)(e + fx^2)^{\frac{3}{2}}}{(c + dx^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(7/2),x)
[Out] Integral((a + b*x**2)*(e + f*x**2)**(3/2)/(c + d*x**2)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)\,\left(f\,x^2+e\right)^{3/2}}{\left(d\,x^2+c\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x)
[Out] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2), x)
```

3.34 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx$

Optimal. Leaf size=531

$$\frac{(d(bc+6ad)e-c(4bc+3ad)f)x\sqrt{e+fx^2}}{35c^2d^2(c+dx^2)^{5/2}}+\frac{(bc(4d^2e^2+cdef-8c^2f^2)+3ad(8d^2e^2-5cdef-2c^2f^2))x\sqrt{}}{105c^3d^2(de-cf)(c+dx^2)^{3/2}}$$

```
[Out] -1/7*(-a*d+b*c)*x*(f*x^2+e)^(3/2)/c/d/(d*x^2+c)^(7/2)-1/105*e^(3/2)*(3*a*d*
(c^2*f^2-11*c*d*e*f+8*d^2*e^2)+2*b*c*(2*c^2*f^2-c*d*e*f+2*d^2*e^2))*(1/(1+f
*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1
/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^4/d^2/(-c*f+d*e)^2/(e*(d*x
^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/35*(d*(6*a*d+b*c)*e-c*(3*a*d+4*b
*c)*f)*x*(f*x^2+e)^(1/2)/c^2/d^2/(d*x^2+c)^(5/2)+1/105*(b*c*(-8*c^2*f^2+c*d
*e*f+4*d^2*e^2)+3*a*d*(-2*c^2*f^2-5*c*d*e*f+8*d^2*e^2))*x*(f*x^2+e)^(1/2)/c
^3/d^2/(-c*f+d*e)/(d*x^2+c)^(3/2)+1/105*(6*a*d*(c^3*f^3+2*c^2*d*e*f^2-12*c*
d^2*e^2*f+8*d^3*e^3)+b*c*(8*c^3*f^3-5*c^2*d*e*f^2-5*c*d^2*e^2*f+8*d^3*e^3))
*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x
^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(7/2)/d^(5/2)/(-c*f+d*e)^2
/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {540, 541, 539, 429, 422}

$$-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,(3ad(c^2f^2-11cdef+8d^2e^2)+2bc(2c^2f^2-cdef+2d^2e^2))\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105c^4d^2\sqrt{e+fx^2}\,(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{e+fx^2}\,(6ad(c^3f^3+2c^2def^2-12cd^2e^2f+8d^3e^3)+bc(8c^3f^3-5c^2def^2-5cd^2e^2f+8d^3e^3))\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{105c^{7/2}d^{5/2}\sqrt{c+dx^2}\,(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{x\sqrt{e+fx^2}\,(de(6ad+bc)-cf(3ad+4bc))}{35c^2d^2\,(c+dx^2)^{5/2}}+\frac{x\sqrt{e+fx^2}\,(3ad(-2c^2f^2-5cdef+8d^2e^2)+bc(-8c^2f^2+cdef+4d^2e^2))}{105c^3d^2\,(c+dx^2)^{3/2}\,(de-cf)}-\frac{x(e+fx^2)^{3/2}\,(bc-ad)}{7cd\,(c+dx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x]
```

```
[Out] ((d*(b*c + 6*a*d)*e - c*(4*b*c + 3*a*d)*f)*x*Sqrt[e + f*x^2])/(35*c^2*d^2*(
c + d*x^2)^(5/2)) + ((b*c*(4*d^2*e^2 + c*d*e*f - 8*c^2*f^2) + 3*a*d*(8*d^2*
e^2 - 5*c*d*e*f - 2*c^2*f^2))*x*Sqrt[e + f*x^2])/(105*c^3*d^2*(d*e - c*f)*(
c + d*x^2)^(3/2)) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(7*c*d*(c + d*x^2)^(7
/2)) + ((6*a*d*(8*d^3*e^3 - 12*c*d^2*e^2*f + 2*c^2*d*e*f^2 + c^3*f^3) + b*c
*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[e + f*x^2]*E
llipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(105*c^(7/2)*d^(5/2
)*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (e
^(3/2)*Sqrt[f]*(3*a*d*(8*d^2*e^2 - 11*c*d*e*f + c^2*f^2) + 2*b*c*(2*d^2*e^2
 - c*d*e*f + 2*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[
```

```
e]], 1 - (d*e)/(c*f)])/(105*c^4*d^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(
e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = -\frac{(bc-ad)x(e+fx^2)^{3/2}}{7cd\,(c+dx^2)^{7/2}} - \frac{\int \frac{\sqrt{e+fx^2}\,(-(bc+6ad)e-(4bc+3ad)fx^2)}{(c+dx^2)^{7/2}}\,dx}{7cd}$$

$$= \frac{(d(bc+6ad)e-c(4bc+3ad)f)x\sqrt{e+fx^2}}{35c^2d^2\,(c+dx^2)^{5/2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{7cd\,(c+dx^2)^{7/2}} + \frac{\int \frac{e(}{}}{}$$

$$= \frac{(d(bc+6ad)e-c(4bc+3ad)f)x\sqrt{e+fx^2}}{35c^2d^2\,(c+dx^2)^{5/2}} + \frac{(bc(4d^2e^2+cdef-8c^2f^2)+3}{105c^3d^2(c}$$

$$= \frac{(d(bc+6ad)e-c(4bc+3ad)f)x\sqrt{e+fx^2}}{35c^2d^2\,(c+dx^2)^{5/2}} + \frac{(bc(4d^2e^2+cdef-8c^2f^2)+3}{105c^3d^2(c}$$

$$= \frac{(d(bc+6ad)e-c(4bc+3ad)f)x\sqrt{e+fx^2}}{35c^2d^2\,(c+dx^2)^{5/2}} + \frac{(bc(4d^2e^2+cdef-8c^2f^2)+3}{105c^3d^2(c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.71, size = 545, normalized size = 1.03

$$\frac{\sqrt{\frac{d}{c}}\left(-\sqrt{\frac{d}{c}}x(e+fx^2)\left(15c^3(bc-ad)(de-cf)^3-3c^2(de-cf)^2(bc(de-9cf)+2ad(3de+cf))(c+dx^2)-c(de-cf)(bc(4d^2e^2+cdef-8c^2f^2)+3ad(8d^2e^2-5cdef-2c^2f^2))(c+dx^2)^2-(6ad(8d^3e^3-12cd^2e^2f+2c^2def^2+c^3f^3)+bc(8d^3e^3-5cd^2e^2f-5c^2def^2+8c^3f^3))(c+dx^2)^3\right)+ie(c+dx^2)^3\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left((6ad(8d^3e^3-12cd^2e^2f+2c^2def^2+c^3f^3)+bc(8d^3e^3-5cd^2e^2f-5c^2def^2+8c^3f^3))E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-(-de+cf)(3ad(-16d^2e^2+16cdef+c^2f^2)+bc(-8d^2e^2+cdef+4c^2f^2))F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)\right)}{105c^3d^3(de-cf)^2(c+dx^2)^{7/2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x]
```

```
[Out] (Sqrt[d/c]*(-(Sqrt[d/c]*x*(e + f*x^2)*(15*c^3*(b*c - a*d)*(d*e - c*f)^3 - 3
*c^2*(d*e - c*f)^2*(b*c*(d*e - 9*c*f) + 2*a*d*(3*d*e + c*f))*(c + d*x^2) -
c*(d*e - c*f)*(b*c*(4*d^2*e^2 + c*d*e*f - 8*c^2*f^2) + 3*a*d*(8*d^2*e^2 - 5
*c*d*e*f - 2*c^2*f^2))*(c + d*x^2)^2 - (6*a*d*(8*d^3*e^3 - 12*c*d^2*e^2*f +
 2*c^2*d*e*f^2 + c^3*f^3) + b*c*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2
+ 8*c^3*f^3))*(c + d*x^2)^3)) + I*e*(c + d*x^2)^3*Sqrt[1 + (d*x^2)/c]*Sqrt[
1 + (f*x^2)/e]*((6*a*d*(8*d^3*e^3 - 12*c*d^2*e^2*f + 2*c^2*d*e*f^2 + c^3*f^
3) + b*c*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))*EllipticE
[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (-(d*e) + c*f)*(3*a*d*(-16*d^2*e^2
+ 16*c*d*e*f + c^2*f^2) + b*c*(-8*d^2*e^2 + c*d*e*f + 4*c^2*f^2))*EllipticF
[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])))/(105*c^3*d^3*(d*e - c*f)^2*(c + d*
x^2)^(7/2)*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5112 vs. 2(559) = 1118.
time = 0.16, size = 5113, normalized size = 9.63

method	result
elliptic	$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\left(-\frac{\left(acdf-a\,d^2e-b\,c^2f+bcde\right)x\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{7d^6c\left(x^2+\frac{c}{d}\right)^4}+\frac{\left(2acdf+6a\,d^2e-9b\,c^2f+bcde\right)x}{3\ldots}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)\,\left(f\,x^2+e\right)^{3/2}}{\left(d\,x^2+c\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x)
[Out] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2), x)
```

3.35 $\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=551

$$\frac{(7adf(8d^2e^2-23cdef+23c^2f^2)-b(48d^3e^3-128cd^2e^2f+103c^2def^2-15c^3f^3))\,x\sqrt{c+dx^2}}{105df^3\sqrt{e+fx^2}}-\frac{(28adf(de$$

```
[Out] 1/105*(7*a*d*f*(23*c^2*f^2-23*c*d*e*f+8*d^2*e^2)-b*(-15*c^3*f^3+103*c^2*d*e
*f^2-128*c*d^2*e^2*f+48*d^3*e^3))*x*(d*x^2+c)^(1/2)/d/f^3/(f*x^2+e)^(1/2)-1
/105*(7*a*d*f*(23*c^2*f^2-23*c*d*e*f+8*d^2*e^2)-b*(-15*c^3*f^3+103*c^2*d*e*
f^2-128*c*d^2*e^2*f+48*d^3*e^3))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*El
lipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x
^2+c)^(1/2)/d/f^(7/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/105
*(7*a*f*(15*c^2*f^2-11*c*d*e*f+4*d^2*e^2)-b*e*(45*c^2*f^2-61*c*d*e*f+24*d^2
*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/
(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/f^(7/2)/(e*(d*
x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/35*(-7*a*d*f-5*b*c*f+6*b*d*e)*x
*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/f^2+1/7*b*x*(d*x^2+c)^(5/2)*(f*x^2+e)^(1/2
)/f-1/105*(28*a*d*f*(-2*c*f+d*e)-b*(15*c^2*f^2-43*c*d*e*f+24*d^2*e^2))*x*(d
*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/f^3
```

Rubi [A]
time = 0.43, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {542, 545, 429, 506, 422}

$$\frac{\sqrt{e}\sqrt{c+dx^2}\,(7af(15c^2f^2-11cdef+4d^2e^2)-be(45c^2f^2-61cdef+24d^2e^2))\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105f^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\sqrt{c+dx^2}\,(7adf(23c^2f^2-23cdef+8d^2e^2)-b(-15c^3f^3+103c^2def^2-128cd^2e^2f+48d^3e^3))\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105df^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\,(28adf(de-2cf)-b(15c^2f^2-43cdef+24d^2e^2))}{105f^3}+\frac{x\sqrt{c+dx^2}\,(7adf(23c^2f^2-23cdef+8d^2e^2)-b(-15c^3f^3+103c^2def^2-128cd^2e^2f+48d^3e^3))}{105df^3\sqrt{e+fx^2}}-\frac{x(c+dx^2)^{3/2}\sqrt{e+fx^2}\,(-7adf-5bcf+6bde)}{35f^2}+\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^(5/2))/Sqrt[e + f*x^2],x]
```

```
[Out] ((7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2) - b*(48*d^3*e^3 - 128*c*d^2
*e^2*f + 103*c^2*d*e*f^2 - 15*c^3*f^3))*x*Sqrt[c + d*x^2])/(105*d*f^3*Sqrt[
e + f*x^2]) - ((28*a*d*f*(d*e - 2*c*f) - b*(24*d^2*e^2 - 43*c*d*e*f + 15*c^
2*f^2))*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(105*f^3) - ((6*b*d*e - 5*b*c*f
- 7*a*d*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(35*f^2) + (b*x*(c + d*x^2)
^(5/2)*Sqrt[e + f*x^2])/(7*f) - (Sqrt[e]*(7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f +
 23*c^2*f^2) - b*(48*d^3*e^3 - 128*c*d^2*e^2*f + 103*c^2*d*e*f^2 - 15*c^3*f
^3))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)
])/(105*d*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) +
```

```
(Sqrt[e]*(7*a*f*(4*d^2*e^2 - 11*c*d*e*f + 15*c^2*f^2) - b*e*(24*d^2*e^2 - 6
1*c*d*e*f + 45*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(105*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*S
qrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx = \frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f} + \frac{\int \frac{(c+dx^2)^{3/2}(-c(be-7af)+(-6bde+5bcf+7adf)x^2)}{\sqrt{e+fx^2}}\,dx}{7f}$$

$$= -\frac{(6bde-5bcf-7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35f^2} + \frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f}$$

$$= -\frac{(28adf(de-2cf)-b(24d^2e^2-43cdef+15c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105f^3}$$

$$= -\frac{(28adf(de-2cf)-b(24d^2e^2-43cdef+15c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105f^3}$$

$$= \frac{(7adf(8d^2e^2-23cdef+23c^2f^2)-b(48d^3e^3-128cd^2e^2f+103c^2def^2-15c}{105df^3\sqrt{e+fx^2}}$$

$$= \frac{(7adf(8d^2e^2-23cdef+23c^2f^2)-b(48d^3e^3-128cd^2e^2f+103c^2def^2-15c}{105df^3\sqrt{e+fx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.27, size = 386, normalized size = 0.70

$$\frac{\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)(7adf(-4de+11cf+3dfx^2)+b(45c^2f^2+cdf(-61e+45fx^2)+3d^2(8e^2-6efx^2+5f^2x^4)))-ie(7adf(8d^2e^2-23cdef+23c^2f^2)+b(-48d^3e^3+128cd^2e^2f-103c^2def^2+15c^3f^3))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+i(-de+cf)(4be(12d^2e^2-26cdef+15c^2f^2)-7af(8d^2e^2-19cdef+15c^2f^2))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)}{105\sqrt{\frac{d}{c}}f^4\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^(5/2))/Sqrt[e + f*x^2],x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(7*a*d*f*(-4*d*e + 11*c*f + 3*d*f*x^
2) + b*(45*c^2*f^2 + c*d*f*(-61*e + 45*f*x^2) + 3*d^2*(8*e^2 - 6*e*f*x^2 +
5*f^2*x^4))) - I*e*(7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2) + b*(-48*
d^3*e^3 + 128*c*d^2*e^2*f - 103*c^2*d*e*f^2 + 15*c^3*f^3))*Sqrt[1 + (d*x^2)
/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*
(-(d*e) + c*f)*(4*b*e*(12*d^2*e^2 - 26*c*d*e*f + 15*c^2*f^2) - 7*a*f*(8*d^2
*e^2 - 19*c*d*e*f + 15*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*El
lipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(105*Sqrt[d/c]*f^4*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1385 vs. 2(573) = 1146.
time = 0.14, size = 1386, normalized size = 2.52

method	result
elliptic	$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\left(\frac{b\,d^2x^5\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{7f}+\frac{\left(a\,d^3+3bc\,d^2-\frac{b\,d^2(6cf+6de)}{7f}\right)x^3\sqrt{df\,x^4+cf\,x^2}}{5df}\right.}{}$
risch	$\frac{x\left(15b\,x^4d^2f^2+21a\,d^2f^2x^2+45bcd\,f^2x^2-18b\,d^2ef\,x^2+77acd\,f^2-28a\,d^2ef+45b\,c^2f^2-61bcdef+24b\,d^2e^2\right)\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+}}{105f^3}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(128*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f-103*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
b*c^2*d*e^2*f^2-152*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f+161*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3-161*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*a*c*d^2*e^2*f^2-238*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3+189*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*f^2+164*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*b*c^2*d*e^2*f^2+45*(-d/c)^(1/2)*b*c^3*e*f^3*x-19*(-d/c)^(1/2)*b*c*d^2
*e*f^3*x^5+70*(-d/c)^(1/2)*a*c*d^2*e*f^3*x^3+29*(-d/c)^(1/2)*b*c^2*d*e*f^3*
x^3-55*(-d/c)^(1/2)*b*c*d^2*e^2*f^2*x^3+60*(-d/c)^(1/2)*b*c*d^2*f^4*x^7-3*(
-d/c)^(1/2)*b*d^3*e*f^3*x^7+98*(-d/c)^(1/2)*a*c*d^2*f^4*x^5-7*(-d/c)^(1/2)*
a*d^3*e*f^3*x^5+90*(-d/c)^(1/2)*b*c^2*d*f^4*x^5+6*(-d/c)^(1/2)*b*d^3*e^2*f^
2*x^5-56*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*a*d^3*e^3*f-60*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
```

```
pticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f^3+56*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^3*f+15*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*b*c^3*e*f^3+77*(-d/c)^(1/2)*a*c^2*d*e*f^3*x-28*(-d/c)^(1/2)*a*c*d^2*
e^2*f^2*x-61*(-d/c)^(1/2)*b*c^2*d*e^2*f^2*x+24*(-d/c)^(1/2)*b*c*d^2*e^3*f*x
+77*(-d/c)^(1/2)*a*c^2*d*f^4*x^3-28*(-d/c)^(1/2)*a*d^3*e^2*f^2*x^3+24*(-d/c
)^(1/2)*b*d^3*e^3*f*x^3+15*(-d/c)^(1/2)*b*d^3*f^4*x^9+21*(-d/c)^(1/2)*a*d^3
*f^4*x^7+45*(-d/c)^(1/2)*b*c^3*f^4*x^3+48*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e^4-48*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*
e^4+105*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*a*c^3*f^4)/f^4/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/(-d/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/sqrt(f*x^2 + e), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)\,(c + dx^2)^{\frac{5}{2}}}{\sqrt{e + fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
[Out] Integral((a + b*x**2)*(c + d*x**2)**(5/2)/sqrt(e + f*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/sqrt(f*x^2 + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{5/2}}{\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(1/2),x)
[Out] int(((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(1/2), x)
```

3.36 $\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=396

$$-\frac{(10adf(de-2cf)-b(8d^2e^2-13cdef+3c^2f^2))\,x\sqrt{c+dx^2}}{15df^2\sqrt{e+fx^2}}-\frac{(4bde-3bcf-5adf)x\sqrt{c+dx^2}\,\sqrt{e+fx}}{15f^2}$$

```
[Out] -1/15*(10*a*d*f*(-2*c*f+d*e)-b*(3*c^2*f^2-13*c*d*e*f+8*d^2*e^2))*x*(d*x^2+c
)^(1/2)/d/f^2/(f*x^2+e)^(1/2)+1/15*(10*a*d*f*(-2*c*f+d*e)-b*(3*c^2*f^2-13*c
*d*e*f+8*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1
/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/d/
f^(5/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/15*(5*a*f*(-3*c*f
+d*e)-b*(-6*c*e*f+4*d*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ellipti
cF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)
^(1/2)/f^(5/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/5*b*x*(d*x
^2+c)^(3/2)*(f*x^2+e)^(1/2)/f-1/15*(-5*a*d*f-3*b*c*f+4*b*d*e)*x*(d*x^2+c)^(
1/2)*(f*x^2+e)^(1/2)/f^2
```

Rubi [A]
time = 0.30, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {542, 545, 429, 506, 422}

$$\frac{\sqrt{e}\,\sqrt{c+dx^2}\,(10adf(de-2cf)-b(3c^2f^2-13cdef+8d^2e^2))\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15df^{5/2}\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\,\sqrt{c+dx^2}\,(5af(de-3cf)-b(4de^2-6cef))\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15f^{5/2}\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{c+dx^2}\,(10adf(de-2cf)-b(3c^2f^2-13cdef+8d^2e^2))}{15df^2\sqrt{e+fx^2}}-\frac{x\sqrt{c+dx^2}\,\sqrt{e+fx^2}\,(-5adf-3bcf+4bde)}{15f^2}+\frac{bx(c+dx^2)^{3/2}\,\sqrt{e+fx^2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^(3/2))/Sqrt[e + f*x^2],x]
```

```
[Out] -1/15*((10*a*d*f*(d*e - 2*c*f) - b*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*x*
Sqrt[c + d*x^2])/(d*f^2*Sqrt[e + f*x^2]) - ((4*b*d*e - 3*b*c*f - 5*a*d*f)*x
*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*f^2) + (b*x*(c + d*x^2)^(3/2)*Sqrt[e
+ f*x^2])/(5*f) + (Sqrt[e]*(10*a*d*f*(d*e - 2*c*f) - b*(8*d^2*e^2 - 13*c*d*
e*f + 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
- (d*e)/(c*f)])/(15*d*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2]) - (Sqrt[e]*(5*a*f*(d*e - 3*c*f) - b*(4*d*e^2 - 6*c*e*f))*Sqrt[c +
 d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*f^(5/2
)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx = \frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5f} + \frac{\int \frac{\sqrt{c+dx^2}(-c(be-5af)+(-4bde+3bcf+5adf)x^2)}{\sqrt{e+fx^2}}\,dx}{5f}$$

$$= -\frac{(4bde-3bcf-5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15f^2} + \frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5f}$$

$$= -\frac{(4bde-3bcf-5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15f^2} + \frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5f}$$

$$= -\frac{(10adf(de-2cf)-b(8d^2e^2-13cdef+3c^2f^2))\,x\sqrt{c+dx^2}}{15df^2\sqrt{e+fx^2}} - \frac{(4bde-3b}{}$$

$$= -\frac{(10adf(de-2cf)-b(8d^2e^2-13cdef+3c^2f^2))\,x\sqrt{c+dx^2}}{15df^2\sqrt{e+fx^2}} - \frac{(4bde-3b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.30, size = 279, normalized size = 0.70

$$\frac{\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)(5adf+b(-4de+6cf+3dfx^2)) - ie(-10adf(de-2cf)+b(8d^2e^2-13cdef+3c^2f^2))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right) + i(-de+cf)(5af(2de-3cf)+be(-8de+9cf))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)}{15\sqrt{\frac{d}{c}}f^3\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^(3/2))/Sqrt[e + f*x^2],x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(5*a*d*f + b*(-4*d*e + 6*c*f + 3*d*f
*x^2)) - I*e*(-10*a*d*f*(d*e - 2*c*f) + b*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f
^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*
x], (c*f)/(d*e)] + I*(-(d*e) + c*f)*(5*a*f*(2*d*e - 3*c*f) + b*e*(-8*d*e +
9*c*f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/
c]*x], (c*f)/(d*e)])/(15*Sqrt[d/c]*f^3*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(424) = 848$.
time = 0.14, size = 924, normalized size = 2.33 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(3*(-d/c)^(1/2)*b*d^2*f^3*x^7+5*(-d/c)
^(1/2)*a*d^2*f^3*x^5+9*(-d/c)^(1/2)*b*c*d*f^3*x^5-(-d/c)^(1/2)*b*d^2*e*f^2*
x^5+5*(-d/c)^(1/2)*a*c*d*f^3*x^3+5*(-d/c)^(1/2)*a*d^2*e*f^2*x^3+6*(-d/c)^(1
/2)*b*c^2*f^3*x^3+5*(-d/c)^(1/2)*b*c*d*e*f^2*x^3-4*(-d/c)^(1/2)*b*d^2*e^2*f
*x^3+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*a*c^2*f^3-25*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipti
cF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f^2+10*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*f-9*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*b*c^2*e*f^2+17*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f-8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^2*e^3+20*((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f
^2-10*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*a*d^2*e^2*f+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
E(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*e*f^2-13*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f+8*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*b*d^2*e^3+5*(-d/c)^(1/2)*a*c*d*e*f^2*x+6*(-d/c)^(1/2)*b*c^2*e*f^2*x-4*(-d
/c)^(1/2)*b*c*d*e^2*f*x)/f^3/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/(-d/c)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/sqrt(f*x^2 + e), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)(c + dx^2)^{\frac{3}{2}}}{\sqrt{e + fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*(c + d*x**2)**(3/2)/sqrt(e + f*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/sqrt(f*x^2 + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)\,\left(d\,x^2+c\right)^{3/2}}{\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(1/2),x)
```

```
[Out] int(((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(1/2), x)
```

3.37 $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=282

$$-\frac{(2bde-bcf-3adf)x\sqrt{c+dx^2}}{3df\sqrt{e+fx^2}}+\frac{bx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3f}+\frac{\sqrt{e}\,(2bde-bcf-3adf)\sqrt{c+dx^2}\,E\Big(\tan^{-1}\Big(}{3df^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx}}$$

```
[Out] -1/3*(-3*a*d*f-b*c*f+2*b*d*e)*x*(d*x^2+c)^(1/2)/d/f/(f*x^2+e)^(1/2)+1/3*(-3
*a*d*f-b*c*f+2*b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f
^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)
/d/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(-3*a*f+b*e)
*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x
^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/f^(3/2)/(e*(d*x^2+c)
/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*b*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)
/f
```

Rubi [A]
time = 0.12, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {542, 545, 429, 506, 422}

$$-\frac{\sqrt{e}\,\sqrt{c+dx^2}\,(be-3af)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3f^{3/2}\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{e}\,\sqrt{c+dx^2}\,(-3adf-bcf+2bde)E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3df^{3/2}\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{c+dx^2}\,(-3adf-bcf+2bde)}{3df\sqrt{e+fx^2}}+\frac{bx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] -1/3*((2*b*d*e - b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(d*f*Sqrt[e + f*x^2])
+ (b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*f) + (Sqrt[e]*(2*b*d*e - b*c*f -
 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/
(c*f)])/(3*d*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
 - (Sqrt[e]*(b*e - 3*a*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt
[e]], 1 - (d*e)/(c*f)])/(3*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sq
rt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \frac{bx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3f} + \frac{\int \frac{-c(be-3af)+(-2bde+bcf+3adf)x^2}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx}{3f}$$

$$= \frac{bx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3f} - \frac{(c(be-3af))\int \frac{1}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx}{3f} + \frac{(-2bde}{}$$

$$= -\frac{(2bde-bcf-3adf)x\sqrt{c+dx^2}}{3df\sqrt{e+fx^2}} + \frac{bx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3f} - \frac{\sqrt{e}\,(be-3af)}{3f^3}$$

$$= -\frac{(2bde-bcf-3adf)x\sqrt{c+dx^2}}{3df\sqrt{e+fx^2}} + \frac{bx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3f} + \frac{\sqrt{e}\,(2bde-bc}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.59, size = 215, normalized size = 0.76

$$\frac{b\sqrt{\frac{d}{c}}\,fx(c+dx^2)(e+fx^2) - ie(-2bde+bcf+3adf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right) + i(2be-3af)(-de+cf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)}{3\sqrt{\frac{d}{c}}\,f^2\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] (b*Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2) - I*e*(-2*b*d*e + b*c*f + 3*a*d*f)
*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x],
(c*f)/(d*e)] + I*(2*b*e - 3*a*f)*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*Sqrt[d/c]*f
^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.13, size = 501, normalized size = 1.78

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{bx\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{3f}+\frac{\left(ac-\frac{ceb}{3f}\right)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticF}\left(x\sqrt{}\right.}{\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+cf\,x^2+de}}\right.}{}$

risch	$\frac{bx\sqrt{dx^2+c}\,\sqrt{fx^2+e}}{3f}+\frac{\left(-\frac{(3adf+bcf-2bde)e\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{dfx^4+cfx^2+dex^2+ce}}\right.}{}$
default	$\frac{\sqrt{dx^2+c}\,\sqrt{fx^2+e}\,\left(\sqrt{-\frac{d}{c}}\,bdf^2x^5+\sqrt{-\frac{d}{c}}\,bcf^2x^3+\sqrt{-\frac{d}{c}}\,bdefx^3+3\sqrt{\frac{dx^2+c}{c}}\,\sqrt{\frac{fx^2+e}{e}}\,\text{EllipticF}\left(x\sqrt{\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((-d/c)^(1/2)*b*d*f^2*x^5+(-d/c)^(1/2)*
b*c*f^2*x^3+(-d/c)^(1/2)*b*d*e*f*x^3+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*f^2-3*((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*f-2*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*b*c*e*f+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*b*d*e^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ell
ipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e*f-2*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d*
e^2+(-d/c)^(1/2)*b*c*e*f*x)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/f^2/(-d/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)\sqrt{c + dx^2}}{\sqrt{e + fx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2 + a)\,\sqrt{d\,x^2 + c}}{\sqrt{f\,x^2 + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2),x)
```

```
[Out] int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2), x)
```

3.38 $\int \frac{a+bx^2}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=206

$$\frac{bx\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{b\sqrt{e}\,\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}}+\frac{a\sqrt{e}\,\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}}$$

```
[Out] b*x*(d*x^2+c)^(1/2)/d/(f*x^2+e)^(1/2)-b*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(
1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/
2)*(d*x^2+c)^(1/2)/d/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2
)+a*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+
f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/c/f^(1/2)/(e*(d*x
^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {545, 429, 506, 422}

$$\frac{a\sqrt{e}\,\sqrt{c+dx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\,\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{b\sqrt{e}\,\sqrt{c+dx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\,\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{bx\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (b*Sqrt[e]*Sqrt[c + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c
+ d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (a*Sqrt[e]*Sqrt[c + d*x^2]*El
lipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*Sqrt[(e*(
c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx = a\int \frac{1}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx + b\int \frac{x^2}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$$

$$= \frac{bx\sqrt{c+dx^2}}{d\sqrt{e+fx^2}} + \frac{a\sqrt{e}\,\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}} - \frac{(be)\int \frac{\sqrt{c+d}}{(e+fx^2)}}{d}$$

$$= \frac{bx\sqrt{c+dx^2}}{d\sqrt{e+fx^2}} - \frac{b\sqrt{e}\,\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}} + \frac{a\sqrt{e}\,\sqrt{c+dx}}{c\sqrt{f}\,\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.95, size = 131, normalized size = 0.64

$$-\frac{i\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\left(beE\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+(-be+af)F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)\right)}{\sqrt{\frac{d}{c}}\,f\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(b*e*EllipticE[I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)] + (-(b*e) + a*f)*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (
c*f)/(d*e)]))/(Sqrt[d/c]*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.12, size = 158, normalized size = 0.77

method	result
default	$\frac{\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)af-\operatorname{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)be+\operatorname{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)be\right)\sqrt{\frac{fx^2+e}{e}}\,\sqrt{\frac{dx^2+c}{c}}}{f\sqrt{-\frac{d}{c}}\,(dfx^4+cfx^2+dex^2+ce)}$
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{a\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{dfx^4+cfx^2+dex^2+ce}}-\frac{be\sqrt{1+\frac{dx^2}{c}}\,\sqrt{}}{}\right.}{\sqrt{dx^2+c}\,\sqrt{fx^2+e}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*f-EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*b*e+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*e)*((f*x^2+e)
/e)^(1/2)*((d*x^2+c)/c)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/f/(-d/c)^(1/2
)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b\,x^2+a}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.39 $\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=209

$$-\frac{(bc-ad)\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\,\sqrt{d}\,(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{\sqrt{e}\,(be-af)\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\,(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] (-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1
/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/c/(-c*f+d*
e)/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(-a*d+b*c)*(1/(1
+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/(-c*f+d*e)/c^(1/2)/d^(1/2)/(d*x^2+
c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {539, 429, 422}

$$\frac{\sqrt{e}\,\sqrt{c+dx^2}\,(be-af)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\,\sqrt{e+fx^2}\,(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e+fx^2}\,(bc-ad)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\,\sqrt{d}\,\sqrt{c+dx^2}\,(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -(((b*c - a*d)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^
2))/(e*(c + d*x^2))])) + (Sqrt[e]*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticF[Arc
Tan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*(d*e - c*f)*Sqrt[(e*
(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rubi steps

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = -\frac{(bc-ad)\int \frac{\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx}{de-cf} + \frac{(be-af)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{de-cf}$$

$$= -\frac{(bc-ad)\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\,(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{\sqrt{e}\,(be-af)\sqrt{c+dx}}{c\sqrt{f}\,(de-cf)\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.30, size = 206, normalized size = 0.99

$$\frac{\sqrt{\frac{d}{c}}\left(\sqrt{\frac{d}{c}}(bc-ad)x(e+fx^2)+i(bc-ad)e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ia(-de+cf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)}{d(-de+cf)\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[d/c]*(Sqrt[d/c]*(b*c - a*d)*x*(e + f*x^2) + I*(b*c - a*d)*e*Sqrt[1 +
(d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e
)] - I*a*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I
*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(d*(-(d*e) + c*f)*Sqrt[c + d*x^2]*Sqr
t[e + f*x^2])
```

Maple [A]
time = 0.12, size = 334, normalized size = 1.60

method	result
default	$\frac{\left(-\sqrt{-\frac{d}{c}}\, a d f\, x^{3}+\sqrt{-\frac{d}{c}}\, b c f\, x^{3}+\sqrt{\frac{d\, x^{2}+c}{c}}\, \sqrt{\frac{f\, x^{2}+e}{e}}\, \mathrm{EllipticF}\left(x \sqrt{-\frac{d}{c}}, \sqrt{\frac{c f}{d e}}\right) a c f-\sqrt{\frac{d\, x^{2}+c}{c}}\, \sqrt{\frac{f\, x^{2}+e}{e}}\, \mathrm{Ellip}\right.}{}$
elliptic	$\frac{\sqrt{\left(d\, x^{2}+c\right)\left(f\, x^{2}+e\right)}\, \left(-\frac{\left(d f\, x^{2}+d e\right) x \left(a d-b c\right)}{d c \left(c f-d e\right) \sqrt{\left(x^{2}+\frac{c}{d}\right)\left(d f\, x^{2}+d e\right)}}+\frac{\left(\frac{b}{d}+\frac{a d-b c}{c d}+\frac{e \left(a d-b c\right)}{c \left(c f-d e\right)}\right) \sqrt{1+\frac{d\, x^{2}}{c}}\, \sqrt{1+\frac{f}{}}}{\sqrt{-\frac{d}{c}}\, \sqrt{d f\, x^{4}+c f}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d/c)^(1/2)*a*d*f*x^3+(-d/c)^(1/2)*b*c*f*x^3+((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*f-((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e+(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*a*d*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*b*c*e-(-d/c)^(1/2)*a*d*e*x+(-d/c)^(1/2)*b*c*e*x)*(f*x^2+
e)^(1/2)*(d*x^2+c)^(1/2)/c/(-d/c)^(1/2)/(c*f-d*e)/(d*f*x^4+c*f*x^2+d*e*x^2+
c*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2}{\left(c + dx^2\right)^{\frac{3}{2}} \sqrt{e + fx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
[Out] Integral((a + b*x**2)/((c + d*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b\,x^2 + a}{{\left(d\,x^2 + c\right)}^{3/2}\,\sqrt{f\,x^2 + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)),x)
[Out] int((a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)), x)
```

3.40 $\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=284

$$-\frac{(bc-ad)x\sqrt{e+fx^2}}{3c(de-cf)\,(c+dx^2)^{3/2}}+\frac{(2ad(de-2cf)+bc(de+cf))\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}\,(de-cf)^2\sqrt{c+dx^2}\,\sqrt{\frac{c\,(e+fx^2)}{e\,(c+dx^2)}}}-\frac{\sqrt{e}\,\sqrt{}}{}$$

```
[Out] -1/3*(-3*a*c*f+a*d*e+2*b*c*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ellip
ticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)
*(d*x^2+c)^(1/2)/c^2/(-c*f+d*e)^2/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)
^(1/2)-1/3*(-a*d+b*c)*x*(f*x^2+e)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(3/2)+1/3*(2
*a*d*(-2*c*f+d*e)+b*c*(c*f+d*e))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*El
lipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1
/2)/c^(3/2)/(-c*f+d*e)^2/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^
(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {541, 539, 429, 422}

$$\frac{\sqrt{e+fx^2}\,(2ad(de-2cf)+bc(cf+de))E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}\,\sqrt{c+dx^2}\,(de-cf)^2\sqrt{\frac{c\,(e+fx^2)}{e\,(c+dx^2)}}}-\frac{\sqrt{e}\,\sqrt{f}\,\sqrt{c+dx^2}\,(-3acf+ade+2bce)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}\,(de-cf)^2\sqrt{\frac{e\,(c+dx^2)}{c\,(e+fx^2)}}}-\frac{x\sqrt{e+fx^2}\,(bc-ad)}{3c\,(c+dx^2)^{3/2}\,(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -1/3*((b*c - a*d)*x*Sqrt[e + f*x^2])/(c*(d*e - c*f)*(c + d*x^2)^(3/2)) + ((
2*a*d*(d*e - 2*c*f) + b*c*(d*e + c*f))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*Sqrt[d]*(d*e - c*f)^2*Sqrt[
c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (Sqrt[e]*Sqrt[f]*(2*b*c
*e + a*d*e - 3*a*c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]]
, 1 - (d*e)/(c*f)])/(3*c^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2
))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = -\frac{(bc-ad)x\sqrt{e+fx^2}}{3c(de-cf)(c+dx^2)^{3/2}} - \frac{\int \frac{-bce-2ade+3acf+(bc-ad)fx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx}{3c(de-cf)}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3c(de-cf)(c+dx^2)^{3/2}} - \frac{(f(2bce+ade-3acf))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}}{3c(de-cf)^2}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3c(de-cf)(c+dx^2)^{3/2}} + \frac{(2ad(de-2cf)+bc(de+cf))\sqrt{e+fx^2}\,E\Big(\mathrm{t}}{3c^{3/2}\sqrt{d}\,(de-cf)^2\sqrt{c+dx^2}\,\sqrt{\frac{c}{e}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.83, size = 302, normalized size = 1.06

$$\frac{\sqrt{\frac{d}{c}}\,x(e+fx^2)\left(bc(2c^2f+d^2ex^2+cdfx^2)+ad(-5c^2f+2d^2ex^2+cd(3e-4fx^2))\right)+ie(2ad(de-2cf)+bc(de+cf))(c+dx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+i(-de+cf)(bce+2ade-3acf)(c+dx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)}{3c^2\sqrt{\frac{d}{c}}\,(de-cf)^2(c+dx^2)^{3/2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[d/c]*x*(e + f*x^2)*(b*c*(2*c^2*f + d^2*e*x^2 + c*d*f*x^2) + a*d*(-5*c
^2*f + 2*d^2*e*x^2 + c*d*(3*e - 4*f*x^2))) + I*e*(2*a*d*(d*e - 2*c*f) + b*c
*(d*e + c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE
[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*(-(d*e) + c*f)*(b*c*e + 2*a*d*e -
 3*a*c*f)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*A
rcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*c^2*Sqrt[d/c]*(d*e - c*f)^2*(c + d*x^
2)^(3/2)*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1351 vs. $2(324) = 648$.
time = 0.12, size = 1352, normalized size = 4.76

method	result
elliptic	$\frac{\sqrt{\left(d\,x^{2}+c\right)\left(f\,x^{2}+e\right)}\left(-\frac{x(ad-bc)\sqrt{df\,x^{4}+cf\,x^{2}+de\,x^{2}+ce}}{3d^{2}c(cf-de)\left(x^{2}+\frac{c}{d}\right)^{2}}-\frac{\left(df\,x^{2}+de\right)x\left(4acdf-2a\,d^{2}e-b\,c^{2}f-bcde\right)}{3d\,c^{2}(cf-de)^{2}\sqrt{\left(x^{2}+\frac{c}{d}\right)\left(df\,x^{2}+de\right.}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-5*(-d/c)^(1/2)*a*c^2*d*e*f*x+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^2*x^2-2*((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^
2*x^2+2*(-d/c)^(1/2)*b*c^3*e*f*x+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^3*f^2+(-d/c)^(1/2)*b*c*d^2*e*
f*x^5-(-d/c)^(1/2)*a*c*d^2*e*f*x^3+(-d/c)^(1/2)*b*c^2*d*e*f*x^3+(-d/c)^(1/2
)*b*c^2*d*f^2*x^5-5*(-d/c)^(1/2)*a*c^2*d*f^2*x^3+(-d/c)^(1/2)*b*c*d^2*e^2*x
^3+3*(-d/c)^(1/2)*a*c*d^2*e^2*x+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ell
ipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^2*x^2-((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^2*x
^2-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*a*c^2*d*e*f+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f+3*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*f^2*x^2+2*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*a*c*d^2*e^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*El
lipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2-2*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2-((
```

```
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*b*c^3*e*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2+2*(-d/c)^(1/2)*a*d^3*e^2*x^3+2*(-d/c)^(1/
2)*b*c^3*f^2*x^3-4*(-d/c)^(1/2)*a*c*d^2*f^2*x^5+2*(-d/c)^(1/2)*a*d^3*e*f*x^
5-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d
/e)^(1/2))*a*c*d^2*e*f*x^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
E(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e*f*x^2+4*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e*f*x^2-
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*b*c^2*d*e*f*x^2)/(f*x^2+e)^(1/2)/(c*f-d*e)^2/c^2/(-d/c)^(1/2)/(d*x^2
+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2}{\left(c + dx^2\right)^{\frac{5}{2}} \sqrt{e + fx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)/((c + d*x**2)**(5/2)*sqrt(e + f*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b\,x^2+a}{{\left(d\,x^2+c\right)}^{5/2}\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)),x)
[Out] int((a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)), x)
```

3.41 $\int \frac{a+bx^2}{\left(c+dx^2\right)^{7/2} \sqrt{e+fx^2}} \, dx$

Optimal. Leaf size=401

$$-\frac{(bc-ad)x\sqrt{e+fx^2}}{5c(de-cf)\left(c+dx^2\right)^{5/2}}+\frac{(4ad(de-2cf)+bc(de+3cf))x\sqrt{e+fx^2}}{15c^2(de-cf)^2\left(c+dx^2\right)^{3/2}}+\frac{(bc(2d^2e^2-7cdef-3c^2f^2)+ad(}{15c^{5/2}}$$

```
[Out] -1/15*(b*c*e*(-9*c*f+d*e)+a*(15*c^2*f^2-11*c*d*e*f+4*d^2*e^2))*(1/(1+f*x^2/
e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(
1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c^3/(-c*f+d*e)^3/(e*(d*x^
2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/5*(-a*d+b*c)*x*(f*x^2+e)^(1/2)/c/
(-c*f+d*e)/(d*x^2+c)^(5/2)+1/15*(4*a*d*(-2*c*f+d*e)+b*c*(3*c*f+d*e))*x*(f*x
^2+e)^(1/2)/c^2/(-c*f+d*e)^2/(d*x^2+c)^(3/2)+1/15*(b*c*(-3*c^2*f^2-7*c*d*e*
f+2*d^2*e^2)+a*d*(23*c^2*f^2-23*c*d*e*f+8*d^2*e^2))*(1/(1+d*x^2/c))^(1/2)*(
1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^
(1/2))*(f*x^2+e)^(1/2)/c^(5/2)/(-c*f+d*e)^3/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x
^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {541, 539, 429, 422}

$$\frac{\sqrt{e+fx^2}\left(ad(23c^2f^2-23cdef+8d^2e^2)+bc(-3c^2f^2-7cdef+2d^2e^2)\right)E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{d}\sqrt{c+dx^2}(de-cf)^3\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\left(a(15c^2f^2-11cdef+4d^2e^2)+bce(de-9cf)\right)F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3\sqrt{e+fx^2}(de-cf)^3\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{x\sqrt{e+fx^2}(4ad(de-2cf)+bc(3cf+de))}{15c^2\left(c+dx^2\right)^{3/2}(de-cf)^2}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5c\left(c+dx^2\right)^{5/2}(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -1/5*((b*c - a*d)*x*Sqrt[e + f*x^2])/(c*(d*e - c*f)*(c + d*x^2)^(5/2)) + ((
4*a*d*(d*e - 2*c*f) + b*c*(d*e + 3*c*f))*x*Sqrt[e + f*x^2])/(15*c^2*(d*e -
c*f)^2*(c + d*x^2)^(3/2)) + ((b*c*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) + a*d
*(8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*c^(5/2)*Sqrt[d]*(d*e - c*f)^3*Sqr
t[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (Sqrt[e]*Sqrt[f]*(b*c
*e*(d*e - 9*c*f) + a*(4*d^2*e^2 - 11*c*d*e*f + 15*c^2*f^2))*Sqrt[c + d*x^2]
*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*c^3*(d*e - c*
f)^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))])])*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx &= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5c(de-cf)\left(c+dx^2\right)^{5/2}}-\frac{\int \frac{-bce-4ade+5acf+3(bc-ad)fx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx}{5c(de-cf)}\\
&= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5c(de-cf)\left(c+dx^2\right)^{5/2}}+\frac{(4ad(de-2cf)+bc(de+3cf))x\sqrt{e+fx^2}}{15c^2(de-cf)^2\left(c+dx^2\right)^{3/2}}\\
&= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5c(de-cf)\left(c+dx^2\right)^{5/2}}+\frac{(4ad(de-2cf)+bc(de+3cf))x\sqrt{e+fx^2}}{15c^2(de-cf)^2\left(c+dx^2\right)^{3/2}}\\
&= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5c(de-cf)\left(c+dx^2\right)^{5/2}}+\frac{(4ad(de-2cf)+bc(de+3cf))x\sqrt{e+fx^2}}{15c^2(de-cf)^2\left(c+dx^2\right)^{3/2}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.92, size = 393, normalized size = 0.98

$$\frac{-\sqrt{\frac{d}{c}}\,x(e+fx^2)\left(3c^2(bc-ad)(de-cf)^2+c(-de+cf)(4ad(de-2cf)+bc(de+3cf))\,(c+dx^2)+(ad(-8d^2e^2+23cdef-23c^2f^2)+bc(-2d^2e^2+7cdef+3c^2f^2))\,(c+dx^2)^2\right)-i(c+dx^2)^2\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(e(ad(-8d^2e^2+23cdef-23c^2f^2)+bc(-2d^2e^2+7cdef+3c^2f^2))\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+(de-cf)\,(2bce(de-3cf)+a(8d^2e^2-19cdef+15c^2f^2))\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)\right)}{15c^3\sqrt{\frac{d}{c}}\,(de-cf)^3\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (-(Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)*(d*e - c*f)^2 + c*(-(d*e) + c
*f)*(4*a*d*(d*e - 2*c*f) + b*c*(d*e + 3*c*f))*(c + d*x^2) + (a*d*(-8*d^2*e^
2 + 23*c*d*e*f - 23*c^2*f^2) + b*c*(-2*d^2*e^2 + 7*c*d*e*f + 3*c^2*f^2))*(c
 + d*x^2)^2)) - I*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(e*
(a*d*(-8*d^2*e^2 + 23*c*d*e*f - 23*c^2*f^2) + b*c*(-2*d^2*e^2 + 7*c*d*e*f +
 3*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (d*e - c*f)*(
2*b*c*e*(d*e - 3*c*f) + a*(8*d^2*e^2 - 19*c*d*e*f + 15*c^2*f^2))*EllipticF[
I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(15*c^3*Sqrt[d/c]*(d*e - c*f)^3*(c +
 d*x^2)^(5/2)*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3038 vs. $2(435) = 870$.
time = 0.13, size = 3039, normalized size = 7.58

method	result
elliptic	$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\left(-\frac{x(ad-bc)\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{5d^3c(cf-de)\left(x^2+\frac{c}{d}\right)^3}-\frac{\left(8acdf-4a\,d^2e-3b\,c^2f-bcde\right)x\sqrt{df\,x^4+cf\,}}{15c^2(cf-de)^2d^2\left(x^2+\frac{c}{d}\right)^2}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(41*(-d/c)^(1/2)*a*c^3*d^2*e^2*f*x-(-d/c)^(1/2)*b*c^4*d*e^2*f*x+23*(-d
/c)^(1/2)*a*c*d^4*e*f^2*x^7+7*(-d/c)^(1/2)*b*c^2*d^3*e*f^2*x^7-2*(-d/c)^(1/
2)*b*c*d^4*e^2*f*x^7+35*(-d/c)^(1/2)*a*c^2*d^3*e*f^2*x^5+3*(-d/c)^(1/2)*a*c
*d^4*e^2*f*x^5+15*(-d/c)^(1/2)*b*c^3*d^2*e*f^2*x^5+2*(-d/c)^(1/2)*b*c^2*d^3
*e^2*f*x^5-13*(-d/c)^(1/2)*a*c^3*d^2*e*f^2*x^3+43*(-d/c)^(1/2)*a*c^2*d^3*e^
2*f*x^3+8*(-d/c)^(1/2)*b*c^4*d*e*f^2*x^3+12*(-d/c)^(1/2)*b*c^3*d^2*e^2*f*x^
3-34*(-d/c)^(1/2)*a*c^4*d*e*f^2*x+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^5*f^3-23*(-d/c)^(1/2)*a*c^2
*d^3*f^3*x^7-8*(-d/c)^(1/2)*a*d^5*e^2*f*x^7+3*(-d/c)^(1/2)*b*c^3*d^2*f^3*x^
7-54*(-d/c)^(1/2)*a*c^3*d^2*f^3*x^5+9*(-d/c)^(1/2)*b*c^4*d*f^3*x^5-2*(-d/c)
^(1/2)*b*c*d^4*e^3*x^5-34*(-d/c)^(1/2)*a*c^4*d*f^3*x^3-20*(-d/c)^(1/2)*a*c*
```

d^4*e^3*x^3-5*(-d/c)^(1/2)*b*c^2*d^3*e^3*x^3-15*(-d/c)^(1/2)*a*c^2*d^3*e^3*
x+9*(-d/c)^(1/2)*b*c^5*e*f^2*x+8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*El
lipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^4*d*e^2*f+23*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^4*d*e*f
^2-23*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*a*c^3*d^2*e^2*f-7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^4*d*e^2*f-8*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^5*e^3*x^4+
8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a*d^5*e^3*x^4-8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(
x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^3*e^3-6*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^5*e*f^2-2*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*b*c^3*d^2*e^3+8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^3*e^3-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^5*e*f^2+2*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d
^2*e^3-8*(-d/c)^(1/2)*a*d^5*e^3*x^5+9*(-d/c)^(1/2)*b*c^5*f^3*x^3+15*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
a*c^3*d^2*f^3*x^4+30*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^4*d*f^3*x^2-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^4*e^3*x^4+2*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*b*c*d^4*e^3*x^4-16*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^4*e^3*x^2-4*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d^3*e^3*x^2+16*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*a*c*d^4*e^3*x^2+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(
x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d^3*e^3*x^2-34*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^4*d*e*f^2+2
7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a*c^3*d^2*e^2*f-34*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipti
cF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^3*e*f^2*x^4+27*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^4*e^
2*f*x^4-6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*b*c^3*d^2*e*f^2*x^4+8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d^3*e^2*f*x^4+23*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a*c^2*d^3*e*f^2*x^4-23*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^4*e^2*f*x^4-3*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e*f^2*x
^4-7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*b*c^2*d^3*e^2*f*x^4-68*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^3*d^2*e*f^2*x^2+54*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c

```
^2*d^3*e^2*f*x^2-12*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*b*c^4*d*e*f^2*x^2+16*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^2*f*x^2+
46*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*a*c^3*d^2*e*f^2*x^2-46*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*El
lipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^3*e^2*f*x^2-6*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^4*
d*e*f^2*x^2-14*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^2*f*x^2)/(f*x^2+e)^(1/2)/(c*f-d*e)^3/c^3/
(-d/c)^(1/2)/(d*x^2+c)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2}{(c + dx^2)^{\frac{7}{2}} \sqrt{e + fx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(7/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)/((c + d*x**2)**(7/2)*sqrt(e + f*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b\,x^2+a}{{\left(d\,x^2+c\right)}^{7/2}\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(1/2)),x)
[Out] int((a + b*x^2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(1/2)), x)
```

3.42 $\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=501

$$-\frac{(5af(8d^2e^2-13cdef+3c^2f^2)-2be(24d^2e^2-44cdef+19c^2f^2))\,x\sqrt{c+dx^2}}{15ef^3\sqrt{e+fx^2}}-\frac{(be-af)x(c+dx^2)^{5/2}}{ef\sqrt{e+fx^2}}-d$$

```
[Out] -(-a*f+b*e)*x*(d*x^2+c)^(5/2)/e/f/(f*x^2+e)^(1/2)-1/15*(5*a*f*(3*c^2*f^2-13
*c*d*e*f+8*d^2*e^2)-2*b*e*(19*c^2*f^2-44*c*d*e*f+24*d^2*e^2))*x*(d*x^2+c)^(
1/2)/e/f^3/(f*x^2+e)^(1/2)+1/15*(5*a*f*(3*c^2*f^2-13*c*d*e*f+8*d^2*e^2)-2*b
*e*(19*c^2*f^2-44*c*d*e*f+24*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1
/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2
+c)^(1/2)/f^(7/2)/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1
/15*(10*a*d*f*(-3*c*f+2*d*e)-b*(15*c^2*f^2-41*c*d*e*f+24*d^2*e^2))*(1/(1+f*
x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/
2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/f^(7/2)/(e*(d*x^2+c)/c/(f*x^2
+e))^(1/2)/(f*x^2+e)^(1/2)+1/5*d*(-5*a*f+6*b*e)*x*(d*x^2+c)^(3/2)*(f*x^2+e)
^(1/2)/e/f^2-1/15*d*(b*e*(-23*c*f+24*d*e)-5*a*f*(-3*c*f+4*d*e))*x*(d*x^2+c)
^(1/2)*(f*x^2+e)^(1/2)/e/f^3
```

Rubi [A]
time = 0.40, antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {540, 542, 545, 429, 506, 422}

$$-\frac{\sqrt{e}\sqrt{c+dx^2}\,(10adf(2de-3cf)-b(15c^2f^2-41cdef+24d^2e^2))\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15f^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{c+dx^2}\,(5af(3c^2f^2-13cdef+8d^2e^2)-2be(19c^2f^2-44cdef+24d^2e^2))\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15\sqrt{e}f^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{c+dx^2}\,(5af(3c^2f^2-13cdef+8d^2e^2)-2be(19c^2f^2-44cdef+24d^2e^2))}{15ef^3\sqrt{e+fx^2}}-\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}\,(be(24de-23cf)-5af(4de-3cf))}{15ef^3}+\frac{dx(c+dx^2)^{3/2}\sqrt{e+fx^2}\,(6be-5af)}{5ef^2}-\frac{x(c+dx^2)^{5/2}(be-af)}{ef\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(3/2),x]
```

```
[Out] -1/15*((5*a*f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) - 2*b*e*(24*d^2*e^2 - 44
*c*d*e*f + 19*c^2*f^2))*x*Sqrt[c + d*x^2])/(e*f^3*Sqrt[e + f*x^2]) - ((b*e
- a*f)*x*(c + d*x^2)^(5/2))/(e*f*Sqrt[e + f*x^2]) - (d*(b*e*(24*d*e - 23*c*
f) - 5*a*f*(4*d*e - 3*c*f))*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*e*f^3) +
 (d*(6*b*e - 5*a*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*e*f^2) + ((5*a*
f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) - 2*b*e*(24*d^2*e^2 - 44*c*d*e*f + 1
9*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(15*Sqrt[e]*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2]) - (Sqrt[e]*(10*a*d*f*(2*d*e - 3*c*f) - b*(24*d^2*e^2 - 41*c*d*e*f
 + 15*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 -
```

```
(d*e)/(c*f)])/(15*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*
x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx = -\frac{(be-af)x(c+dx^2)^{5/2}}{ef\sqrt{e+fx^2}} - \frac{\int \frac{(c+dx^2)^{3/2}(-bce-d(6be-5af)x^2)}{\sqrt{e+fx^2}}\,dx}{ef}$$

$$= -\frac{(be-af)x(c+dx^2)^{5/2}}{ef\sqrt{e+fx^2}} + \frac{d(6be-5af)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5ef^2} - \frac{\int \sqrt{c+}}{}$$

$$= -\frac{(be-af)x(c+dx^2)^{5/2}}{ef\sqrt{e+fx^2}} - \frac{d(be(24de-23cf)-5af(4de-3cf))x\sqrt{c+dx^2}}{15ef^3}$$

$$= -\frac{(be-af)x(c+dx^2)^{5/2}}{ef\sqrt{e+fx^2}} - \frac{d(be(24de-23cf)-5af(4de-3cf))x\sqrt{c+dx^2}}{15ef^3}$$

$$= -\frac{(5af(8d^2e^2-13cdef+3c^2f^2)-2be(24d^2e^2-44cdef+19c^2f^2))\,x\sqrt{c+dx}}{15ef^3\sqrt{e+fx^2}}$$

$$= -\frac{(5af(8d^2e^2-13cdef+3c^2f^2)-2be(24d^2e^2-44cdef+19c^2f^2))\,x\sqrt{c+dx}}{15ef^3\sqrt{e+fx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.19, size = 369, normalized size = 0.74

$$\frac{\sqrt{\frac{d}{c}}fx(c+dx^2)(5af(-6cdef+3c^2f^2+d^2e(4e+fx^2))+be(-15c^2f^2+cdf(41e+11fx^2)-3d^2(8e^2+2efx^2-f^2x^4)))-ide(-5af(8d^2e^2-13cdef+3c^2f^2)+2be(24d^2e^2-44cdef+19c^2f^2))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ie(-de+cf)(5adf(-8de+9cf)+b(48d^2e^2-64cdef+15c^2f^2))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)}{15\sqrt{\frac{d}{c}}ef^4\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(5*a*f*(-6*c*d*e*f + 3*c^2*f^2 + d^2*e*(4*e + f*
x^2)) + b*e*(-15*c^2*f^2 + c*d*f*(41*e + 11*f*x^2) - 3*d^2*(8*e^2 + 2*e*f*x
^2 - f^2*x^4))) - I*d*e*(-5*a*f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) + 2*b*
e*(24*d^2*e^2 - 44*c*d*e*f + 19*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x
^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(d*e) + c*f)*
```

```
(5*a*d*f*(-8*d*e + 9*c*f) + b*(48*d^2*e^2 - 64*c*d*e*f + 15*c^2*f^2))*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)])/(15*Sqrt[d/c]*e*f^4*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1168 vs. $2(525) = 1050$.
time = 0.18, size = 1169, normalized size = 2.33 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(-88*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f+38*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*
c^2*d*e^2*f^2+112*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f-15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3+65*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*
c*d^2*e^2*f^2+45*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3-85*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*f^2-79*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b
*c^2*d*e^2*f^2+15*(-d/c)^(1/2)*a*c^3*f^4*x-15*(-d/c)^(1/2)*b*c^3*e*f^3*x+14
*(-d/c)^(1/2)*b*c*d^2*e*f^3*x^5-25*(-d/c)^(1/2)*a*c*d^2*e*f^3*x^3-4*(-d/c)^
(1/2)*b*c^2*d*e*f^3*x^3+35*(-d/c)^(1/2)*b*c*d^2*e^2*f^2*x^3+3*(-d/c)^(1/2)*
b*d^3*e*f^3*x^7+5*(-d/c)^(1/2)*a*d^3*e*f^3*x^5-6*(-d/c)^(1/2)*b*d^3*e^2*f^2
*x^5+40*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*a*d^3*e^3*f+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellip
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f^3-40*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^3*f-30*(
-d/c)^(1/2)*a*c^2*d*e*f^3*x+20*(-d/c)^(1/2)*a*c*d^2*e^2*f^2*x+41*(-d/c)^(1/
2)*b*c^2*d*e^2*f^2*x-24*(-d/c)^(1/2)*b*c*d^2*e^3*f*x+15*(-d/c)^(1/2)*a*c^2*
d*f^4*x^3+20*(-d/c)^(1/2)*a*d^3*e^2*f^2*x^3-24*(-d/c)^(1/2)*b*d^3*e^3*f*x^3
-48*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d
/e)^(1/2))*b*d^3*e^4+48*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e^4)/f^4/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
/(-d/c)^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)\,(c+dx^2)^{\frac{5}{2}}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*(c + d*x**2)**(5/2)/(e + f*x**2)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)\,{(d\,x^2+c)}^{5/2}}{{(f\,x^2+e)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(3/2),x)
```

```
[Out] int(((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(3/2), x)
```

3.43 $\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}} \, dx$

Optimal. Leaf size=358

$$-\frac{(be(8de-7cf)-3af(2de-cf))x\sqrt{c+dx^2}}{3ef^2\sqrt{e+fx^2}}-\frac{(be-af)x(c+dx^2)^{3/2}}{ef\sqrt{e+fx^2}}+\frac{d(4be-3af)x\sqrt{c+dx^2}\,\sqrt{e+f}}{3ef^2}$$

```
[Out] -(-a*f+b*e)*x*(d*x^2+c)^(3/2)/e/f/(f*x^2+e)^(1/2)-1/3*(b*e*(-7*c*f+8*d*e)-3
*a*f*(-c*f+2*d*e))*x*(d*x^2+c)^(1/2)/e/f^2/(f*x^2+e)^(1/2)+1/3*(b*e*(-7*c*f
+8*d*e)-3*a*f*(-c*f+2*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ellipti
cE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/f
^(5/2)/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(-3*a*d*
f-3*b*c*f+4*b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1
/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/f^
(5/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d*(-3*a*f+4*b*e)*
x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/e/f^2
```

Rubi [A]
time = 0.25, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {540, 542, 545, 429, 506, 422}

$$-\frac{\sqrt{e}\sqrt{c+dx^2}\,(-3adf-3bcf+4bde)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{c+dx^2}\,(be(8de-7cf)-3af(2de-cf))E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3\sqrt{e}\,f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{dx\sqrt{c+dx^2}\,\sqrt{e+fx^2}\,(4be-3af)}{3ef^2}-\frac{x\sqrt{c+dx^2}\,(be(8de-7cf)-3af(2de-cf))}{3ef^2\sqrt{e+fx^2}}-\frac{x(c+dx^2)^{3/2}\,(be-af)}{ef\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x]
```

```
[Out] -1/3*((b*e*(8*d*e - 7*c*f) - 3*a*f*(2*d*e - c*f))*x*Sqrt[c + d*x^2])/(e*f^2
*Sqrt[e + f*x^2]) - ((b*e - a*f)*x*(c + d*x^2)^(3/2))/(e*f*Sqrt[e + f*x^2])
 + (d*(4*b*e - 3*a*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*e*f^2) + ((b*e*
(8*d*e - 7*c*f) - 3*a*f*(2*d*e - c*f))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sq
rt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*Sqrt[e]*f^(5/2)*Sqrt[(e*(c + d*x^2)
)/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (Sqrt[e]*(4*b*d*e - 3*b*c*f - 3*a*d*f
)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/
(3*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = -\frac{(be-af)x(c+dx^2)^{3/2}}{ef\sqrt{e+fx^2}} - \frac{\int \frac{\sqrt{c+dx^2}\,(-bce-d(4be-3af)x^2)}{\sqrt{e+fx^2}}\,dx}{ef}$$

$$= -\frac{(be-af)x(c+dx^2)^{3/2}}{ef\sqrt{e+fx^2}} + \frac{d(4be-3af)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ef^2} - \frac{\int \frac{ce(4bde-}{\ldots}}{\ldots}$$

$$= -\frac{(be-af)x(c+dx^2)^{3/2}}{ef\sqrt{e+fx^2}} + \frac{d(4be-3af)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ef^2} - \frac{(c(4bde-}{\ldots}$$

$$= -\frac{(be(8de-7cf)-3af(2de-cf))x\sqrt{c+dx^2}}{3ef^2\sqrt{e+fx^2}} - \frac{(be-af)x(c+dx^2)^{3/2}}{ef\sqrt{e+fx^2}} + $$

$$= -\frac{(be(8de-7cf)-3af(2de-cf))x\sqrt{c+dx^2}}{3ef^2\sqrt{e+fx^2}} - \frac{(be-af)x(c+dx^2)^{3/2}}{ef\sqrt{e+fx^2}} + $$

Mathematica [C] Result contains complex when optimal does not.
time = 5.21, size = 260, normalized size = 0.73

$$\frac{\sqrt{\frac{d}{c}}fx(c+dx^2)\left(3af(-de+cf)+be(4de-3cf+dfx^2)\right) - ide(-3af(-2de+cf)+be(-8de+7cf))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)\middle|\frac{cf}{de}\right) - ie(-de+cf)(-8bde+3bcf+6adf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)\middle|\frac{cf}{de}\right)}{3\sqrt{\frac{d}{c}}ef^3\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(3*a*f*(-(d*e) + c*f) + b*e*(4*d*e - 3*c*f + d*f
*x^2)) - I*d*e*(-3*a*f*(-2*d*e + c*f) + b*e*(-8*d*e + 7*c*f))*Sqrt[1 + (d*x
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] -
 I*e*(-(d*e) + c*f)*(-8*b*d*e + 3*b*c*f + 6*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt
[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*Sqrt[d/c
]*e*f^3*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.15, size = 750, normalized size = 2.09 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((-d/c)^(1/2)*b*d^2*e*f^2*x^5+3*(-d/c)^
(1/2)*a*c*d*f^3*x^3-3*(-d/c)^(1/2)*a*d^2*e*f^2*x^3-2*(-d/c)^(1/2)*b*c*d*e*f
^2*x^3+4*(-d/c)^(1/2)*b*d^2*e^2*f*x^3+6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f^2-6*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e
^2*f+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*b*c^2*e*f^2-11*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipt
icF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f+8*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^2*e^3-3*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a*c*d*e*f^2+6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*f+7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f-8*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^2*e^3+3
*(-d/c)^(1/2)*a*c^2*f^3*x-3*(-d/c)^(1/2)*a*c*d*e*f^2*x-3*(-d/c)^(1/2)*b*c^2
*e*f^2*x+4*(-d/c)^(1/2)*b*c*d*e^2*f*x)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/f^3/e/
(-d/c)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)(c + dx^2)^{\frac{3}{2}}}{(e + fx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
[Out] Integral((a + b*x**2)*(c + d*x**2)**(3/2)/(e + f*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)\,\left(d\,x^2+c\right)^{3/2}}{\left(f\,x^2+e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x)
[Out] int(((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2), x)
```

3.44 $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=258

$$-\frac{(be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}}+\frac{(2be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}}-\frac{(2be-af)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\,f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}+\frac{b\sqrt{e}\,\sqrt{}}{}$$

```
[Out] -(-a*f+b*e)*x*(d*x^2+c)^(1/2)/e/f/(f*x^2+e)^(1/2)+(-a*f+2*b*e)*x*(d*x^2+c)^
(1/2)/e/f/(f*x^2+e)^(1/2)-(-a*f+2*b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1
/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2
+c)^(1/2)/f^(3/2)/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+b
*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x
^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/f^(3/2)/(e*(d*x^2+c)
/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {540, 545, 429, 506, 422}

$$-\frac{\sqrt{c+dx^2}(2be-af)E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\,f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{c+dx^2}(be-af)}{ef\sqrt{e+fx^2}}+\frac{x\sqrt{c+dx^2}(2be-af)}{ef\sqrt{e+fx^2}}+\frac{b\sqrt{e}\,\sqrt{c+dx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

```
[Out] -(((b*e - a*f)*x*Sqrt[c + d*x^2])/(e*f*Sqrt[e + f*x^2])) + ((2*b*e - a*f)*x
*Sqrt[c + d*x^2])/(e*f*Sqrt[e + f*x^2]) - ((2*b*e - a*f)*Sqrt[c + d*x^2]*El
lipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[e]*f^(3/2)*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b*Sqrt[e]*Sqrt[c + d
*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(f^(3/2)*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = -\frac{(be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} - \frac{\int \frac{-bce-d(2be-af)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{ef}$$

$$= -\frac{(be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} + \frac{(bc)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{f} + \frac{(d(2be-af))\int -}{}$$

$$= -\frac{(be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} + \frac{(2be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} + \frac{b\sqrt{e}\sqrt{c+dx^2}\,F\Big(\tan^{-1}\Big(}{f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$$= -\frac{(be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} + \frac{(2be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} - \frac{(2be-af)\sqrt{c+dx^2}\,E\Big(\mathrm{ta}}{\sqrt{e}\,f^{3/2}\sqrt{\frac{e(c+d}{c(e+f}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.75, size = 208, normalized size = 0.81

$$\frac{\sqrt{\frac{d}{c}}f(-be+af)x(c+dx^2) - ide(2be-af)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right) - ie(-2bde+bcf+adf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)}{\sqrt{\frac{d}{c}}ef^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d/c]*f*(-(b*e) + a*f)*x*(c + d*x^2) - I*d*e*(2*b*e - a*f)*Sqrt[1 + (d
*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
 - I*e*(-2*b*d*e + b*c*f + a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(Sqrt[d/c]*e*f^2*Sqrt[c + d*
x^2]*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.12, size = 393, normalized size = 1.52

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{(dfx^2+cf)(af-be)x}{f^2e\sqrt{\left(x^2+\frac{e}{f}\right)(dfx^2+cf)}} + \frac{\left(\frac{adf+bcf-bde}{f^2}+\frac{(af-be)(cf-de)}{f^2e}-\frac{c(af-be)}{fe}\right)\sqrt{1+\frac{dx^2}{c}}}{\sqrt{-\frac{d}{c}}\sqrt{dfx^4+c}}\right.}{}$

default	$\frac{\sqrt{d\,x^{2}+c}\,\sqrt{f\,x^{2}+e}\,\left(\sqrt{-\frac{d}{c}}\,a d\,f^{2} x^{3}-\sqrt{-\frac{d}{c}}\,b d e f\,x^{3}+\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{f\,x^{2}+e}{e}}\,\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{c f}{d e}}\right) a d e \ldots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((-d/c)^(1/2)*a*d*f^2*x^3-(-d/c)^(1/2)*b*d*
e*f*x^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*a*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e*f-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d*e^2-((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*f+2*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*b*d*e^2+(-d/c)^(1/2)*a*c*f^2*x-(-d/c)^(1/2)*b*c*e*f*x)/(d*f*x^4+c*f*x^2
+d*e*x^2+c*e)/f^2/e/(-d/c)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^{2})\sqrt{c+dx^{2}}}{(e+fx^{2})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}}{{\left(f\,x^2+e\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2),x)
```

```
[Out] int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2), x)
```

3.45 $\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{(be-af)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\,\sqrt{f}\,(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}}-\frac{(bc-ad)\sqrt{e}\,\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\,(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}}$$

```
[Out] (-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1
/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/(-c*f+d*e)/e^(1/2)
/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(-a*d+b*c)*(1/(1+f
*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1
/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/c/(-c*f+d*e)/f^(1/2)/(e*(d*x
^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {539, 429, 422}

$$\frac{\sqrt{c+dx^2}\,(be-af)E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\,\sqrt{f}\,\sqrt{e+fx^2}\,(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\,\sqrt{c+dx^2}\,(bc-ad)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\,\sqrt{e+fx^2}\,(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((b*e - a*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*
e)/(c*f)])/(Sqrt[e]*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2]) - ((b*c - a*d)*Sqrt[e]*Sqrt[c + d*x^2]*EllipticF[ArcTan
[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c
+ d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rubi steps

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = -\frac{(bc-ad)\int \frac{1}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx}{de-cf} + \frac{(be-af)\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{de-cf}$$

$$= \frac{(be-af)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\,\sqrt{f}\,(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}} - \frac{(bc-ad)\sqrt{e}\,\sqrt{c+dx^2}\,\ldots}{c\sqrt{f}\,(de-cf)\sqrt{\ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.03, size = 212, normalized size = 1.01

$$\frac{\sqrt{\frac{d}{c}}\,f(-be+af)x(c+dx^2) - ide(be-af)\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right) - ibe(-de+cf)\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)}{\sqrt{\frac{d}{c}}\,ef(-de+cf)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*f*(-(b*e) + a*f)*x*(c + d*x^2) - I*d*e*(b*e - a*f)*Sqrt[1 + (d*x
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] -
 I*b*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*A
rcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(Sqrt[d/c]*e*f*(-(d*e) + c*f)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.14, size = 349, normalized size = 1.67

method	result
default	$\frac{\left(\sqrt{-\frac{d}{c}}\, a d\, f^{2} x^{3}-\sqrt{-\frac{d}{c}}\, b d e f\, x^{3}+\sqrt{\frac{d\,x^{2}+c}{c}}\, \sqrt{\frac{f\,x^{2}+e}{e}}\, \operatorname{EllipticF}\left(x \sqrt{-\frac{d}{c}}, \sqrt{\frac{c f}{d e}}\right) b c e f-\sqrt{\frac{d\,x^{2}+c}{c}}\, \sqrt{\frac{f\,x^{2}+e}{e}}\, \mathrm{Ell}}{}$
elliptic	$\frac{\sqrt{\left(d\,x^{2}+c\right)\left(f\,x^{2}+e\right)}\, \left(\frac{\left(d f\,x^{2}+c f\right) x \left(a f-b e\right)}{f e \left(c f-d e\right) \sqrt{\left(x^{2}+\frac{e}{f}\right)\left(d f\,x^{2}+c f\right)}}+\frac{\left(\frac{b}{f}+\frac{a f-b e}{f e}-\frac{c \left(a f-b e\right)}{e \left(c f-d e\right)}\right) \sqrt{1+\frac{d\,x^{2}}{c}}\, \sqrt{1+\frac{f}{}}}{\sqrt{-\frac{d}{c}}\, \sqrt{d f\,x^{4}+c f}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-d/c)^(1/2)*a*d*f^2*x^3-(-d/c)^(1/2)*b*d*e*f*x^3+((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e*f-((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*
d*e^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*a*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*b*d*e^2+(-d/c)^(1/2)*a*c*f^2*x-(-d/c)^(1/2)*b*c
*e*f*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/(-d/c)^(1/2)/e/f/(c*f-d*e)/(d*f*x^4
+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2}{\sqrt{c + dx^2}\,(e + fx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
[Out] Integral((a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b\,x^2 + a}{\sqrt{d\,x^2 + c}\,\left(f\,x^2 + e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
[Out] int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

3.46 $\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=272

$$-\frac{(bc-ad)x}{c(de-cf)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}-\frac{\sqrt{f}\,(2bce-ade-acf)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{e}\,(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}}+\frac{\sqrt{e}\,(\ldots}{}$$

```
[Out] -(-a*d+b*c)*x/c/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)+(-2*a*d*f+b*c*f+
b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/
(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/c/(-c*f+d*e)^2
/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(-a*c*f-a*d*e+2*b*
c*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1
+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c/(-c*f+d*e)^2/e
^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {541, 539, 429, 422}

$$\frac{\sqrt{e}\,\sqrt{c+dx^2}\,(-2adf+bcf+bde)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\,\sqrt{e+fx^2}\,(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{f}\,\sqrt{c+dx^2}\,(-acf-ade+2bce)E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{e}\,\sqrt{e+fx^2}\,(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x(bc-ad)}{c\sqrt{c+dx^2}\,\sqrt{e+fx^2}\,(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] -(((b*c - a*d)*x)/(c*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])) - (Sqrt[
f]*(2*b*c*e - a*d*e - a*c*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[e]*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c
*(e + f*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[e]*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[c
 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[
f]*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx^2}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx &= -\frac{(bc-ad)x}{c(de-cf)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}-\frac{\int \frac{-c(be-af)+(bc-ad)fx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx}{c(de-cf)}\\
&= -\frac{(bc-ad)x}{c(de-cf)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}-\frac{(f(2bce-ade-acf))\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,d}{c(de-cf)^2}\\
&= -\frac{(bc-ad)x}{c(de-cf)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}-\frac{\sqrt{f}\,(2bce-ade-acf)\sqrt{c+dx^2}\,E\Big(}{c\sqrt{e}\,(de-cf)^2\sqrt{\frac{e\,(c+d}{c\,(e+f}}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.51, size = 262, normalized size = 0.96

$$
\frac{\sqrt{\frac{d}{c}}\left(\sqrt{\frac{d}{c}}\,x\left(a\left(c^2f^2+cdf^2x^2+d^2e(e+fx^2)\right)-bce\left(cf+d(e+2fx^2)\right)\right)-ide\left(2bce-a(de+cf)\right)\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)\middle|\frac{cf}{de}\right)-i(bc-ad)e(-de+cf)\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)\middle|\frac{cf}{de}\right)\right)}{de(de-cf)^2\sqrt{c+dx^2}\,\sqrt{e+fx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*(Sqrt[d/c]*x*(a*(c^2*f^2 + c*d*f^2*x^2 + d^2*e*(e + f*x^2)) - b*
c*e*(c*f + d*(e + 2*f*x^2))) - I*d*e*(2*b*c*e - a*(d*e + c*f))*Sqrt[1 + (d*
x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
- I*(b*c - a*d)*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*El
lipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(d*e*(d*e - c*f)^2*Sqrt[c +
d*x^2]*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.13, size = 581, normalized size = 2.14

method	result
elliptic	$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\left(-\frac{2df\left(-\frac{(acf+ade-2bce)x^3}{2ce\left(c^2f^2-2cdef+d^2e^2\right)}-\frac{\left(c^2a\,f^2+a\,d^2e^2-b\,c^2ef-bcd\,e^2\right)x}{2ce\left(c^2f^2-2cdef+d^2e^2\right)df}\right)}{\sqrt{\left(x^4+\frac{(cf+de)x^2}{df}+\frac{ce}{df}\right)df}}+\frac{\left(\frac{a}{ce}-\frac{c^2a\,f^2+a\,d^2e^2-b\,c^2ef-\cdots}{ce\left(c^2f^2-2cdef+d^2e^{\cdots}\right.}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\left(\sqrt{-\frac{d}{c}}\,acd\,f^2x^3+\sqrt{-\frac{d}{c}}\,a\,d^2ef\,x^3-2\sqrt{-\frac{d}{c}}\,bcdef\,x^3-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)acdef+\sqrt{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-d/c)^(1/2)*a*c*d*f^2*x^3+(-d/c)^(1/2)*a*d^2*e*f*x^3-2*(-d/c)^(1/2)*b*c*d
*e*f*x^3-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*a*c*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*e*f-((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^
2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a*c*d*e*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2+(-d/c)^(1/2)*a*c^2*f
^2*x+(-d/c)^(1/2)*a*d^2*e^2*x-(-d/c)^(1/2)*b*c^2*e*f*x-(-d/c)^(1/2)*b*c*d*e
^2*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/c/(-d/c)^(1/2)/e/(c*f-d*e)^2/(d*f*x^4
+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)/((c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b\,x^2+a}{{\left(d\,x^2+c\right)}^{3/2}\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x)
```

```
[Out] int((a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x)
```

3.47 $\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=375

$$-\frac{(bc-ad)x}{3c(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}+\frac{(2ad(de-3cf)+bc(de+3cf))x}{3c^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}+\frac{\sqrt{f}\,(bce(de+7cf)+a(2d^2e^2}{3c^2\sqrt{e}}$$

```
[Out] -1/3*(-a*d+b*c)*x/c/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2)+1/3*(2*a*d*(
-3*c*f+d*e)+b*c*(3*c*f+d*e))*x/c^2/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(
1/2)+1/3*(b*c*e*(7*c*f+d*e)+a*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e^2))*(1/(1+f*x^2
/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),
(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^2/(-c*f+d*e)^3/e^(1/2)/(e*(d*x
^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(a*d*(-9*c*f+d*e)+b*c*(3*c*f+5
*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/
(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c^2/(-
c*f+d*e)^3/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {541, 539, 429, 422}

$$\frac{\sqrt{f}\sqrt{c+dx^2}\left(a\left(-3c^2f^2-7cdef+2d^2e^2\right)+bce(7cf+de)\right)E\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e}\sqrt{e+fx^2}(de-cf)^3\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(ad(de-9cf)+bc(3cf+5de))F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(de-cf)^3\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{x(2ad(de-3cf)+bc(3cf+de))}{3c^2\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)^2}-\frac{x(bc-ad)}{3c\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] -1/3*((b*c - a*d)*x)/(c*(d*e - c*f)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]) + ((
2*a*d*(d*e - 3*c*f) + b*c*(d*e + 3*c*f))*x)/(3*c^2*(d*e - c*f)^2*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]) + (Sqrt[f]*(b*c*e*(d*e + 7*c*f) + a*(2*d^2*e^2 - 7*c
*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
 1 - (d*e)/(c*f)])/(3*c^2*Sqrt[e]*(d*e - c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e
+ f*x^2))]*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[f]*(a*d*(d*e - 9*c*f) + b*c*(5*
d*e + 3*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d
*e)/(c*f)])/(3*c^2*(d*e - c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = -\frac{(bc-ad)x}{3c(de-cf)(c+dx^2)^{3/2}\sqrt{e+fx^2}} - \frac{\int \frac{-bce-2ade+3acf+3(bc-ad)fx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx}{3c(de-cf)}$$

$$= -\frac{(bc-ad)x}{3c(de-cf)(c+dx^2)^{3/2}\sqrt{e+fx^2}} + \frac{(2ad(de-3cf)+bc(de+3cf))x}{3c^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$= -\frac{(bc-ad)x}{3c(de-cf)(c+dx^2)^{3/2}\sqrt{e+fx^2}} + \frac{(2ad(de-3cf)+bc(de+3cf))x}{3c^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$= -\frac{(bc-ad)x}{3c(de-cf)(c+dx^2)^{3/2}\sqrt{e+fx^2}} + \frac{(2ad(de-3cf)+bc(de+3cf))x}{3c^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.49, size = 428, normalized size = 1.14

$$\frac{\sqrt{\frac{d}{c}}\,x\left(-bce\left(3c^3f^2+d^3ex^2\left(e+fx^2\right)+cd^2fx^2\left(4e+7fx^2\right)+c^2df\left(5e+11fx^2\right)\right)+a\left(3c^4f^3+6c^3df^3x^2-2d^4e^2x^2\left(e+fx^2\right)+c^2d^2f\left(8e^2+8efx^2+3f^2x^4\right)+cd^3e\left(-3e^2+4efx^2+7f^2x^4\right)\right)\right)-ide\left(bce(de+7cf)+a\left(2d^2e^2-7cdef-3c^2f^2\right)\right)\left(c+dx^2\right)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)-ie(-de+cf)(2ad(de-3cf)+bc(de+3cf))\left(c+dx^2\right)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)}{3c^2\sqrt{\frac{d}{c}}\,e(-de+cf)^3\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*x*(-(b*c*e*(3*c^3*f^2 + d^3*e*x^2*(e + f*x^2) + c*d^2*f*x^2*(4*e
 + 7*f*x^2) + c^2*d*f*(5*e + 11*f*x^2))) + a*(3*c^4*f^3 + 6*c^3*d*f^3*x^2 -
 2*d^4*e^2*x^2*(e + f*x^2) + c^2*d^2*f*(8*e^2 + 8*e*f*x^2 + 3*f^2*x^4) + c*
d^3*e*(-3*e^2 + 4*e*f*x^2 + 7*f^2*x^4))) - I*d*e*(b*c*e*(d*e + 7*c*f) + a*(
2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(d*e) +
 c*f)*(2*a*d*(d*e - 3*c*f) + b*c*(d*e + 3*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3
*c^2*Sqrt[d/c]*e*(-(d*e) + c*f)^3*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1741 vs. $2(409) = 818$.
time = 0.14, size = 1742, normalized size = 4.65

method	result
elliptic	$\frac{\sqrt{\left(d\,x^2+c\right)\left(f\,x^2+e\right)}\left(\frac{\left(df\,x^2+cf\right)fx(af-be)}{e(cf-de)^3\sqrt{\left(x^2+\frac{e}{f}\right)\left(df\,x^2+cf\right)}}+\frac{x(ad-bc)\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{3dc(cf-de)^2\left(x^2+\frac{c}{d}\right)^2}+\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(7*(-d/c)^(1/2)*b*c^2*d^2*e*f^2*x^5+(-d/c)^(1/2)*b*c*d^3*e^2*f*x^5-8*(
-d/c)^(1/2)*a*c^2*d^2*e*f^2*x^3-4*(-d/c)^(1/2)*a*c*d^3*e^2*f*x^3+11*(-d/c)^
(1/2)*b*c^3*d*e*f^2*x^3+4*(-d/c)^(1/2)*b*c^2*d^2*e^2*f*x^3-8*(-d/c)^(1/2)*a
*c^2*d^2*e^2*f*x+5*(-d/c)^(1/2)*b*c^3*d*e^2*f*x+2*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^4*e^3*x^2-2*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*a*d^4*e^3*x^2+2*(-d/c)^(1/2)*a*d^4*e^3*x^3-3*(-d/c)^(1/2)*a*c^4*f^3*x+
2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a*c*d^3*e^3-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^4*e*f^2+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d^2*e^3-2*((d*x^2+c)
```

```
/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c
*d^3*e^3-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*b*c^2*d^2*e^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellip
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^3*e^3*x^2-((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^3*e^3*x^2
+6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*a*c^3*d*e*f^2-8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^2*e^2*f+2*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d*e^2*f+3*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*a*c^3*d*e*f^2+7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^2*e^2*f-7*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d*e^2*f-6*(-d/c
)^(1/2)*a*c^3*d*f^3*x^3+(-d/c)^(1/2)*b*c*d^3*e^3*x^3+3*(-d/c)^(1/2)*a*c*d^3
*e^3*x+3*(-d/c)^(1/2)*b*c^4*e*f^2*x-7*(-d/c)^(1/2)*a*c*d^3*e*f^2*x^5-3*(-d/
c)^(1/2)*a*c^2*d^2*f^3*x^5+2*(-d/c)^(1/2)*a*d^4*e^2*f*x^5+6*((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^
2*e*f^2*x^2-8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*a*c*d^3*e^2*f*x^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d*e*f^2*x^2+2*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
b*c^2*d^2*e^2*f*x^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^2*e*f^2*x^2+7*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^3*e^2*f*x^2
-7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*b*c^2*d^2*e^2*f*x^2)/(f*x^2+e)^(1/2)/(c*f-d*e)^3/e/(-d/c)^(1/2)/c
^2/(d*x^2+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2}{\left(c + dx^2\right)^{\frac{5}{2}} \left(e + fx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)/((c + d*x**2)**(5/2)*(e + f*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b\,x^2 + a}{{\left(d\,x^2 + c\right)}^{5/2}\,{\left(f\,x^2 + e\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x)
```

```
[Out] int((a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)), x)
```

3.48 $\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{(de-cf)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{d}\,(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}+\frac{\sqrt{c}\,(be-af)\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}$$

```
[Out] -(-c*f+d*e)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(
1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/(-a*d+b*c)/c^(1/2
)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+(-a*f+b*e)*(1/(1+
d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(
1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/a/(-a*d+b*c)/d^(1/2)/(c*(b*
x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {539, 429, 422}

$$\frac{\sqrt{c}\,\sqrt{a+bx^2}\,(be-af)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{c+dx^2}\,(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}\,(de-cf)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{d}\,\sqrt{c+dx^2}\,(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(((d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*e - a*f)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rubi steps

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{(be-af)\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{bc-ad} - \frac{(de-cf)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{bc-ad}$$

$$= -\frac{(de-cf)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{d}\,(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}} + \frac{\sqrt{c}\,(be-af)\sqrt{a+bx}}{a\sqrt{d}\,(bc-ad)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.33, size = 212, normalized size = 1.01

$$\frac{\sqrt{\frac{b}{a}}\,d(de-cf)x(a+bx^2)-ibc(-de+cf)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-ic(-bc+ad)f\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\,cd(-bc+ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[b/a]*d*(d*e - c*f)*x*(a + b*x^2) - I*b*c*(-(d*e) + c*f)*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
 I*c*(-(b*c) + a*d)*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c*d*(-(b*c) + a*d)*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.13, size = 349, normalized size = 1.67

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, bcdf\, x^3+\sqrt{-\frac{b}{a}}\, b\, d^2 e\, x^3+\sqrt{\frac{b\, x^2+a}{a}}\, \sqrt{\frac{d\, x^2+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) acdf-\sqrt{\frac{b\, x^2+a}{a}}\, \sqrt{\frac{d\, x^2+c}{c}}\, \text{Elli}}{}$
elliptic	$\frac{\sqrt{\left(b\, x^2+a\right)\left(d\, x^2+c\right)}\left(-\frac{\left(bd\, x^2+ad\right) x\left(cf-de\right)}{dc\left(ad-bc\right)\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\, x^2+ad\right)}}+\frac{\left(\frac{f}{d}-\frac{cf-de}{dc}+\frac{a\left(cf-de\right)}{c\left(ad-bc\right)}\right)\sqrt{1+\frac{b\, x^2}{a}}\, \sqrt{1+\frac{d\, x^2}{c}}}{\sqrt{-\frac{b}{a}}\, \sqrt{bd\, x^4+ad\, x}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b/a)^(1/2)*b*c*d*f*x^3+(-b/a)^(1/2)*b*d^2*e*x^3+((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d*f-((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b
*c^2*f+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))*b*c^2*f-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*d*e-(-b/a)^(1/2)*a*c*d*f*x+(-b/a)^(1/2)*a*
d^2*e*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/c/d/(-b/a)^(1/2)/(a*d-b*c)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx^2}{\sqrt{a + bx^2}\,(c + dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)/(d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
[Out] Integral((e + f*x**2)/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2 + e}{\sqrt{b\,x^2 + a}\,{\left(d\,x^2 + c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)),x)
[Out] int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)), x)
```

3.49 $\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=247

$$\frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}}+\frac{\sqrt{a}\,\sqrt{b}\,(de-cf)\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{cd(bc+ad)\sqrt{a-bx^2}\,\sqrt{1+\frac{dx^2}{c}}}+\frac{\sqrt{a}\,f\sqrt{1-\frac{bx^2}{a}}\,\cdots}{\sqrt{b}\,\cdots}$$

```
[Out] (-c*f+d*e)*x*(-b*x^2+a)^(1/2)/c/(a*d+b*c)/(d*x^2+c)^(1/2)+(-c*f+d*e)*Ellipt
icE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*b^(1/2)*(1-b*x^2/a)^(1/2)*(
d*x^2+c)^(1/2)/c/d/(a*d+b*c)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+f*EllipticF
(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/d/b^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {541, 538, 438, 437, 435, 432, 430}

$$\frac{\sqrt{a}\,\sqrt{b}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c+dx^2}\,(de-cf)E\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{cd\sqrt{a-bx^2}\,\sqrt{\frac{dx^2}{c}+1}\,(ad+bc)}+\frac{\sqrt{a}\,f\sqrt{1-\frac{bx^2}{a}}\,\sqrt{\frac{dx^2}{c}+1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\,d\sqrt{a-bx^2}\,\sqrt{c+dx^2}}+\frac{x\sqrt{a-bx^2}\,(de-cf)}{c\sqrt{c+dx^2}\,(ad+bc)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x^2)/(Sqrt[a - b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((d*e - c*f)*x*Sqrt[a - b*x^2])/(c*(b*c + a*d)*Sqrt[c + d*x^2]) + (Sqrt[a]*
Sqrt[b]*(d*e - c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(c*d*(b*c + a*d)*Sqrt[a - b*x^2]*Sqrt[
1 + (d*x^2)/c]) + (Sqrt[a]*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2
]*Sqrt[c + d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}} - \frac{\int \frac{-c(be+af)-b(de-cf)x^2}{\sqrt{a-bx^2}\,\sqrt{c+dx^2}}\,dx}{c(bc+ad)}$$

$$= \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}} + \frac{f\int \frac{1}{\sqrt{a-bx^2}\,\sqrt{c+dx^2}}\,dx}{d} + \frac{(b(de-cf))\int \frac{\sqrt{c+}}{\sqrt{a-}}}{cd(bc+ad)}$$

$$= \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}} + \frac{\left(b(de-cf)\sqrt{1-\frac{bx^2}{a}}\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{cd(bc+ad)\sqrt{a-bx^2}} + \frac{\left(f\sqrt{\right.}{}$$

$$= \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}} + \frac{\left(b(de-cf)\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c+dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,d}{cd(bc+ad)\sqrt{a-bx^2}\,\sqrt{1+\frac{dx^2}{c}}}$$

$$= \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}} + \frac{\sqrt{a}\,\sqrt{b}\,(de-cf)\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\right.\right.}{cd(bc+ad)\sqrt{a-bx^2}\,\sqrt{1+\frac{dx^2}{c}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.54, size = 220, normalized size = 0.89

$$\frac{\sqrt{-\frac{b}{a}}\,d(de-cf)x(a-bx^2)+ibc(-de+cf)\sqrt{1-\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|-\frac{ad}{bc}\right)-ic(bc+ad)f\sqrt{1-\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\,cd(bc+ad)\sqrt{a-bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x^2)/(Sqrt[a - b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[-(b/a)]*d*(d*e - c*f)*x*(a - b*x^2) + I*b*c*(-(d*e) + c*f)*Sqrt[1 - (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/
(b*c))] - I*c*(b*c + a*d)*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(Sqrt[-(b/a)]*c*d*(b*c + a*d
)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.12, size = 349, normalized size = 1.41

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, bcdf\, x^3-\sqrt{\frac{b}{a}}\, b\, d^2 e\, x^3+\sqrt{\frac{-b\,x^2+a}{a}}\, \sqrt{\frac{d\,x^2+c}{c}}\, \text{EllipticF}\left(x\sqrt{\frac{b}{a}}, \sqrt{-\frac{ad}{bc}}\right)acdf+\sqrt{\frac{-b\,x^2+a}{a}}\, \sqrt{\frac{d\,x^2+c}{c}}\, \text{Ellipt}\right.}{}$
elliptic	$\frac{\sqrt{\left(-b\,x^2+a\right)\left(d\,x^2+c\right)}\left(-\frac{\left(-bd\,x^2+ad\right)x\left(cf-de\right)}{dc\left(ad+bc\right)\sqrt{\left(x^2+\frac{c}{d}\right)\left(-bd\,x^2+ad\right)}}+\frac{\left(\frac{f}{d}-\frac{cf-de}{dc}+\frac{a\left(cf-de\right)}{c\left(ad+bc\right)}\right)\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1}}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b/a)^(1/2)*b*c*d*f*x^3-(b/a)^(1/2)*b*d^2*e*x^3+((-b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*c*d*f+((-b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*
c^2*f-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*
d/b/c)^(1/2))*b*c^2*f+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*
(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c*d*e-(b/a)^(1/2)*a*c*d*f*x+(b/a)^(1/2)*a*d
^2*e*x)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/d/(b/a)^(1/2)/(a*d+b*c)/(-b*d*x^
4+a*d*x^2-b*c*x^2+a*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)/(d*x**2+c)**(3/2)/(-b*x**2+a)**(1/2),x)
[Out] Integral((e + f*x**2)/(sqrt(a - b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e}{\sqrt{a-b\,x^2}\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^2)/((a - b*x^2)^(1/2)*(c + d*x^2)^(3/2)),x)
[Out] int((e + f*x^2)/((a - b*x^2)^(1/2)*(c + d*x^2)^(3/2)), x)
```

3.50 $\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=237

$$\frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}}-\frac{(de+cf)\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{d}\,(bc+ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{c-dx^2}}+\frac{e\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\ \ldots}{\sqrt{c}\,\sqrt{d}\,\sqrt{a+\ldots}}$$

```
[Out] (c*f+d*e)*x*(b*x^2+a)^(1/2)/c/(a*d+b*c)/(-d*x^2+c)^(1/2)-(c*f+d*e)*Elliptic
E(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)*(1-d*x^2/c)^(1/2)/(a*
d+b*c)/c^(1/2)/d^(1/2)/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)+e*EllipticF(x*d^(
1/2)/c^(1/2),(-b*c/a/d)^(1/2))*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)/c^(1/2)/
d^(1/2)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {541, 538, 438, 437, 435, 432, 430}

$$-\frac{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,(cf+de)E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{d}\,\sqrt{\frac{bx^2}{a}+1}\,\sqrt{c-dx^2}\,(ad+bc)}+\frac{e\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,\sqrt{c-dx^2}}+\frac{x\sqrt{a+bx^2}\,(cf+de)}{c\sqrt{c-dx^2}\,(ad+bc)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x^2)/(Sqrt[a + b*x^2]*(c - d*x^2)^(3/2)),x]
```

```
[Out] ((d*e + c*f)*x*Sqrt[a + b*x^2])/(c*(b*c + a*d)*Sqrt[c - d*x^2]) - ((d*e + c
*f)*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c
]], -((b*c)/(a*d))])/(Sqrt[c]*Sqrt[d]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[
c - d*x^2]) + (e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(
Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt
[c - d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}} + \frac{\int \frac{c(be-af)-b(de+cf)x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx}{c(bc+ad)}$$

$$= \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}} + \frac{e\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx}{c} - \frac{(de+cf)\int \frac{\sqrt{a+b\cdots}}{\sqrt{c-d\cdots}}}{c(bc+ad)}$$

$$= \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}} + \frac{\left(e\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{c\sqrt{c-dx^2}} - \cdots$$

$$= \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}} - \frac{\left((de+cf)\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\cdots}{c(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

$$= \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}} - \frac{(de+cf)\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\cdots\right.}{\sqrt{c}\sqrt{d}\,(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.38, size = 213, normalized size = 0.90

$$\frac{\sqrt{\frac{b}{a}}\,d(de+cf)x(a+bx^2) - ibc(de+cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|-\frac{ad}{bc}\right) + ic(bc+ad)f\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\,cd(bc+ad)\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*(c - d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[b/a]*d*(d*e + c*f)*x*(a + b*x^2) - I*b*c*(d*e + c*f)*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))] +
 I*c*(b*c + a*d)*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[I*ArcS
inh[Sqrt[b/a]*x], -((a*d)/(b*c))])/(Sqrt[b/a]*c*d*(b*c + a*d)*Sqrt[a + b*x^
2]*Sqrt[c - d*x^2])
```

Maple [A]
time = 0.12, size = 333, normalized size = 1.41

method	result
default	$\frac{\left(\sqrt{\frac{d}{c}}\, bcf\, x^{3}+\sqrt{\frac{d}{c}}\, bde\, x^{3}+\sqrt{\frac{-d\, x^{2}+c}{c}}\, \sqrt{\frac{b\, x^{2}+a}{a}}\, \mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}}, \sqrt{-\frac{bc}{ad}}\right) ade+\sqrt{\frac{-d\, x^{2}+c}{c}}\, \sqrt{\frac{b\, x^{2}+a}{a}}\, \mathrm{EllipticF}\left(\right.\right.}{}$
elliptic	$\frac{\sqrt{\left(b\, x^{2}+a\right)\left(-d\, x^{2}+c\right)}\, \left(-\frac{\left(-bd\, x^{2}-ad\right) x\left(cf+de\right)}{dc\left(ad+bc\right)\sqrt{\left(x^{2}-\frac{c}{d}\right)\left(-bd\, x^{2}-ad\right)}}+\frac{\left(-\frac{f}{d}+\frac{cf+de}{dc}-\frac{a\left(cf+de\right)}{c\left(ad+bc\right)}\right)\sqrt{1-\frac{d\, x^{2}}{c}}\, \sqrt{1-}}{\sqrt{\frac{d}{c}}\, \sqrt{-bd\, x^{4}-}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d/c)^(1/2)*b*c*f*x^3+(d/c)^(1/2)*b*d*e*x^3+((-d*x^2+c)/c)^(1/2)*((b*x^2+a
)/a)^(1/2)*EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))*a*d*e+((-d*x^2+c)/c)^(
1/2)*((b*x^2+a)/a)^(1/2)*EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))*b*c*e-((
-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(
1/2))*a*c*f-((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)^(1/2
),(-b*c/a/d)^(1/2))*a*d*e+(d/c)^(1/2)*a*c*f*x+(d/c)^(1/2)*a*d*e*x)*(b*x^2+a
)^(1/2)*(-d*x^2+c)^(1/2)/c/(d/c)^(1/2)/(a*d+b*c)/(-b*d*x^4-a*d*x^2+b*c*x^2+
a*c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)/(-d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
[Out] Integral((e + f*x**2)/(sqrt(a + b*x**2)*(c - d*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e}{\sqrt{b\,x^2+a}\,{\left(c-d\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^2)/((a + b*x^2)^(1/2)*(c - d*x^2)^(3/2)),x)
[Out] int((e + f*x^2)/((a + b*x^2)^(1/2)*(c - d*x^2)^(3/2)), x)
```

3.51 $\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=242

$$-\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}}+\frac{(de+cf)\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\,(bc-ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}+\frac{e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,F}{\sqrt{c}\sqrt{d}\sqrt{a-bx}}$$

```
[Out] -(c*f+d*e)*x*(-b*x^2+a)^(1/2)/c/(-a*d+b*c)/(-d*x^2+c)^(1/2)+(c*f+d*e)*Ellip
ticE(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*(-b*x^2+a)^(1/2)*(1-d*x^2/c)^(1/2)/
(-a*d+b*c)/c^(1/2)/d^(1/2)/(1-b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)+e*EllipticF(x
*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*(1-b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)/c^(1/
2)/d^(1/2)/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {541, 538, 438, 437, 435, 432, 430}

$$\frac{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}(bc-ad)}+\frac{e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x^2)/(Sqrt[a - b*x^2]*(c - d*x^2)^(3/2)),x]
```

```
[Out] -(((d*e + c*f)*x*Sqrt[a - b*x^2])/(c*(b*c - a*d)*Sqrt[c - d*x^2])) + ((d*e
+ c*f)*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqr
t[c]], (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[
c - d*x^2]) + (e*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(
Sqrt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[a - b*x^2]*Sqrt[c
- d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx = -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}} - \frac{\int \frac{-c(be+af)+b(de+cf)x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx}{c(bc-ad)}$$

$$= -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}} + \frac{e\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx}{c} + \frac{(de+cf)\int \frac{\sqrt{a-\ldots}}{\sqrt{c-\ldots}}}{c(bc-ad)}$$

$$= -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}} + \frac{\left(e\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{c\sqrt{c-dx^2}} + \ldots$$

$$= -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}} + \frac{\left((de+cf)\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}} \ldots}{c(bc-ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

$$= -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}} + \frac{(de+cf)\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right) \ldots\right.}{\sqrt{c}\sqrt{d}\,(bc-ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.44, size = 221, normalized size = 0.91

$$\frac{\sqrt{-\frac{b}{a}}\,d(de+cf)x(a-bx^2) + ibc(de+cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right) + ic(-bc+ad)f\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\,cd(-bc+ad)\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x^2)/(Sqrt[a - b*x^2]*(c - d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[-(b/a)]*d*(d*e + c*f)*x*(a - b*x^2) + I*b*c*(d*e + c*f)*Sqrt[1 - (b*x
^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], (a*d)/(b*c)
] + I*c*(-(b*c) + a*d)*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[
I*ArcSinh[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*c*d*(-(b*c) + a*d)*S
qrt[a - b*x^2]*Sqrt[c - d*x^2])
```

Maple [A]
time = 0.12, size = 338, normalized size = 1.40

method	result
default	$\frac{\left(-\sqrt{\frac{d}{c}}\, bcf\, x^{3}-\sqrt{\frac{d}{c}}\, bde\, x^{3}+\sqrt{\frac{-d\, x^{2}+c}{c}}\, \sqrt{\frac{-b\, x^{2}+a}{a}}\, \mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}}, \sqrt{\frac{bc}{ad}}\right) ade-\sqrt{\frac{-d\, x^{2}+c}{c}}\, \sqrt{\frac{-b\, x^{2}+a}{a}}\, \mathrm{Ellipt}\right.}{}$
elliptic	$\frac{\sqrt{\left(-b\, x^{2}+a\right)\left(-d\, x^{2}+c\right)}\, \left(-\frac{\left(bd\, x^{2}-ad\right) x(cf+de)}{dc(ad-bc)\sqrt{\left(x^{2}-\frac{c}{d}\right)\left(bd\, x^{2}-ad\right)}}+\frac{\left(-\frac{f}{d}+\frac{cf+de}{dc}-\frac{a(cf+de)}{c(ad-bc)}\right)\sqrt{1-\frac{d\, x^{2}}{c}}\, \sqrt{1}}{\sqrt{\frac{d}{c}}\, \sqrt{bd\, x^{4}-}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(d/c)^(1/2)*b*c*f*x^3-(d/c)^(1/2)*b*d*e*x^3+((-d*x^2+c)/c)^(1/2)*((-b*x^2
+a)/a)^(1/2)*EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))*a*d*e-((-d*x^2+c)/c)^
(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))*b*c*e-(
(-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)^(1/2),(b*c/a/d)^
(1/2))*a*c*f-((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)^(1
/2),(b*c/a/d)^(1/2))*a*d*e+(d/c)^(1/2)*a*c*f*x+(d/c)^(1/2)*a*d*e*x)*(-b*x^2
+a)^(1/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/c/(a*d-b*c)/(b*d*x^4-a*d*x^2-b*c*x^2
+a*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx^2}{\sqrt{a - bx^2}\,(c - dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)/(-d*x**2+c)**(3/2)/(-b*x**2+a)**(1/2),x)
[Out] Integral((e + f*x**2)/(sqrt(a - b*x**2)*(c - d*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2 + e}{\sqrt{a - b\,x^2}\,{\left(c - d\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^2)/((a - b*x^2)^(1/2)*(c - d*x^2)^(3/2)),x)
[Out] int((e + f*x^2)/((a - b*x^2)^(1/2)*(c - d*x^2)^(3/2)), x)
```

3.52 $\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx$

Optimal. Leaf size=191

$$\frac{bx\sqrt{2+dx^2}}{d\sqrt{3+fx^2}}-\frac{\sqrt{2}\,b\sqrt{2+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}+\frac{a\sqrt{2+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}$$

```
[Out] b*x*(d*x^2+2)^(1/2)/d/(f*x^2+3)^(1/2)+1/2*a*(1/(3*f*x^2+9))^(1/2)*(3*f*x^2+
9)^(1/2)*EllipticF(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1/2*(4-6*d/f)^(1/2))
*(d*x^2+2)^(1/2)*2^(1/2)/f^(1/2)/((d*x^2+2)/(f*x^2+3))^(1/2)/(f*x^2+3)^(1/2
)-b*(1/(3*f*x^2+9))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticE(x*f^(1/2)*3^(1/2)/(3*
f*x^2+9)^(1/2),1/2*(4-6*d/f)^(1/2))*2^(1/2)*(d*x^2+2)^(1/2)/d/f^(1/2)/((d*x
^2+2)/(f*x^2+3))^(1/2)/(f*x^2+3)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {545, 429, 506, 422}

$$\frac{a\sqrt{dx^2+2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}-\frac{\sqrt{2}\,b\sqrt{dx^2+2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{bx\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

```
[Out] (b*x*Sqrt[2 + d*x^2])/(d*Sqrt[3 + f*x^2]) - (Sqrt[2]*b*Sqrt[2 + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(d*Sqrt[f]*Sqrt[(2 + d
*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) + (a*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*Sqrt[f]*Sqrt[(2 + d*x^2)/(3
 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{\sqrt{2+dx^2}\,\sqrt{3+fx^2}}\,dx &= a\int \frac{1}{\sqrt{2+dx^2}\,\sqrt{3+fx^2}}\,dx + b\int \frac{x^2}{\sqrt{2+dx^2}\,\sqrt{3+fx^2}}\,dx\\ &= \frac{bx\sqrt{2+dx^2}}{d\sqrt{3+fx^2}} + \frac{a\sqrt{2+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}\,\sqrt{f}\,\sqrt{\frac{2+dx^2}{3+fx^2}}\,\sqrt{3+fx^2}} - \frac{(3b)\int \frac{\sqrt{2+dx^2}}{(3+fx^2)^{3/2}}}{d}\\ &= \frac{bx\sqrt{2+dx^2}}{d\sqrt{3+fx^2}} - \frac{\sqrt{2}\,b\sqrt{2+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\,\sqrt{\frac{2+dx^2}{3+fx^2}}\,\sqrt{3+fx^2}} + \frac{a\sqrt{2+dx^2}\,F}{\sqrt{2}\,\sqrt{f}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.74, size = 81, normalized size = 0.42

$$-\frac{i\left(3bE\left(i\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+(-3b+af)F\left(i\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)\right)}{\sqrt{3}\,\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

```
[Out] ((-I)*(3*b*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (-3*b +
 a*f)*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)]))/(Sqrt[3]*Sqr
t[d]*f)
```

Maple [A]
time = 0.13, size = 105, normalized size = 0.55

method	result
default	$\frac{\sqrt{2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3}, \frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{d}{f}}}{2}\right) a d-2\,\mathrm{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3}, \frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{d}{f}}}{2}\right) b+2\,\mathrm{EllipticE}\left(\frac{x\sqrt{\ldots}}{\ldots}\right.\ldots}{2\sqrt{-f}\, d}$
elliptic	$\frac{\sqrt{\left(f\,x^{2}+3\right)\left(d\,x^{2}+2\right)}\,\left(\frac{a\sqrt{3 f\,x^{2}+9}\,\sqrt{2 d\,x^{2}+4}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3 f}}{3}, \frac{\sqrt{-4+\frac{6 d+4 f}{f}}}{2}\right)}{2\sqrt{-3 f}\,\sqrt{d f\,x^{4}+3 d\,x^{2}+2 f\,x^{2}+6}}-\frac{b\sqrt{3 f\,x^{2}+\ldots}}{\ldots}\right.\ldots}{\sqrt{f\,x^{2}+3}\,\sqrt{d\,x\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(
1/2))*a*d-2*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1
/2))*b+2*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2)
)*b)/(-f)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{\sqrt{dx^2+2}\,\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)/(sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b\,x^2+a}{\sqrt{d\,x^2+2}\,\sqrt{f\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2)),x)
```

```
[Out] int((a + b*x^2)/((d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2)), x)
```

3.53 $\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx$

Optimal. Leaf size=262

$$-\frac{(6bd-2bf-3adf)x\sqrt{2+dx^2}}{3df\sqrt{3+fx^2}}+\frac{bx\sqrt{2+dx^2}\sqrt{3+fx^2}}{3f}+\frac{\sqrt{2}(6bd-2bf-3adf)\sqrt{2+dx^2}\,E\Big(\tan^{-1}\Big(}{3df^{3/2}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}$$

```
[Out] -1/3*(-3*a*d*f+6*b*d-2*b*f)*x*(d*x^2+2)^(1/2)/d/f/(f*x^2+3)^(1/2)+1/3*(-3*a
*d*f+6*b*d-2*b*f)*(1/(3*f*x^2+9))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticE(x*f^(1/
2)*3^(1/2)/(3*f*x^2+9)^(1/2),1/2*(4-6*d/f)^(1/2))*2^(1/2)*(d*x^2+2)^(1/2)/d
/f^(3/2)/((d*x^2+2)/(f*x^2+3))^(1/2)/(f*x^2+3)^(1/2)-(-a*f+b)*(1/(3*f*x^2+9
))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticF(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1/
2*(4-6*d/f)^(1/2))*2^(1/2)*(d*x^2+2)^(1/2)/f^(3/2)/((d*x^2+2)/(f*x^2+3))^(1
/2)/(f*x^2+3)^(1/2)+1/3*b*x*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/f
```

Rubi [A]
time = 0.12, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {542, 545, 429, 506, 422}

$$-\frac{\sqrt{2}\sqrt{dx^2+2}\,(b-af)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{f^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{\sqrt{2}\sqrt{dx^2+2}\,(-3adf+6bd-2bf)E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{3df^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}-\frac{x\sqrt{dx^2+2}\,(-3adf+6bd-2bf)}{3df\sqrt{fx^2+3}}+\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sqrt[2 + d*x^2])/Sqrt[3 + f*x^2],x]
```

```
[Out] -1/3*((6*b*d - 2*b*f - 3*a*d*f)*x*Sqrt[2 + d*x^2])/(d*f*Sqrt[3 + f*x^2]) +
(b*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(3*f) + (Sqrt[2]*(6*b*d - 2*b*f - 3*a
*d*f)*Sqrt[2 + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f
)])/(3*d*f^(3/2)*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) - (Sqrt[2]*
(b - a*f)*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/
(2*f)])/(f^(3/2)*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \frac{bx\sqrt{2+dx^2}\,\sqrt{3+fx^2}}{3f} + \frac{\int \frac{-6(b-af)+(-6bd+2bf+3adf)x^2}{\sqrt{2+dx^2}\,\sqrt{3+fx^2}}\,dx}{3f}$$

$$= \frac{bx\sqrt{2+dx^2}\,\sqrt{3+fx^2}}{3f} - \frac{(2(b-af))\int \frac{1}{\sqrt{2+dx^2}\,\sqrt{3+fx^2}}\,dx}{f} - \frac{(6bd-$$

$$= -\frac{(6bd-2bf-3adf)x\sqrt{2+dx^2}}{3df\sqrt{3+fx^2}} + \frac{bx\sqrt{2+dx^2}\,\sqrt{3+fx^2}}{3f} - \frac{\sqrt{2}\,(b-af)}{f}$$

$$= -\frac{(6bd-2bf-3adf)x\sqrt{2+dx^2}}{3df\sqrt{3+fx^2}} + \frac{bx\sqrt{2+dx^2}\,\sqrt{3+fx^2}}{3f} + \frac{\sqrt{2}\,(6bd-2b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.25, size = 142, normalized size = 0.54

$$\frac{b\sqrt{d}\,fx\sqrt{2+dx^2}\,\sqrt{3+fx^2}+i\sqrt{3}\,(6bd-2bf-3adf)E\left(i\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+i\sqrt{3}\,(3d-2f)(-2b+af)F\left(i\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)}{3\sqrt{d}\,f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sqrt[2 + d*x^2])/Sqrt[3 + f*x^2],x]
```

```
[Out] (b*Sqrt[d]*f*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2] + I*Sqrt[3]*(6*b*d - 2*b*f -
 3*a*d*f)*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + I*Sqrt[3
]*(3*d - 2*f)*(-2*b + a*f)*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/
(3*d)])/(3*Sqrt[d]*f^2)
```

Maple [A]
time = 0.14, size = 367, normalized size = 1.40

method	result

elliptic	$\frac{\sqrt{(f x^2+3)(d x^2+2)}\left(\frac{bx\sqrt{df x^4+3d x^2+2f x^2+6}}{3f}+\frac{\left(2a-\frac{2b}{f}\right)\sqrt{3f x^2+9}\sqrt{2d x^2+4}\,\mathrm{EllipticF}\left(\frac{x\sqrt{\cdots}}{\cdots}\right.}{2\sqrt{-3f}\sqrt{df x^4+3d x^2+\cdots}}\right.}{\cdots}$
risch	$\frac{bx\sqrt{d x^2+2}\sqrt{f x^2+3}}{3f}+\frac{\left(-\frac{(3adf-6bd+2bf)\sqrt{3f x^2+9}\sqrt{2d x^2+4}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\cdots\right.\right.}{\sqrt{-3f}\sqrt{df x^4+3d x^2+2f x^2+\cdots}}\right.}{\cdots}$
default	$\frac{\sqrt{d x^2+2}\sqrt{f x^2+3}\left(b d^2 f x^5\sqrt{-f}+3\sqrt{2}\,\mathrm{EllipticE}\left(\frac{x\sqrt{3}\sqrt{-f}}{3},\frac{\sqrt{2}\sqrt{3}\sqrt{\frac{d}{f}}}{2}\right)adf\sqrt{d x^2+2}\sqrt{f x\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)*(b*d^2*f*x^5*(-f)^(1/2)+3*2^(1/2)*Ellip
ticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*a*d*f*(d*x^2
+2)^(1/2)*(f*x^2+3)^(1/2)+3*b*d^2*x^3*(-f)^(1/2)+2*b*d*f*x^3*(-f)^(1/2)-6*2
^(1/2)*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*
b*d*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)+2*2^(1/2)*EllipticE(1/3*x*3^(1/2)*(-f)^
(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*b*f*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)+
3*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2
))*b*d*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)-2*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-
f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*b*f*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/
2)+6*b*d*x*(-f)^(1/2))/(d*f*x^4+3*d*x^2+2*f*x^2+6)/f/(-f)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)\sqrt{dx^2 + 2}}{\sqrt{fx^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
[Out] Integral((a + b*x**2)*sqrt(d*x**2 + 2)/sqrt(f*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2 + a)\,\sqrt{d\,x^2 + 2}}{\sqrt{f\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(d*x^2 + 2)^(1/2))/(f*x^2 + 3)^(1/2),x)
[Out] int(((a + b*x^2)*(d*x^2 + 2)^(1/2))/(f*x^2 + 3)^(1/2), x)
```

3.54 $\int (a+bx^2)\sqrt{2+dx^2}\,\sqrt{3+fx^2}\,dx$

Optimal. Leaf size=356

$$\frac{(5adf(3d+2f)-2b(9d^2-6df+4f^2))\,x\sqrt{2+dx^2}}{15d^2f\sqrt{3+fx^2}}+\frac{(3bd-4bf+5adf)x\sqrt{2+dx^2}\,\sqrt{3+fx^2}}{15df}+\frac{bx(2+dx$$

```
[Out] 1/15*(5*a*d*f*(3*d+2*f)-2*b*(9*d^2-6*d*f+4*f^2))*x*(d*x^2+2)^(1/2)/d^2/f/(f
*x^2+3)^(1/2)-1/15*(5*a*d*f*(3*d+2*f)-2*b*(9*d^2-6*d*f+4*f^2))*(1/(3*f*x^2+
9))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticE(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1
/2*(4-6*d/f)^(1/2))*2^(1/2)*(d*x^2+2)^(1/2)/d^2/f^(3/2)/((d*x^2+2)/(f*x^2+3
))^(1/2)/(f*x^2+3)^(1/2)-1/5*(-10*a*d*f+3*b*d+2*b*f)*(1/(3*f*x^2+9))^(1/2)*
(3*f*x^2+9)^(1/2)*EllipticF(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1/2*(4-6*d/
f)^(1/2))*2^(1/2)*(d*x^2+2)^(1/2)/d/f^(3/2)/((d*x^2+2)/(f*x^2+3))^(1/2)/(f*
x^2+3)^(1/2)+1/5*b*x*(d*x^2+2)^(3/2)*(f*x^2+3)^(1/2)/d+1/15*(5*a*d*f+3*b*d-
4*b*f)*x*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/d/f
```

Rubi [A]
time = 0.21, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {542, 545, 429, 506, 422}

$$-\frac{\sqrt{2}\,\sqrt{dx^2+2}\,(5adf(3d+2f)-2b(9d^2-6df+4f^2))\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{15d^2f^{3/2}\sqrt{fx^2+3}\,\sqrt{\frac{dx^2+2}{fx^2+3}}}-\frac{\sqrt{2}\,\sqrt{dx^2+2}\,(-10adf+3bd+2bf)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{5df^{3/2}\sqrt{fx^2+3}\,\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{x\sqrt{dx^2+2}\,(5adf(3d+2f)-2b(9d^2-6df+4f^2))}{15d^2f\sqrt{fx^2+3}}+\frac{x\sqrt{dx^2+2}\,\sqrt{fx^2+3}\,(5adf+3bd-4bf)}{15df}+\frac{bx(dx^2+2)^{3/2}\sqrt{fx^2+3}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2],x]
```

```
[Out] ((5*a*d*f*(3*d + 2*f) - 2*b*(9*d^2 - 6*d*f + 4*f^2))*x*Sqrt[2 + d*x^2])/(15
*d^2*f*Sqrt[3 + f*x^2]) + ((3*b*d - 4*b*f + 5*a*d*f)*x*Sqrt[2 + d*x^2]*Sqrt
[3 + f*x^2])/(15*d*f) + (b*x*(2 + d*x^2)^(3/2)*Sqrt[3 + f*x^2])/(5*d) - (Sq
rt[2]*(5*a*d*f*(3*d + 2*f) - 2*b*(9*d^2 - 6*d*f + 4*f^2))*Sqrt[2 + d*x^2]*E
llipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(15*d^2*f^(3/2)*Sqr
t[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) - (Sqrt[2]*(3*b*d + 2*b*f - 10*
a*d*f)*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*
f)])/(5*d*f^(3/2)*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx = \frac{bx(2+dx^2)^{3/2}\sqrt{3+fx^2}}{5d} + \frac{\int \frac{\sqrt{2+dx^2}\,(-3(2b-5ad)+(3bd-4bf+5adf)x^2)}{\sqrt{3+fx^2}}}{5d}$$

$$= \frac{(3bd-4bf+5adf)x\sqrt{2+dx^2}\sqrt{3+fx^2}}{15df} + \frac{bx(2+dx^2)^{3/2}\sqrt{3+}}{5d}$$

$$= \frac{(3bd-4bf+5adf)x\sqrt{2+dx^2}\sqrt{3+fx^2}}{15df} + \frac{bx(2+dx^2)^{3/2}\sqrt{3+}}{5d}$$

$$= \frac{(5adf(3d+2f)-2b(9d^2-6df+4f^2))\,x\sqrt{2+dx^2}}{15d^2f\sqrt{3+fx^2}} + \frac{(3bd-4bf+}{}$$

$$= \frac{(5adf(3d+2f)-2b(9d^2-6df+4f^2))\,x\sqrt{2+dx^2}}{15d^2f\sqrt{3+fx^2}} + \frac{(3bd-4bf+}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.26, size = 186, normalized size = 0.52

$$\frac{\sqrt{d}\,fx\sqrt{2+dx^2}\sqrt{3+fx^2}\,(2bf+5adf+3bd(1+fx^2))+i\sqrt{3}\,(-5adf(3d+2f)+2b(9d^2-6df+4f^2))\,E\left(i\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+i\sqrt{3}\,(3d-2f)(-6bd+2bf+5adf)F\left(i\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)}{15d^{3/2}f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2],x]
```

```
[Out] (Sqrt[d]*f*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]*(2*b*f + 5*a*d*f + 3*b*d*(1 +
f*x^2)) + I*Sqrt[3]*(-5*a*d*f*(3*d + 2*f) + 2*b*(9*d^2 - 6*d*f + 4*f^2))*El
lipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + I*Sqrt[3]*(3*d - 2*f
)*(-6*b*d + 2*b*f + 5*a*d*f)*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f
)/(3*d)])/(15*d^(3/2)*f^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 774 vs. 2(372) = 744.
time = 0.13, size = 775, normalized size = 2.18

method	result

elliptic	$\frac{\sqrt{(f x^2+3)(d x^2+2)}\left(\frac{b x^3\sqrt{df x^4+3d x^2+2f x^2+6}}{5}+\frac{\left(adf+3bd+2bf-\frac{b(12d+8f)}{5}\right)x\sqrt{df x^4+3d x^2+\cdots}}{3df}\right.}{}$
risch	$\frac{x(3bd x^2 f+5adf+3bd+2bf)\sqrt{f x^2+3}\,\sqrt{d x^2+2}}{15df}+\frac{\left(-\frac{(15a d^2 f+10ad f^2-18b d^2+12bdf-8b f^2)\sqrt{3f x^2+9}\,\sqrt{2d x^2\cdots}}{\sqrt{-3\cdots}}\right.}{}$
default	$\frac{\sqrt{d x^2+2}\,\sqrt{f x^2+3}\left(3b d^3 f^2 x^7\sqrt{-f}+5a d^3 f^2 x^5\sqrt{-f}+12b d^3 f x^5\sqrt{-f}+8b d^2 f^2 x^5\sqrt{-f}+15a d^3 f x^3\sqrt{-\cdots}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)*(3*b*d^3*f^2*x^7*(-f)^(1/2)+5*a*d^3*f^
2*x^5*(-f)^(1/2)+12*b*d^3*f*x^5*(-f)^(1/2)+8*b*d^2*f^2*x^5*(-f)^(1/2)+15*a*
d^3*f*x^3*(-f)^(1/2)+10*a*d^2*f^2*x^3*(-f)^(1/2)+15*2^(1/2)*EllipticF(1/3*x
*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*a*d^2*f*(f*x^2+3)^(1/2
)*(d*x^2+2)^(1/2)-10*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)
*3^(1/2)*(d/f)^(1/2))*a*d*f^2*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)+15*2^(1/2)*El
lipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*a*d^2*f*(
f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)+10*2^(1/2)*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2
),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*a*d*f^2*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)+
9*b*d^3*x^3*(-f)^(1/2)+30*b*d^2*f*x^3*(-f)^(1/2)+4*b*d*f^2*x^3*(-f)^(1/2)+9
*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2)
)*b*d^2*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)-18*2^(1/2)*EllipticF(1/3*x*3^(1/2)*
(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*b*d*f*(f*x^2+3)^(1/2)*(d*x^2+2)
^(1/2)+8*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/
f)^(1/2))*b*f^2*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)-18*2^(1/2)*EllipticE(1/3*x*
3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*b*d^2*(f*x^2+3)^(1/2)*(
d*x^2+2)^(1/2)+12*2^(1/2)*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^
(1/2)*(d/f)^(1/2))*b*d*f*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)-8*2^(1/2)*Elliptic
E(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*b*f^2*(f*x^2+3)
^(1/2)*(d*x^2+2)^(1/2)+30*a*d^2*f*x*(-f)^(1/2)+18*b*d^2*x*(-f)^(1/2)+12*b*d
```

```
*f*x*(-f)^(1/2))/(d*f*x^4+3*d*x^2+2*f*x^2+6)/d^2/f/(-f)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx^{2}\right) \sqrt{dx^{2} + 2} \sqrt{fx^{2} + 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+2)**(1/2)*(f*x**2+3)**(1/2),x)
[Out] Integral((a + b*x**2)*sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^2+a\right)\,\sqrt{d\,x^2+2}\,\sqrt{f\,x^2+3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2),x)
[Out] int((a + b*x^2)*(d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2), x)
```

3.55 $\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=113

$$-\frac{\sqrt{b-\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[Out] -1/2*EllipticE(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2),((b-(-4*a*c+b
^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b+(-4*a*c+b^2)^(1/2))*(b-(-4*a*c
+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 87, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {21, 435}

$$-\frac{\sqrt{b-\sqrt{b^2-4ac}}\left(\sqrt{b^2-4ac}+b\right)E\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(-b - Sqrt[b^2 -
 4*a*c])]*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*(b + Sqrt[b^2 - 4*a*c])*EllipticE[ArcSin[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]], (b - Sqrt[b^2 - 4*a*c])/(b +
 Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx = \left(-b-\sqrt{b^2-4ac}\right)\int \frac{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)E\left(\sin^{-1}\right.}{\sqrt{2}\sqrt{c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.56, size = 104, normalized size = 0.92

$$-2i\sqrt{2}\,a\sqrt{\frac{c}{-b+\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{-b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(-b - Sqrt
[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]),x]
```

```
[Out] (-2*I)*Sqrt[2]*a*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[
2]*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b
^2 - 4*a*c])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2537 vs. 2(92) = 184.
time = 0.47, size = 2538, normalized size = 22.46

method	result	size
elliptic	Expression too large to display	2538

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2-(-4*a*c+b^2)^(1/2)-b)/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)))^(1/2
)/(1+2*c/(-b+(-4*a*c+b^2)^(1/2))*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*((-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(2*c*x
^2+(-4*a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(1/2)*(-(2*c*x^2+(-4*a*c+b^2)^(1/
```

```
2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)/((-2*c*x^2+(-4*a*c+b^2)^(1
/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/((2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-b+(-
4*a*c+b^2)^(1/2)))^(1/2)/(-2*((-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(2*c*x^2+(-4*
a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(1/2)*c*x^2-4*(-(2*c*x^2+(-4*a*c+b^2)^(1
/2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*a*c+(-(2*c*x^2+(-4*a*c+b^
2)^(1/2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*b^2+((-2*c*x^2+(-4*a
*c+b^2)^(1/2)+b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(1/2)*b)*(
1/2*(4*a*c-b^2)/(-2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2+16*a^2*b*c^2
-4*a*b^3*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2))^(
1/2)*(4+2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2+16*a^2*b*c^2-4*a*b^3*c
)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2)*x^2)^(1/2)*(
4-2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2-16*a^2*b*c^2+4*a*b^3*c)/(-b+
(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2)*x^2)^(1/2)/(-4*a*c
+b^2-8*c^2/(-b+(-4*a*c+b^2)^(1/2))*x^2*a+2*c/(-b+(-4*a*c+b^2)^(1/2))*x^2*b^
2-8*c^2*x^2/(-b-(-4*a*c+b^2)^(1/2))*a+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2))*b^2-1
6*c^3*x^4/(-b-(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))*a+4*c^2*x^4/(-b-(
-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))*b^2)^(1/2)*EllipticF(1/2*x*(-2*(
(-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2+16*a^2*b*c^2-4*a*b^3*c)/(-b+(-4*a
*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2))^(1/2),1/2*(-4-2*(-8*c^
2/(-b+(-4*a*c+b^2)^(1/2))*a+2*c/(-b+(-4*a*c+b^2)^(1/2))*b^2-8*c^2/(-b-(-4*a
*c+b^2)^(1/2))*a+2*c/(-b-(-4*a*c+b^2)^(1/2))*b^2)*((-4*a*c+b^2)^(5/2)-(-4*a
*c+b^2)^(3/2)*b^2-16*a^2*b*c^2+4*a*b^3*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*
c+b^2)^(1/2))/a/(4*a*c-b^2)/(-16*c^3/(-b-(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^
2)^(1/2))*a+4*c^2/(-b-(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))*b^2))^(1/
2))-1/2*b/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*
c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*((-4*a*c+b^2)^(3/2)-(-4
*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(
-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(1+2*c/(-b+(-4*a*c+
b^2)^(1/2))*x^2+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2))+4*c^2*x^4/(-b-(-4*a*c+b^2)^
(1/2))/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(1/2*x*(-2*((-4*a*c+b^2)^(3/
2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/4*(-16-2*(2*c/(-b+(-4*a*c+b^2)^(1/2))+2*c/(-b-(-4*a*c+b^2
)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^
2)^(1/2))/a/c^2*(-b-(-4*a*c+b^2)^(1/2)))^(1/2))-2*c/(-2*((-4*a*c+b^2)^(3/2)
-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2)*(4+2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(
-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^
(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)/(1+2*c/(-b+(-4*a*c+b^2)^(1/2))*x^2+2*c*x^2/(-b-(-4*a
*c+b^2)^(1/2))+4*c^2*x^4/(-b-(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2)))^(
1/2)/(2*c/(-b+(-4*a*c+b^2)^(1/2))+2*c/(-b-(-4*a*c+b^2)^(1/2))-(-4*a*c+b^2)^
(1/2)/a)*(EllipticF(1/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*
a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(
2*c/(-b+(-4*a*c+b^2)^(1/2))+2*c/(-b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2
```

```
)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/a/c^2*(-b-(-4*a*c+
b^2)^(1/2)))^(1/2))-EllipticE(1/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1
/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/
4*(-16-2*(2*c/(-b+(-4*a*c+b^2)^(1/2))+2*c/(-b-(-4*a*c+b^2)^(1/2)))*((-4*a*c
+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/a/c^2*(-
b-(-4*a*c+b^2)^(1/2)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*x^2 - b - sqrt(b^2 - 4*a*c))/(sqrt(-2*c*x^2/(b + sqrt(b^2 -
4*a*c)) + 1)*sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-b + 2cx^2 - \sqrt{-4ac + b^2}}{\sqrt{\frac{-b + 2cx^2 - \sqrt{-4ac + b^2}}{-b - \sqrt{-4ac + b^2}}}\sqrt{\frac{-b + 2cx^2 + \sqrt{-4ac + b^2}}{-b + \sqrt{-4ac + b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+2*c*x**2-(-4*a*c+b**2)**(1/2))/(1+2*c*x**2/(-b-(-4*a*c+b**2)*
*(1/2)))**(1/2)/(1+2*c*x**2/(-b+(-4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral((-b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(sqrt((-b + 2*c*x**2 - sqrt(
-4*a*c + b**2))/(-b - sqrt(-4*a*c + b**2)))*sqrt((-b + 2*c*x**2 + sqrt(-4*a
*c + b**2))/(-b + sqrt(-4*a*c + b**2)))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*x^2 - b - sqrt(b^2 - 4*a*c))/(sqrt(-2*c*x^2/(b + sqrt(b^2 -
4*a*c)) + 1)*sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{b-2\,c\,x^2+\sqrt{b^2-4\,a\,c}}{\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - 2*c*x^2 + (b^2 - 4*a*c)^(1/2))/((1 - (2*c*x^2)/(b - (b^2 - 4*a*c)
^(1/2)))^(1/2)*(1 - (2*c*x^2)/(b + (b^2 - 4*a*c)^(1/2)))^(1/2)),x)
```

```
[Out] int(-(b - 2*c*x^2 + (b^2 - 4*a*c)^(1/2))/((1 - (2*c*x^2)/(b - (b^2 - 4*a*c)
^(1/2)))^(1/2)*(1 - (2*c*x^2)/(b + (b^2 - 4*a*c)^(1/2)))^(1/2)), x)
```

3.56 $\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=526

$$\frac{\left(b-\sqrt{b^2-4ac}\right)x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}-\frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,E}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1}}$$

```
[Out] x*(b-(-4*a*c+b^2)^(1/2))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^
2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*(1/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^
(1/2)*EllipticE(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(1+2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))^(1/2),(-2*(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2)))
^(1/2))*(b-(-4*a*c+b^2)^(1/2))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(b+
(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
))/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)+1/2*(1/(1+2*c*x^2/(b+(-4*a*c+b
^2)^(1/2))))^(1/2)*EllipticF(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),(-2*(-4*a*c+b^2)^(1/2)/(b-(-4*a*c
+b^2)^(1/2)))^(1/2))*(b-(-4*a*c+b^2)^(1/2))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2
)))^(1/2)*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/((1+2*c*x^2/(b-(-4*a
*c+b^2)^(1/2)))/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 526, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 81, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {21, 433, 429, 506, 422}

$$\frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,F\left(\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,E\left(\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{x\left(b-\sqrt{b^2-4ac}\right)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]),x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])*x*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/Sqr
t[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])] - ((b - Sqrt[b^2 - 4*a*c])*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b - Sq
```

```
rt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]) + ((b - Sqrt[b^2 - 4*a*c])*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \left(b-\sqrt{b^2-4ac}\right)\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$$

$$= (2c)\int \frac{x^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\left(b-\right.}{}$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} - \frac{\left(b-\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.60, size = 203, normalized size = 0.39

$$-\frac{i\left(\left(b+\sqrt{b^2-4ac}\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b-\sqrt{b^2-4ac}}}\,x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)-2\sqrt{b^2-4ac}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b-\sqrt{b^2-4ac}}}\,x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)\right)}{\sqrt{2}\sqrt{\frac{c}{b-\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]),x]
```

```
[Out] ((-I)*((b + Sqrt[b^2 - 4*a*c])*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b - Sqrt
[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])] - 2*Sq
rt[b^2 - 4*a*c]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b - Sqrt[b^2 - 4*a*c])]
*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b
- Sqrt[b^2 - 4*a*c])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2476 vs. 2(499) = 998.
time = 0.32, size = 2477, normalized size = 4.71

method	result	size
elliptic	Expression too large to display	2477

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)
/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*((-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(2*c*
x^2+(-4*a*c+b^2)^(1/2)+b)*(4*a*c-b^2)/a/c)^(1/2)*(-(-2*c*x^2+(-4*a*c+b^2)^(
1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)/((-2*c*x^2+(-4*a*c+b^2)^(
1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/((2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/(2*((-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(2*c*x^2+(-4*
a*c+b^2)^(1/2)+b)*(4*a*c-b^2)/a/c)^(1/2)*c*x^2+4*(-(-2*c*x^2+(-4*a*c+b^2)^(
1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*a*c-(-(-2*c*x^2+(-4*a*c+b
^2)^(1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*b^2+((-2*c*x^2+(-4*a
*c+b^2)^(1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(4*a*c-b^2)/a/c)^(1/2)*b)*(
1/2*(4*a*c-b^2)/(-2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2-16*a^2*b*c^2
+4*a*b^3*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2))^(
1/2)*(4+2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2-16*a^2*b*c^2+4*a*b^3*c
)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2)*x^2)^(1/2)*(
4-2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2+16*a^2*b*c^2-4*a*b^3*c)/(-b+
(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2)*x^2)^(1/2)/(-4*a*c
+b^2-8*c^2*x^2/(b+(-4*a*c+b^2)^(1/2))*a+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))*b^2-
8*c^2*x^2/(b-(-4*a*c+b^2)^(1/2))*a+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))*b^2-16*c^
3/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4*a+4*c^2/(b-(-4*a*c+b^2)
^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4*b^2)^(1/2)*EllipticF(1/2*x*(-2*((-4*a*c+
b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2-16*a^2*b*c^2+4*a*b^3*c)/(-b+(-4*a*c+b^2)^
(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2))^(1/2),1/2*(-4-2*(-8*c^2/(b+(-4
*a*c+b^2)^(1/2))*a+2*c/(b+(-4*a*c+b^2)^(1/2))*b^2-8*c^2/(b-(-4*a*c+b^2)^(1/
2))*a+2*c/(b-(-4*a*c+b^2)^(1/2))*b^2)*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2
)*b^2+16*a^2*b*c^2-4*a*b^3*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)
)/a/(4*a*c-b^2)/(-16*c^3/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*a+4*
c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*b^2))^(1/2))+1/2*b/(-2*((
-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*
b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4
-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2
))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))+2
*c*x^2/(b-(-4*a*c+b^2)^(1/2))+4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^
(1/2))*x^4)^(1/2)*EllipticF(1/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2
)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*
(-16-2*(2*c/(b+(-4*a*c+b^2)^(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2
)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-
```

```
4*a*c+b^2)^(1/2)))^(1/2))-2*c/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^
2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*((-
4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b
+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/
2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))+4*c^2/(b-
(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)/(2*c/(b+(-4*a*c+b^2)^
(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2))-(-4*a*c+b^2)^(1/2)/a)*(EllipticF(1/2*x*(-
2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2)
)/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(2*c/(b+(-4*a*c+b^2)^(1/2))+2
*c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b
*c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))-EllipticE(
1/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^
2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(2*c/(b+(-4*a*c+b^2)^
(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b
^2+4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/(sqrt(2*c*x^2/(b + sqrt(b^2 - 4
*a*c)) + 1)*sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b + 2cx^2 - \sqrt{-4ac + b^2}}{\sqrt{\frac{b + 2cx^2 - \sqrt{-4ac + b^2}}{b - \sqrt{-4ac + b^2}}}\sqrt{\frac{b + 2cx^2 + \sqrt{-4ac + b^2}}{b + \sqrt{-4ac + b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2-(-4*a*c+b**2)**(1/2)+b)/(1+2*c*x**2/(b-(-4*a*c+b**2)**(
1/2)))**(1/2)/(1+2*c*x**2/(b+(-4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(sqrt((b + 2*c*x**2 - sqrt(-4
*a*c + b**2))/(b - sqrt(-4*a*c + b**2)))*sqrt((b + 2*c*x**2 + sqrt(-4*a*c +
 b**2))/(b + sqrt(-4*a*c + b**2)))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/(sqrt(2*c*x^2/(b + sqrt(b^2 - 4
*a*c)) + 1)*sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b+2\,c\,x^2-\sqrt{b^2-4\,a\,c}}{\sqrt{\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}+1}\,\sqrt{\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^2 - (b^2 - 4*a*c)^(1/2))/(((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2
)) + 1)^(1/2)*((2*c*x^2)/(b + (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)),x)
```

```
[Out] int((b + 2*c*x^2 - (b^2 - 4*a*c)^(1/2))/(((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2
)) + 1)^(1/2)*((2*c*x^2)/(b + (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)), x)
```

3.57 $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx$

Optimal. Leaf size=128

$$\frac{bx\sqrt{c+dx^2}}{2f}-\frac{(2bde-bcf-2adf)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,f^2}+\frac{(be-af)\sqrt{de-cf}\,\tanh^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\,\sqrt{c+dx^2}}\right)}{\sqrt{e}\,f^2}$$

```
[Out] -1/2*(-2*a*d*f-b*c*f+2*b*d*e)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/f^2/d^(1/2
)+(-a*f+b*e)*arctanh(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1/2))*(-c*f+d*e)
^(1/2)/f^2/e^(1/2)+1/2*b*x*(d*x^2+c)^(1/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {542, 537, 223, 212, 385, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)(-2adf-bcf+2bde)}{2\sqrt{d}\,f^2}+\frac{(be-af)\sqrt{de-cf}\,\tanh^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{e}\,\sqrt{c+dx^2}}\right)}{\sqrt{e}\,f^2}+\frac{bx\sqrt{c+dx^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*f) - ((2*b*d*e - b*c*f - 2*a*d*f)*ArcTanh[(Sqrt[d]
*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*f^2) + ((b*e - a*f)*Sqrt[d*e - c*f]*ArcTan
h[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])])/(Sqrt[e]*f^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx &= \frac{bx\sqrt{c+dx^2}}{2f} + \frac{\int \frac{-c(be-2af)+(-2bde+bcf+2adf)x^2}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx}{2f} \\
&= \frac{bx\sqrt{c+dx^2}}{2f} + \frac{((be-af)(de-cf))\int \frac{1}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx}{f^2} - \frac{(2bde-bcf-\\
&= \frac{bx\sqrt{c+dx^2}}{2f} + \frac{((be-af)(de-cf))\text{Subst}\left(\int \frac{1}{e-(de-cf)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{f^2} - \\
&= \frac{bx\sqrt{c+dx^2}}{2f} - \frac{(2bde-bcf-2adf)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,f^2} + \frac{(be-af)\sqrt{de-
\end{aligned}$$

Mathematica [A]
time = 0.38, size = 141, normalized size = 1.10

$$\frac{bfx\sqrt{c+dx^2} + \frac{2(be-af)\sqrt{-de+cf}\tan^{-1}\left(\frac{-fx\sqrt{c+dx^2}+\sqrt{d}\,(e+fx^2)}{\sqrt{e}\sqrt{-de+cf}}\right)}{\sqrt{e}} + \frac{(2bde-bcf-2adf)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{\sqrt{d}}}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

```
[Out] (b*f*x*Sqrt[c + d*x^2] + (2*(b*e - a*f)*Sqrt[-(d*e) + c*f]*ArcTan[(-(f*x*Sq
rt[c + d*x^2]) + Sqrt[d]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(d*e) + c*f])])/Sqrt[e
] + ((2*b*d*e - b*c*f - 2*a*d*f)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/Sqrt[
d])/(2*f^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(106) = 212$.
time = 0.18, size = 707, normalized size = 5.52 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

```
[Out] b/f*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))+1/2
*(a*f-b*e)/(-f*e)^(1/2)/f*(((x-(-f*e)^(1/2)/f)^2*d+2*d*(-f*e)^(1/2)/f*(x-(-
f*e)^(1/2)/f)+(c*f-d*e)/f)^(1/2)+d^(1/2)*(-f*e)^(1/2)/f*ln((d*(-f*e)^(1/2)/
f+d*(x-(-f*e)^(1/2)/f))/d^(1/2)+((x-(-f*e)^(1/2)/f)^2*d+2*d*(-f*e)^(1/2)/f*
(x-(-f*e)^(1/2)/f)+(c*f-d*e)/f)^(1/2))-(c*f-d*e)/f/((c*f-d*e)/f)^(1/2)*ln((
2*(c*f-d*e)/f+2*d*(-f*e)^(1/2)/f*(x-(-f*e)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*(
(x-(-f*e)^(1/2)/f)^2*d+2*d*(-f*e)^(1/2)/f*(x-(-f*e)^(1/2)/f)+(c*f-d*e)/f)^(
1/2))/(x-(-f*e)^(1/2)/f)))+1/2*(-a*f+b*e)/(-f*e)^(1/2)/f*(((x+(-f*e)^(1/2)/
f)^2*d-2*d*(-f*e)^(1/2)/f*(x+(-f*e)^(1/2)/f)+(c*f-d*e)/f)^(1/2)-d^(1/2)*(-f
*e)^(1/2)/f*ln((-d*(-f*e)^(1/2)/f+d*(x+(-f*e)^(1/2)/f))/d^(1/2)+((x+(-f*e)^
(1/2)/f)^2*d-2*d*(-f*e)^(1/2)/f*(x+(-f*e)^(1/2)/f)+(c*f-d*e)/f)^(1/2))-(c*f
-d*e)/f/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f-2*d*(-f*e)^(1/2)/f*(x+(-f*e)^
(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x+(-f*e)^(1/2)/f)^2*d-2*d*(-f*e)^(1/2)/f*(
x+(-f*e)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x+(-f*e)^(1/2)/f)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e), x)
```

Fricas [A]
time = 2.25, size = 786, normalized size = 6.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(d*x^2 + c)*b*d*f*x + (2*b*d*e - (b*c + 2*a*d)*f)*sqrt(d)*log(-
2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (a*d*f - b*d*e)*sqrt(-(c*f - d
*e)*e^(-1))*log((c^2*f^2*x^4 - 4*(c*f*x^3*e - (2*d*x^3 + c*x)*e^2)*sqrt(d*x
^2 + c)*sqrt(-(c*f - d*e)*e^(-1)) + (8*d^2*x^4 + 8*c*d*x^2 + c^2)*e^2 - 2*(
4*c*d*f*x^4 + 3*c^2*f*x^2)*e)/(f^2*x^4 + 2*f*x^2*e + e^2)))/(d*f^2), 1/4*(2
*sqrt(d*x^2 + c)*b*d*f*x + 2*(2*b*d*e - (b*c + 2*a*d)*f)*sqrt(-d)*arctan(sq
rt(-d)*x/sqrt(d*x^2 + c)) - (a*d*f - b*d*e)*sqrt(-(c*f - d*e)*e^(-1))*log((
c^2*f^2*x^4 - 4*(c*f*x^3*e - (2*d*x^3 + c*x)*e^2)*sqrt(d*x^2 + c)*sqrt(-(c*
f - d*e)*e^(-1)) + (8*d^2*x^4 + 8*c*d*x^2 + c^2)*e^2 - 2*(4*c*d*f*x^4 + 3*c
^2*f*x^2)*e)/(f^2*x^4 + 2*f*x^2*e + e^2)))/(d*f^2), 1/4*(2*sqrt(d*x^2 + c)*
b*d*f*x + 2*(a*d*f - b*d*e)*sqrt(c*f - d*e)*arctan(1/2*(c*f*x^2 - (2*d*x^2
+ c)*e)*sqrt(d*x^2 + c)*sqrt(c*f - d*e)*e^(-1/2)/(c*d*f*x^3 + c^2*f*x - (d^
2*x^3 + c*d*x)*e))*e^(-1/2) + (2*b*d*e - (b*c + 2*a*d)*f)*sqrt(d)*log(-2*d*
x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(d*f^2), 1/2*(sqrt(d*x^2 + c)*b*d*f
*x + (a*d*f - b*d*e)*sqrt(c*f - d*e)*arctan(1/2*(c*f*x^2 - (2*d*x^2 + c)*e)
*sqrt(d*x^2 + c)*sqrt(c*f - d*e)*e^(-1/2)/(c*d*f*x^3 + c^2*f*x - (d^2*x^3 +
 c*d*x)*e))*e^(-1/2) + (2*b*d*e - (b*c + 2*a*d)*f)*sqrt(-d)*arctan(sqrt(-d)
*x/sqrt(d*x^2 + c)))/(d*f^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)\sqrt{c + dx^2}}{e + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)\,\sqrt{d\,x^2+c}}{f\,x^2+e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2),x)
[Out] int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2), x)
```

3.58 $\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=304

$$-\frac{b^2(bc-ad)x\sqrt{e+fx^2}}{2d^2f}-\frac{3b^2(be-2af)x\sqrt{e+fx^2}}{8df^2}+\frac{b^2x(a+bx^2)\sqrt{e+fx^2}}{4df}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{d\cdots}}{\sqrt{c}\,\sqrt{\cdots}}\right.}{\sqrt{c}\,d^3\sqrt{de-c\cdots}}$$

```
[Out] 1/2*b*(-a*d+b*c)*(-2*a*f+b*e)*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/d^2/f^(3/2
)+1/8*b*(8*a^2*f^2-8*a*b*e*f+3*b^2*e^2)*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/
d/f^(5/2)+b*(-a*d+b*c)^2*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/d^3/f^(1/2)-(-a
*d+b*c)^3*arctan(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/d^3/c^(1/2)/(-
c*f+d*e)^(1/2)-1/2*b^2*(-a*d+b*c)*x*(f*x^2+e)^(1/2)/d^2/f-3/8*b^2*(-2*a*f+b
*e)*x*(f*x^2+e)^(1/2)/d/f^2+1/4*b^2*x*(b*x^2+a)*(f*x^2+e)^(1/2)/d/f
```

Rubi [A]
time = 0.21, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {559, 427, 396, 223, 212, 537, 385, 211}

$$\frac{b(8a^2f^2-8abef+3b^2e^2)\tanh^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e+fx^2}}\right)}{8df^{5/2}}-\frac{(bc-ad)^3\text{ArcTan}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\,\sqrt{e+fx^2}}\right)}{\sqrt{c}\,d^3\sqrt{de-cf}}-\frac{b^2x\sqrt{e+fx^2}\,(bc-ad)}{2d^2f}-\frac{3b^2x\sqrt{e+fx^2}\,(be-2af)}{8df^2}+\frac{b^2x(a+bx^2)\sqrt{e+fx^2}}{4df}+\frac{b(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e+fx^2}}\right)}{d^3\sqrt{f}}+\frac{b(bc-ad)(be-2af)\tanh^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e+fx^2}}\right)}{2d^2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -1/2*(b^2*(b*c - a*d)*x*Sqrt[e + f*x^2])/(d^2*f) - (3*b^2*(b*e - 2*a*f)*x*S
qrt[e + f*x^2])/(8*d*f^2) + (b^2*x*(a + b*x^2)*Sqrt[e + f*x^2])/(4*d*f) - (
(b*c - a*d)^3*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[
c]*d^3*Sqrt[d*e - c*f]) + (b*(b*c - a*d)^2*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x
^2]])/(d^3*Sqrt[f]) + (b*(b*c - a*d)*(b*e - 2*a*f)*ArcTanh[(Sqrt[f]*x)/Sqrt
[e + f*x^2]])/(2*d^2*f^(3/2)) + (b*(3*b^2*e^2 - 8*a*b*e*f + 8*a^2*f^2)*ArcT
anh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(8*d*f^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 559

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rubi steps

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{b\int \frac{(a+bx^2)^2}{\sqrt{e+fx^2}}\,dx}{d} + \frac{(-bc+ad)\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx}{d}$$

$$= \frac{b^2x(a+bx^2)\sqrt{e+fx^2}}{4df} - \frac{(b(bc-ad))\int \frac{a+bx^2}{\sqrt{e+fx^2}}\,dx}{d^2} + \frac{(bc-ad)^2\int}{d^2}$$

$$= -\frac{b^2(bc-ad)x\sqrt{e+fx^2}}{2d^2f} - \frac{3b^2(be-2af)x\sqrt{e+fx^2}}{8df^2} + \frac{b^2x(a+bx^2)\sqrt{e+}}{4df}$$

$$= -\frac{b^2(bc-ad)x\sqrt{e+fx^2}}{2d^2f} - \frac{3b^2(be-2af)x\sqrt{e+fx^2}}{8df^2} + \frac{b^2x(a+bx^2)\sqrt{e+}}{4df}$$

$$= -\frac{b^2(bc-ad)x\sqrt{e+fx^2}}{2d^2f} - \frac{3b^2(be-2af)x\sqrt{e+fx^2}}{8df^2} + \frac{b^2x(a+bx^2)\sqrt{e+}}{4df}$$

Mathematica [A]
time = 0.59, size = 214, normalized size = 0.70

$$\frac{\frac{b^2dx\sqrt{e+fx^2}\,(12adf+b(-3de-4cf+2dfx^2))}{f^2} + \frac{8(bc-ad)^3\tan^{-1}\left(\frac{c\sqrt{f}+dx\left(\sqrt{f}\,x-\sqrt{e+fx^2}\right)}{\sqrt{c}\,\sqrt{de-cf}}\right)}{\sqrt{c}\,\sqrt{de-cf}} - \frac{b(24a^2d^2f^2-12abdf(de+2cf)+b^2(3d^2e^2+4cdef+8c^2f^2))\log\left(-\sqrt{f}\,x+\sqrt{e+fx^2}\right)}{f^{5/2}}}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((b^2*d*x*Sqrt[e + f*x^2]*(12*a*d*f + b*(-3*d*e - 4*c*f + 2*d*f*x^2)))/f^2
+ (8*(b*c - a*d)^3*ArcTan[(c*Sqrt[f] + d*x*(Sqrt[f]*x - Sqrt[e + f*x^2]))/(
Sqrt[c]*Sqrt[d*e - c*f])])/(Sqrt[c]*Sqrt[d*e - c*f]) - (b*(24*a^2*d^2*f^2 -
 12*a*b*d*f*(d*e + 2*c*f) + b^2*(3*d^2*e^2 + 4*c*d*e*f + 8*c^2*f^2))*Log[-(
Sqrt[f]*x) + Sqrt[e + f*x^2]])/f^(5/2))/(8*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(260) = 520$.
time = 0.18, size = 591, normalized size = 1.94

method	result

default	$\frac{b\left(b^{2}d^{2}\left(\frac{x^{3}\sqrt{f\,x^{2}+e}}{4f}-\frac{3e\left(\frac{x\sqrt{f\,x^{2}+e}}{2f}-\frac{e\ln\left(\sqrt{f}\,x+\sqrt{f\,x^{2}+e}\right)}{2f^{\frac{3}{2}}}\right)}{4f}\right)+\left(3ab\,d^{2}-b^{2}cd\right)\left(\frac{x\sqrt{f\,x^{2}+e}}{2f}-\frac{e\ln\left(\sqrt{f}\right.}{}\right.\right.}{d}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^3*(b^2*d^2*(1/4*x^3/f*(f*x^2+e)^(1/2)-3/4*e/f*(1/2*x/f*(f*x^2+e)^(1/2)-
1/2*e/f^(3/2)*ln(f^(1/2)*x+(f*x^2+e)^(1/2))))+(3*a*b*d^2-b^2*c*d)*(1/2*x/f*
(f*x^2+e)^(1/2)-1/2*e/f^(3/2)*ln(f^(1/2)*x+(f*x^2+e)^(1/2)))+3*a^2*d^2*ln(f
^(1/2)*x+(f*x^2+e)^(1/2))/f^(1/2)-3*a*b*c*d*ln(f^(1/2)*x+(f*x^2+e)^(1/2))/f
^(1/2)+b^2*c^2*ln(f^(1/2)*x+(f*x^2+e)^(1/2))/f^(1/2))-1/2/d^3*(a^3*d^3-3*a^
2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(
c*f-d*e)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x
-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/
2))/(x-(-c*d)^(1/2)/d))-1/2*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/
d^3/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d
*(x+(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*
d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^3/((d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [A]
time = 10.19, size = 1772, normalized size = 5.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c^2*f -
c*d*e)*f^3*log((8*c^2*f^2*x^4 + 4*(2*c*f*x^3 - (d*x^3 - c*x)*e)*sqrt(c^2*f
- c*d*e)*sqrt(f*x^2 + e) + (d^2*x^4 - 6*c*d*x^2 + c^2)*e^2 - 8*(c*d*f*x^4 -
 c^2*f*x^2)*e)/(d^2*x^4 + 2*c*d*x^2 + c^2)) + (3*b^3*c*d^3*e^3 - 8*(b^3*c^4
 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2)*f^3 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 +
6*a^2*b*c*d^3)*f^2*e + (b^3*c^2*d^2 - 12*a*b^2*c*d^3)*f*e^2)*sqrt(f)*log(-2
*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*x - e) - 2*(2*b^3*c^2*d^2*f^3*x^3 + 3*b^
3*c*d^3*f*x*e^2 - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2)*f^3*x - (2*b^3*c*d^3*f^2*
x^3 - (b^3*c^2*d^2 - 12*a*b^2*c*d^3)*f^2*x)*e)*sqrt(f*x^2 + e))/(c^2*d^3*f^
4 - c*d^4*f^3*e), -1/8*(2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*sqrt(c^2*f - c*d*e)*f^3*log((8*c^2*f^2*x^4 + 4*(2*c*f*x^3 - (d*x^3 - c*x
)*e)*sqrt(c^2*f - c*d*e)*sqrt(f*x^2 + e) + (d^2*x^4 - 6*c*d*x^2 + c^2)*e^2
- 8*(c*d*f*x^4 - c^2*f*x^2)*e)/(d^2*x^4 + 2*c*d*x^2 + c^2)) - (3*b^3*c*d^3*
e^3 - 8*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2)*f^3 + 4*(b^3*c^3*d - 3*
a*b^2*c^2*d^2 + 6*a^2*b*c*d^3)*f^2*e + (b^3*c^2*d^2 - 12*a*b^2*c*d^3)*f*e^2
)*sqrt(-f)*arctan(sqrt(-f)*x/sqrt(f*x^2 + e)) - (2*b^3*c^2*d^2*f^3*x^3 + 3*
b^3*c*d^3*f*x*e^2 - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2)*f^3*x - (2*b^3*c*d^3*f^
2*x^3 - (b^3*c^2*d^2 - 12*a*b^2*c*d^3)*f^2*x)*e)*sqrt(f*x^2 + e))/(c^2*d^3*
f^4 - c*d^4*f^3*e), 1/16*(8*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*sqrt(-c^2*f + c*d*e)*f^3*arctan(1/2*(2*c*f*x^2 - (d*x^2 - c)*e)*sqrt(-
c^2*f + c*d*e)*sqrt(f*x^2 + e)/(c^2*f^2*x^3 - c*d*x*e^2 - (c*d*f*x^3 - c^2*
f*x)*e)) - (3*b^3*c*d^3*e^3 - 8*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2)
*f^3 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 6*a^2*b*c*d^3)*f^2*e + (b^3*c^2*d^2
 - 12*a*b^2*c*d^3)*f*e^2)*sqrt(f)*log(-2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*
x - e) + 2*(2*b^3*c^2*d^2*f^3*x^3 + 3*b^3*c*d^3*f*x*e^2 - 4*(b^3*c^3*d - 3*
a*b^2*c^2*d^2)*f^3*x - (2*b^3*c*d^3*f^2*x^3 - (b^3*c^2*d^2 - 12*a*b^2*c*d^3
)*f^2*x)*e)*sqrt(f*x^2 + e))/(c^2*d^3*f^4 - c*d^4*f^3*e), 1/8*(4*(b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c^2*f + c*d*e)*f^3*arctan(1
/2*(2*c*f*x^2 - (d*x^2 - c)*e)*sqrt(-c^2*f + c*d*e)*sqrt(f*x^2 + e)/(c^2*f^
2*x^3 - c*d*x*e^2 - (c*d*f*x^3 - c^2*f*x)*e)) + (3*b^3*c*d^3*e^3 - 8*(b^3*c
^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2)*f^3 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2
+ 6*a^2*b*c*d^3)*f^2*e + (b^3*c^2*d^2 - 12*a*b^2*c*d^3)*f*e^2)*sqrt(-f)*arc
tan(sqrt(-f)*x/sqrt(f*x^2 + e)) + (2*b^3*c^2*d^2*f^3*x^3 + 3*b^3*c*d^3*f*x*
e^2 - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2)*f^3*x - (2*b^3*c*d^3*f^2*x^3 - (b^3*c
^2*d^2 - 12*a*b^2*c*d^3)*f^2*x)*e)*sqrt(f*x^2 + e))/(c^2*d^3*f^4 - c*d^4*f^
3*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**3/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)^3}{(d\,x^2+c)\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/((c + d*x^2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^3/((c + d*x^2)*(e + f*x^2)^(1/2)), x)
```

3.59 $\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=166

$$\frac{b^2x\sqrt{e+fx^2}}{2df}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\,\sqrt{e+fx^2}}\right)}{\sqrt{c}\,d^2\sqrt{de-cf}}-\frac{b(bc-ad)\tanh^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e+fx^2}}\right)}{d^2\sqrt{f}}-\frac{b(be-2af)\tanh}{2dj}$$

```
[Out] -1/2*b*(-2*a*f+b*e)*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/d/f^(3/2)-b*(-a*d+b*
c)*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/d^2/f^(1/2)+(-a*d+b*c)^2*arctan(x*(-c
*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/d^2/c^(1/2)/(-c*f+d*e)^(1/2)+1/2*b^2
*x*(f*x^2+e)^(1/2)/d/f
```

Rubi [A]
time = 0.08, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {559, 396, 223, 212, 537, 385, 211}

$$\frac{(bc-ad)^2\text{ArcTan}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\,\sqrt{e+fx^2}}\right)}{\sqrt{c}\,d^2\sqrt{de-cf}}-\frac{b(bc-ad)\tanh^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e+fx^2}}\right)}{d^2\sqrt{f}}-\frac{b(be-2af)\tanh^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e+fx^2}}\right)}{2df^{3/2}}+\frac{b^2x\sqrt{e+fx^2}}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (b^2*x*Sqrt[e + f*x^2])/(2*d*f) + ((b*c - a*d)^2*ArcTan[(Sqrt[d*e - c*f]*x)
/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*d^2*Sqrt[d*e - c*f]) - (b*(b*c - a*d)
*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(d^2*Sqrt[f]) - (b*(b*e - 2*a*f)*Arc
Tanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(2*d*f^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 559

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx &= \frac{b\int \frac{a+bx^2}{\sqrt{e+fx^2}}\,dx}{d} + \frac{(-bc+ad)\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx}{d}\\
&= \frac{b^2x\sqrt{e+fx^2}}{2df} - \frac{(b(bc-ad))\int \frac{1}{\sqrt{e+fx^2}}\,dx}{d^2} + \frac{(bc-ad)^2\int \frac{1}{(c+dx^2)\sqrt{e+fx}}}{d^2}\\
&= \frac{b^2x\sqrt{e+fx^2}}{2df} - \frac{(b(bc-ad))\text{Subst}\left(\int \frac{1}{1-fx^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{d^2} + \frac{(bc-ad)^2}{}\\
&= \frac{b^2x\sqrt{e+fx^2}}{2df} + \frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\,\sqrt{e+fx^2}}\right)}{\sqrt{c}\,d^2\sqrt{de-cf}} - \frac{b(bc-ad)\tanh^{-1}\left(\right.}{d^2\sqrt{f}}
\end{aligned}$$

Mathematica [A]
time = 0.32, size = 151, normalized size = 0.91

$$\frac{\frac{b^2dx\sqrt{e+fx^2}}{f}-\frac{2(bc-ad)^2\tan^{-1}\left(\frac{c\sqrt{f}+dx\left(\sqrt{f}x-\sqrt{e+fx^2}\right)}{\sqrt{c}\sqrt{de-cf}}\right)}{\sqrt{c}\sqrt{de-cf}}+\frac{b(bde+2bcf-4adf)\log\left(-\sqrt{f}x+\sqrt{e+fx^2}\right)}{f^{3/2}}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((b^2*d*x*Sqrt[e + f*x^2])/f - (2*(b*c - a*d)^2*ArcTan[(c*Sqrt[f] + d*x*(Sq
rt[f]*x - Sqrt[e + f*x^2]))/(Sqrt[c]*Sqrt[d*e - c*f])])/(Sqrt[c]*Sqrt[d*e -
 c*f]) + (b*(b*d*e + 2*b*c*f - 4*a*d*f)*Log[-(Sqrt[f]*x) + Sqrt[e + f*x^2]]
)/f^(3/2))/(2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(138) = 276.
time = 0.14, size = 449, normalized size = 2.70

method	result
default	$\frac{b\left(bd\left(\frac{x\sqrt{f\,x^2+e}}{2f}-\frac{e\ln\left(\sqrt{f}\,x+\sqrt{f\,x^2+e}\right)}{2f^{\frac{3}{2}}}\right)+\frac{2ad\ln\left(\sqrt{f}\,x+\sqrt{f\,x^2+e}\right)}{\sqrt{f}}-\frac{bc\ln\left(\sqrt{f}\,x+\sqrt{f\,x^2+e}\right)}{\sqrt{f}}\right)}{d^2}$ −
risch	$\frac{b^2x\sqrt{f\,x^2+e}}{2df}+\frac{2b\ln\left(\sqrt{f}\,x+\sqrt{f\,x^2+e}\right)a}{d\sqrt{f}}-\frac{b^2\ln\left(\sqrt{f}\,x+\sqrt{f\,x^2+e}\right)c}{d^2\sqrt{f}}-\frac{b^2\ln\left(\sqrt{f}\,x+\sqrt{f\,x^2+e}\right.}{2d\,f^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^2*(b*d*(1/2*x/f*(f*x^2+e)^(1/2)-1/2*e/f^(3/2)*ln(f^(1/2)*x+(f*x^2+e)^(1
/2)))+2*a*d*ln(f^(1/2)*x+(f*x^2+e)^(1/2))/f^(1/2)-b*c*ln(f^(1/2)*x+(f*x^2+e
)^(1/2))/f^(1/2))-1/2/d^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(-c*d)^(1/2)/(-(c*f-d
```

```
*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(
c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2
)/d)-(c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/2*(-a^2*d^2+2*a*b*c*d-b^2*c^
2)/d^2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2
)/d*(x+(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(
-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [**A**]
time = 2.52, size = 1166, normalized size = 7.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
[Out] [1/4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c^2*f - c*d*e)*f^2*log((8*c^2*f^
2*x^4 + 4*(2*c*f*x^3 - (d*x^3 - c*x)*e)*sqrt(c^2*f - c*d*e)*sqrt(f*x^2 + e)
 + (d^2*x^4 - 6*c*d*x^2 + c^2)*e^2 - 8*(c*d*f*x^4 - c^2*f*x^2)*e)/(d^2*x^4
+ 2*c*d*x^2 + c^2)) - (b^2*c*d^2*e^2 - 2*(b^2*c^3 - 2*a*b*c^2*d)*f^2 + (b^2
*c^2*d - 4*a*b*c*d^2)*f*e)*sqrt(f)*log(-2*f*x^2 + 2*sqrt(f*x^2 + e)*sqrt(f)
*x - e) + 2*(b^2*c^2*d*f^2*x - b^2*c*d^2*f*x*e)*sqrt(f*x^2 + e))/(c^2*d^2*f
^3 - c*d^3*f^2*e), 1/4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c^2*f - c*d*e)
*f^2*log((8*c^2*f^2*x^4 + 4*(2*c*f*x^3 - (d*x^3 - c*x)*e)*sqrt(c^2*f - c*d*
e)*sqrt(f*x^2 + e) + (d^2*x^4 - 6*c*d*x^2 + c^2)*e^2 - 8*(c*d*f*x^4 - c^2*f
*x^2)*e)/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 2*(b^2*c*d^2*e^2 - 2*(b^2*c^3 - 2*a
*b*c^2*d)*f^2 + (b^2*c^2*d - 4*a*b*c*d^2)*f*e)*sqrt(-f)*arctan(sqrt(-f)*x/s
qrt(f*x^2 + e)) + 2*(b^2*c^2*d*f^2*x - b^2*c*d^2*f*x*e)*sqrt(f*x^2 + e))/(c
^2*d^2*f^3 - c*d^3*f^2*e), -1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c^
2*f + c*d*e)*f^2*arctan(1/2*(2*c*f*x^2 - (d*x^2 - c)*e)*sqrt(-c^2*f + c*d*e
)*sqrt(f*x^2 + e)/(c^2*f^2*x^3 - c*d*x*e^2 - (c*d*f*x^3 - c^2*f*x)*e)) + (b
^2*c*d^2*e^2 - 2*(b^2*c^3 - 2*a*b*c^2*d)*f^2 + (b^2*c^2*d - 4*a*b*c*d^2)*f*
e)*sqrt(f)*log(-2*f*x^2 + 2*sqrt(f*x^2 + e)*sqrt(f)*x - e) - 2*(b^2*c^2*d*f
^2*x - b^2*c*d^2*f*x*e)*sqrt(f*x^2 + e))/(c^2*d^2*f^3 - c*d^3*f^2*e), -1/2*
((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c^2*f + c*d*e)*f^2*arctan(1/2*(2*c*f
*x^2 - (d*x^2 - c)*e)*sqrt(-c^2*f + c*d*e)*sqrt(f*x^2 + e)/(c^2*f^2*x^3 - c
*d*x*e^2 - (c*d*f*x^3 - c^2*f*x)*e)) + (b^2*c*d^2*e^2 - 2*(b^2*c^3 - 2*a*b*
```

```
c^2*d)*f^2 + (b^2*c^2*d - 4*a*b*c*d^2)*f*e)*sqrt(-f)*arctan(sqrt(-f)*x/sqrt
(f*x^2 + e)) - (b^2*c^2*d*f^2*x - b^2*c*d^2*f*x*e)*sqrt(f*x^2 + e))/(c^2*d^
2*f^3 - c*d^3*f^2*e)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^2}{(c + dx^2)\sqrt{e + fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**2/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^2}{\left(d\,x^2+c\right)\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/((c + d*x^2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^2/((c + d*x^2)*(e + f*x^2)^(1/2)), x)
```

3.60 $\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=91

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\,\sqrt{e+fx^2}}\right)}{\sqrt{c}\,d\sqrt{de-cf}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}$$

```
[Out] b*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/d/f^(1/2)-(-a*d+b*c)*arctan(x*(-c*f+d*
e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/d/c^(1/2)/(-c*f+d*e)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {537, 223, 212, 385, 211}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}-\frac{(bc-ad)\text{ArcTan}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\,\sqrt{e+fx^2}}\right)}{\sqrt{c}\,d\sqrt{de-cf}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -(((b*c - a*d)*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt
[c]*d*Sqrt[d*e - c*f])) + (b*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(d*Sqrt[
f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{b\int \frac{1}{\sqrt{e+fx^2}}\,dx}{d} + \frac{(-bc+ad)\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx}{d}$$

$$= \frac{b\text{Subst}\left(\int \frac{1}{1-fx^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{d} + \frac{(-bc+ad)\text{Subst}\left(\int \frac{1}{c-(-de+cf)x^2}\,dx, x\right.}{d}$$

$$= -\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\,\sqrt{e+fx^2}}\right)}{\sqrt{c}\,d\sqrt{de-cf}} + \frac{b\tanh^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}$$

Mathematica [**A**]
time = 0.25, size = 111, normalized size = 1.22

$$\frac{\frac{(bc-ad)\tan^{-1}\left(\frac{c\sqrt{f}+dx\left(\sqrt{f}\,x-\sqrt{e+fx^2}\right)}{\sqrt{c}\,\sqrt{de-cf}}\right)}{\sqrt{c}\,\sqrt{de-cf}} - \frac{b\log\left(-\sqrt{f}\,x+\sqrt{e+fx^2}\right)}{\sqrt{f}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (((b*c - a*d)*ArcTan[(c*Sqrt[f] + d*x*(Sqrt[f]*x - Sqrt[e + f*x^2]))/(Sqrt[
c]*Sqrt[d*e - c*f])])/(Sqrt[c]*Sqrt[d*e - c*f]) - (b*Log[-(Sqrt[f]*x) + Sqr
t[e + f*x^2]])/Sqrt[f])/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(75) = 150$.
time = 0.11, size = 352, normalized size = 3.87

method	result
default	$\frac{b\ln\left(\sqrt{f}\,x+\sqrt{f\,x^2+e}\right)}{d\sqrt{f}}-\frac{(ad-bc)\ln\left(\frac{-\frac{2(cf-de)}{d}+\frac{2f\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{-\frac{cf-de}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2}}{x-\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\,d\sqrt{-\frac{cf-de}{d}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b/d*ln(f^(1/2)*x+(f*x^2+e)^(1/2))/f^(1/2)-1/2*(a*d-b*c)/(-c*d)^(1/2)/d/(-(c
*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2
*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^
(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/2*(-a*d+b*c)/(-c*d)^(1/2
)/d/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1
/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x
+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(79) = 158$.
time = 1.73, size = 782, normalized size = 8.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(c^2*f - c*d*e)*(b*c - a*d)*f*log((8*c^2*f^2*x^4 + 4*(2*c*f*x^3
- (d*x^3 - c*x)*e)*sqrt(c^2*f - c*d*e)*sqrt(f*x^2 + e) + (d^2*x^4 - 6*c*d*x
^2 + c^2)*e^2 - 8*(c*d*f*x^4 - c^2*f*x^2)*e)/(d^2*x^4 + 2*c*d*x^2 + c^2)) -
 2*(b*c^2*f - b*c*d*e)*sqrt(f)*log(-2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*x -
```

```
 e))/(c^2*d*f^2 - c*d^2*f*e), -1/4*(sqrt(c^2*f - c*d*e)*(b*c - a*d)*f*log((
8*c^2*f^2*x^4 + 4*(2*c*f*x^3 - (d*x^3 - c*x)*e)*sqrt(c^2*f - c*d*e)*sqrt(f*
x^2 + e) + (d^2*x^4 - 6*c*d*x^2 + c^2)*e^2 - 8*(c*d*f*x^4 - c^2*f*x^2)*e)/(
d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*(b*c^2*f - b*c*d*e)*sqrt(-f)*arctan(sqrt(-f
)*x/sqrt(f*x^2 + e)))/(c^2*d*f^2 - c*d^2*f*e), 1/2*(sqrt(-c^2*f + c*d*e)*(b
*c - a*d)*f*arctan(1/2*(2*c*f*x^2 - (d*x^2 - c)*e)*sqrt(-c^2*f + c*d*e)*sqr
t(f*x^2 + e)/(c^2*f^2*x^3 - c*d*x*e^2 - (c*d*f*x^3 - c^2*f*x)*e)) + (b*c^2*
f - b*c*d*e)*sqrt(f)*log(-2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*x - e))/(c^2*
d*f^2 - c*d^2*f*e), 1/2*(sqrt(-c^2*f + c*d*e)*(b*c - a*d)*f*arctan(1/2*(2*c
*f*x^2 - (d*x^2 - c)*e)*sqrt(-c^2*f + c*d*e)*sqrt(f*x^2 + e)/(c^2*f^2*x^3 -
 c*d*x*e^2 - (c*d*f*x^3 - c^2*f*x)*e)) - 2*(b*c^2*f - b*c*d*e)*sqrt(-f)*arc
tan(sqrt(-f)*x/sqrt(f*x^2 + e)))/(c^2*d*f^2 - c*d^2*f*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2}{(c + dx^2)\sqrt{e + fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b\,x^2 + a}{(d\,x^2 + c)\,\sqrt{f\,x^2 + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c + d*x^2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)/((c + d*x^2)*(e + f*x^2)^(1/2)), x)
```

3.61 $\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\,\sqrt{e+fx^2}}\right)}{\sqrt{c}\,\sqrt{de-cf}}$$

```
[Out] arctan(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/c^(1/2)/(-c*f+d*e)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {385, 211}

$$\frac{\text{ArcTan}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\,\sqrt{e+fx^2}}\right)}{\sqrt{c}\,\sqrt{de-cf}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])]/(Sqrt[c]*Sqrt[d*e - c
*f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{c-(-de+cf)x^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\,\sqrt{e+fx^2}}\right)}{\sqrt{c}\,\sqrt{de-cf}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 70, normalized size = 1.43

$$-\frac{\tan^{-1}\left(\frac{c\sqrt{f}+dx\left(\sqrt{f}\,x-\sqrt{e+fx^2}\right)}{\sqrt{c}\,\sqrt{de-cf}}\right)}{\sqrt{c}\,\sqrt{de-cf}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -(ArcTan[(c*Sqrt[f] + d*x*(Sqrt[f]*x - Sqrt[e + f*x^2]))/(Sqrt[c]*Sqrt[d*e
- c*f])]/(Sqrt[c]*Sqrt[d*e - c*f]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(39) = 78.
time = 0.11, size = 306, normalized size = 6.24

method	result
default	$-\frac{\ln\left(\frac{-\frac{2(cf-de)}{d}+\frac{2f\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{-\frac{cf-de}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 f+\frac{2f\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}-}}{x-\frac{\sqrt{-cd}}{d}}\right.}{2\sqrt{-cd}\,\sqrt{-\frac{cf-de}{d}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)/
d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c
*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+1/2/
(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+
(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(
1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(42) = 84$.
time = 0.95, size = 258, normalized size = 5.27

$$\left[\frac{\log\left(\frac{8\,c^2f^2x^4+4\,(2\,cfx^3-(dx^3-cx)e)\,\sqrt{c^2f-cde}\,\sqrt{fx^2+e}+(d^2x^4-6\,cdx^2+c^2)e^2-8\,(cdfx^4-c^2fx^2)e}{d^2x^4+2\,cdx^2+c^2}\right)}{4\,\sqrt{c^2f-cde}}, -\frac{\sqrt{-c^2f+cde}\,\arctan\left(\frac{(2\,cfx^2-(dx^2-c)e)\,\sqrt{-c^2f+cde}\,\sqrt{fx^2+e}}{2\,(c^2f^2x^3-cdxe^2-(cdfx^3-c^2fx)e)}\right)}{2\,(c^2f-cde)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log((8*c^2*f^2*x^4 + 4*(2*c*f*x^3 - (d*x^3 - c*x)*e)*sqrt(c^2*f - c*d*
e)*sqrt(f*x^2 + e) + (d^2*x^4 - 6*c*d*x^2 + c^2)*e^2 - 8*(c*d*f*x^4 - c^2*f
*x^2)*e)/(d^2*x^4 + 2*c*d*x^2 + c^2))/sqrt(c^2*f - c*d*e), -1/2*sqrt(-c^2*f
 + c*d*e)*arctan(1/2*(2*c*f*x^2 - (d*x^2 - c)*e)*sqrt(-c^2*f + c*d*e)*sqrt(
f*x^2 + e)/(c^2*f^2*x^3 - c*d*x*e^2 - (c*d*f*x^3 - c^2*f*x)*e))/(c^2*f - c*
d*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx^2)\,\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [A]
time = 1.03, size = 74, normalized size = 1.51

$$-\frac{\sqrt{f}\,\arctan\left(\frac{\left(\sqrt{f}\,x-\sqrt{fx^2+e}\right)^2d+2\,cf-de}{2\,\sqrt{-c^2f^2+cdfe}}\right)}{\sqrt{-c^2f^2+cdfe}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(f)*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d + 2*c*f - d*e)/sqrt(
-c^2*f^2 + c*d*f*e))/sqrt(-c^2*f^2 + c*d*f*e)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases} \frac{\operatorname{atan}\left(\frac{x\sqrt{d\,e-c\,f}}{\sqrt{c}\,\sqrt{f\,x^2+e}}\right)}{\sqrt{-c\,(c\,f-d\,e)}} & \text{if } 0<d\,e-c\,f \\ \frac{\ln\left(\frac{\sqrt{c\,(f\,x^2+e)}+x\,\sqrt{c\,f-d\,e}}{\sqrt{c\,(f\,x^2+e)}-x\,\sqrt{c\,f-d\,e}}\right)}{2\,\sqrt{c\,(c\,f-d\,e)}} & \text{if } d\,e-c\,f<0 \\ \int \frac{1}{(d\,x^2+c)\,\sqrt{f\,x^2+e}}\,dx & \text{if } d\,e-c\,f\notin\mathbb{R}\vee c\,f=d\,e \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] piecewise(0 < - c*f + d*e, atan((x*(- c*f + d*e)^(1/2))/(c^(1/2)*(e + f*x^2
)^(1/2)))/(-c*(c*f - d*e))^(1/2), - c*f + d*e < 0, log(((c*(e + f*x^2))^(1/
2) + x*(c*f - d*e)^(1/2))/((c*(e + f*x^2))^(1/2) - x*(c*f - d*e)^(1/2)))/(2
*(c*(c*f - d*e))^(1/2)), ~in(- c*f + d*e, 'real') | c*f == d*e, int(1/((c +
 d*x^2)*(e + f*x^2)^(1/2)), x))
```

3.62 $\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=122

$$\frac{b\tan^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{a}\,\sqrt{e+fx^2}}\right)}{\sqrt{a}\,(bc-ad)\sqrt{be-af}}-\frac{d\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\,\sqrt{e+fx^2}}\right)}{\sqrt{c}\,(bc-ad)\sqrt{de-cf}}$$

```
[Out] b*arctan(x*(-a*f+b*e)^(1/2)/a^(1/2)/(f*x^2+e)^(1/2))/(-a*d+b*c)/a^(1/2)/(-a
*f+b*e)^(1/2)-d*arctan(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/(-a*d+b*
c)/c^(1/2)/(-c*f+d*e)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {546, 385, 211}

$$\frac{b\text{ArcTan}\left(\frac{x\sqrt{be-af}}{\sqrt{a}\,\sqrt{e+fx^2}}\right)}{\sqrt{a}\,(bc-ad)\sqrt{be-af}}-\frac{d\text{ArcTan}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\,\sqrt{e+fx^2}}\right)}{\sqrt{c}\,(bc-ad)\sqrt{de-cf}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (b*ArcTan[(Sqrt[b*e - a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])])/(Sqrt[a]*(b*c - a
*d)*Sqrt[b*e - a*f]) - (d*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^
2])])/(Sqrt[c]*(b*c - a*d)*Sqrt[d*e - c*f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 546

```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/((a + b*x^2)*Sqrt[e + f*x^2]), x
], x] - Dist[d/(b*c - a*d), Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /;
```

```
FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{b\int \frac{1}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{bc-ad} - \frac{d\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx}{bc-ad}$$

$$= \frac{b\text{Subst}\left(\int \frac{1}{a-(-be+af)x^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{bc-ad} - \frac{d\text{Subst}\left(\int \frac{1}{c-(-de+cf)x^2}\right.}{bc-}$$

$$= \frac{b\tan^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{a}\sqrt{e+fx^2}}\right)}{\sqrt{a}\,(bc-ad)\sqrt{be-af}} - \frac{d\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\,(bc-ad)\sqrt{de-cf}}$$

Mathematica [A]
time = 0.38, size = 153, normalized size = 1.25

$$\frac{-\frac{b\tan^{-1}\left(\frac{a\sqrt{f}+bx\left(\sqrt{f}\,x-\sqrt{e+fx^2}\right)}{\sqrt{a}\sqrt{be-af}}\right)}{\sqrt{a}\sqrt{be-af}}+\frac{d\tan^{-1}\left(\frac{c\sqrt{f}+dx\left(\sqrt{f}\,x-\sqrt{e+fx^2}\right)}{\sqrt{c}\sqrt{de-cf}}\right)}{\sqrt{c}\sqrt{de-cf}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (-((b*ArcTan[(a*Sqrt[f] + b*x*(Sqrt[f]*x - Sqrt[e + f*x^2]))/(Sqrt[a]*Sqrt[
b*e - a*f])])/(Sqrt[a]*Sqrt[b*e - a*f])) + (d*ArcTan[(c*Sqrt[f] + d*x*(Sqrt
[f]*x - Sqrt[e + f*x^2]))/(Sqrt[c]*Sqrt[d*e - c*f])])/(Sqrt[c]*Sqrt[d*e - c
*f]))/(b*c - a*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(102) = 204$.
time = 0.12, size = 782, normalized size = 6.41

method	result

default	$\frac{b\,d^2 \ln\left(\frac{-\frac{2(cf-de)}{d}+\frac{2f\sqrt{-cd}\,\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{-\frac{cf-de}{d}}\,\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 f+\frac{2f\sqrt{-cd}\,\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}-}}{x-\frac{\sqrt{-cd}}{d}}\right.}{2\left(b\sqrt{-cd}+\sqrt{-ab}\,d\right)\left(\sqrt{-ab}\,d-b\sqrt{-cd}\right)\sqrt{-cd}\,\sqrt{-\frac{cf-de}{d}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*d^2/(b*(-c*d)^(1/2)+(-a*b)^(1/2)*d)/((-a*b)^(1/2)*d-b*(-c*d)^(1/2))/(
-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)/d*(x-(
-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1
/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/2*b^2*d/
(-a*b)^(1/2)/(b*(-c*d)^(1/2)+(-a*b)^(1/2)*d)/((-a*b)^(1/2)*d-b*(-c*d)^(1/2)
)/(-(a*f-b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b+2*f*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))+2*(-(a*f-b*e)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*f+2*f*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/2*b^2*d/(-
a*b)^(1/2)/(b*(-c*d)^(1/2)+(-a*b)^(1/2)*d)/((-a*b)^(1/2)*d-b*(-c*d)^(1/2))/
(-(a*f-b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b-2*f*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))+2*(-(a*f-b*e)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*f-2*f*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/2*b*d^2/(b*(
-c*d)^(1/2)+(-a*b)^(1/2)*d)/((-a*b)^(1/2)*d-b*(-c*d)^(1/2))/(-c*d)^(1/2)/(-
(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x+(-c*d
)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(108) = 216$.
time = 58.27, size = 1377, normalized size = 11.29

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")

[Out] [-1/4*((b*c^2*f - b*c*d*e)*sqrt(a^2*f - a*b*e)*log((8*a^2*f^2*x^4 - 4*(2*a*f*x^3 - (b*x^3 - a*x)*e)*sqrt(a^2*f - a*b*e)*sqrt(f*x^2 + e) + (b^2*x^4 - 6*a*b*x^2 + a^2)*e^2 - 8*(a*b*f*x^4 - a^2*f*x^2)*e)/(b^2*x^4 + 2*a*b*x^2 + a^2)) + (a^2*d*f - a*b*d*e)*sqrt(c^2*f - c*d*e)*log((8*c^2*f^2*x^4 + 4*(2*c*f*x^3 - (d*x^3 - c*x)*e)*sqrt(c^2*f - c*d*e)*sqrt(f*x^2 + e) + (d^2*x^4 - 6*c*d*x^2 + c^2)*e^2 - 8*(c*d*f*x^4 - c^2*f*x^2)*e)/(d^2*x^4 + 2*c*d*x^2 + c^2)))/((a^2*b*c^3 - a^3*c^2*d)*f^2 - (a*b^2*c^3 - a^3*c*d^2)*f*e + (a*b^2*c^2*d - a^2*b*c*d^2)*e^2), 1/4*(2*(a^2*d*f - a*b*d*e)*sqrt(-c^2*f + c*d*e)*arctan(1/2*(2*c*f*x^2 - (d*x^2 - c)*e)*sqrt(-c^2*f + c*d*e)*sqrt(f*x^2 + e)/(c^2*f^2*x^3 - c*d*x*e^2 - (c*d*f*x^3 - c^2*f*x)*e)) - (b*c^2*f - b*c*d*e)*sqrt(a^2*f - a*b*e)*log((8*a^2*f^2*x^4 - 4*(2*a*f*x^3 - (b*x^3 - a*x)*e)*sqrt(a^2*f - a*b*e)*sqrt(f*x^2 + e) + (b^2*x^4 - 6*a*b*x^2 + a^2)*e^2 - 8*(a*b*f*x^4 - a^2*f*x^2)*e)/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((a^2*b*c^3 - a^3*c^2*d)*f^2 - (a*b^2*c^3 - a^3*c*d^2)*f*e + (a*b^2*c^2*d - a^2*b*c*d^2)*e^2), -1/4*(2*(b*c^2*f - b*c*d*e)*sqrt(-a^2*f + a*b*e)*arctan(1/2*(2*a*f*x^2 - (b*x^2 - a)*e)*sqrt(-a^2*f + a*b*e)*sqrt(f*x^2 + e)/(a^2*f^2*x^3 - a*b*x*e^2 - (a*b*f*x^3 - a^2*f*x)*e)) + (a^2*d*f - a*b*d*e)*sqrt(c^2*f - c*d*e)*log((8*c^2*f^2*x^4 + 4*(2*c*f*x^3 - (d*x^3 - c*x)*e)*sqrt(c^2*f - c*d*e)*sqrt(f*x^2 + e) + (d^2*x^4 - 6*c*d*x^2 + c^2)*e^2 - 8*(c*d*f*x^4 - c^2*f*x^2)*e)/(d^2*x^4 + 2*c*d*x^2 + c^2)))/((a^2*b*c^3 - a^3*c^2*d)*f^2 - (a*b^2*c^3 - a^3*c*d^2)*f*e + (a*b^2*c^2*d - a^2*b*c*d^2)*e^2), -1/2*((b*c^2*f - b*c*d*e)*sqrt(-a^2*f + a*b*e)*arctan(1/2*(2*a*f*x^2 - (b*x^2 - a)*e)*sqrt(-a^2*f + a*b*e)*sqrt(f*x^2 + e)/(a^2*f^2*x^3 - a*b*x*e^2 - (a*b*f*x^3 - a^2*f*x)*e)) - (a^2*d*f - a*b*d*e)*sqrt(-c^2*f + c*d*e)*arctan(1/2*(2*c*f*x^2 - (d*x^2 - c)*e)*sqrt(-c^2*f + c*d*e)*sqrt(f*x^2 + e)/(c^2*f^2*x^3 - c*d*x*e^2 - (c*d*f*x^3 - c^2*f*x)*e)))/((a^2*b*c^3 - a^3*c^2*d)*f^2 - (a*b^2*c^3 - a^3*c*d^2)*f*e + (a*b^2*c^2*d - a^2*b*c*d^2)*e^2)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)\,\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**2+a)/(d*x**2+c)/(f*x**2+e)**(1/2),x)

[Out] Integral(1/((a + b*x**2)*(c + d*x**2)*sqrt(e + f*x**2)), x)

Giac [A]

time = 1.27, size = 173, normalized size = 1.42

$$-f^{\frac{3}{2}}\left(\frac{b\arctan\left(\frac{\left(\sqrt{f}\,x-\sqrt{fx^2+e}\right)^2 b+2\,af-be}{2\,\sqrt{-a^2f^2+abfe}}\right)}{\sqrt{-a^2f^2+abfe}\,(bcf-adf)}-\frac{d\arctan\left(\frac{\left(\sqrt{f}\,x-\sqrt{fx^2+e}\right)^2 d+2\,cf-de}{2\,\sqrt{-c^2f^2+cdfe}}\right)}{\sqrt{-c^2f^2+cdfe}\,(bcf-adf)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] -f^(3/2)*(b*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*b + 2*a*f - b*e)/sq
rt(-a^2*f^2 + a*b*f*e))/(sqrt(-a^2*f^2 + a*b*f*e)*(b*c*f - a*d*f)) - d*arct
an(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d + 2*c*f - d*e)/sqrt(-c^2*f^2 + c*
d*f*e))/(sqrt(-c^2*f^2 + c*d*f*e)*(b*c*f - a*d*f)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(b\,x^2+a)\,\left(d\,x^2+c\right)\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^(1/2)), x)
```

3.63 $\int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=203

$$\frac{b^2x\sqrt{e+fx^2}}{2a(bc-ad)(be-af)(a+bx^2)}+\frac{b(b^2ce-3abde-2abcf+4a^2df)\tan^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{a}\,\sqrt{e+fx^2}}\right)}{2a^{3/2}(bc-ad)^2(be-af)^{3/2}}+\frac{d^2\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{\sqrt{c}\,(bc-ad}$$

```
[Out] 1/2*b*(4*a^2*d*f-2*a*b*c*f-3*a*b*d*e+b^2*c*e)*arctan(x*(-a*f+b*e)^(1/2)/a^(
1/2)/(f*x^2+e)^(1/2))/a^(3/2)/(-a*d+b*c)^2/(-a*f+b*e)^(3/2)+d^2*arctan(x*(-
c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/(-a*d+b*c)^2/c^(1/2)/(-c*f+d*e)^(1/
2)+1/2*b^2*x*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)
```

Rubi [A]
time = 0.19, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {560, 385, 211, 541, 12}

$$\frac{b\,\text{ArcTan}\left(\frac{x\sqrt{be-af}}{\sqrt{a}\,\sqrt{e+fx^2}}\right)(4a^2df-2abcf-3abde+b^2ce)}{2a^{3/2}(bc-ad)^2(be-af)^{3/2}}+\frac{d^2\,\text{ArcTan}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\,\sqrt{e+fx^2}}\right)}{\sqrt{c}\,(bc-ad)^2\sqrt{de-cf}}+\frac{b^2x\sqrt{e+fx^2}}{2a\,(a+bx^2)\,(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (b^2*x*Sqrt[e + f*x^2])/(2*a*(b*c - a*d)*(b*e - a*f)*(a + b*x^2)) + (b*(b^2
*c*e - 3*a*b*d*e - 2*a*b*c*f + 4*a^2*d*f)*ArcTan[(Sqrt[b*e - a*f]*x)/(Sqrt[
a]*Sqrt[e + f*x^2])])/(2*a^(3/2)*(b*c - a*d)^2*(b*e - a*f)^(3/2)) + (d^2*Ar
cTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*(b*c - a*d)^2
*Sqrt[d*e - c*f])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx = -\frac{b\int \frac{-bc+2ad+bdx^2}{(a+bx^2)^2\sqrt{e+fx^2}}\,dx}{(bc-ad)^2} + \frac{d^2\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx}{(bc-ad)^2}$$

$$= \frac{b^2x\sqrt{e+fx^2}}{2a(bc-ad)(be-af)(a+bx^2)} + \frac{d^2\text{Subst}\left(\int \frac{1}{c-(-de+cf)x^2}\,dx, x, \frac{}{\sqrt{e}}\right.}{(bc-ad)^2}$$

$$= \frac{b^2x\sqrt{e+fx^2}}{2a(bc-ad)(be-af)(a+bx^2)} + \frac{d^2\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\,(bc-ad)^2\sqrt{de-cf}} + \frac{(b(}{}$$

$$= \frac{b^2x\sqrt{e+fx^2}}{2a(bc-ad)(be-af)(a+bx^2)} + \frac{d^2\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\,(bc-ad)^2\sqrt{de-cf}} + \frac{(b(}{}$$

$$= \frac{b^2x\sqrt{e+fx^2}}{2a(bc-ad)(be-af)(a+bx^2)} + \frac{b(b^2ce-3abde-2abcf+4a^2df)}{2a^{3/2}(bc-ad)^2(be}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 14.00, size = 531, normalized size = 2.62

$$
\frac{-\frac{30bd\tan^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{a}\,\sqrt{e+fx^2}}\right)}{\sqrt{a}\,\sqrt{be-af}}+\frac{30d^2\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\,\sqrt{e+fx^2}}\right)}{\sqrt{c}\,\sqrt{de-cf}}+\frac{b(bc-ad)x\sqrt{e+fx^2}\left(-45e\sqrt{\frac{a(be-af)x^2(e+fx^2)}{e^2(a+bx^2)^2}}-30fx^2\sqrt{\frac{a(be-af)x^2(e+fx^2)}{e^2(a+bx^2)^2}}+45e\sin^{-1}\left(\sqrt{\frac{(be-af)x^2}{e(a+bx^2)}}\right)+30fx^2\sin^{-1}\left(\sqrt{\frac{(be-af)x^2}{e(a+bx^2)}}\right)+16e\left(\frac{(be-af)x^2}{e(a+bx^2)}\right)^{5/2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(be-af)x^2}{e(a+bx^2)}\right)+16fx^2\left(\frac{(be-af)x^2}{e(a+bx^2)}\right)^{5/2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(be-af)x^2}{e(a+bx^2)}\right)\right)}{e^2\left(\frac{(be-af)x^2}{e(a+bx^2)}\right)^{3/2}(a+bx^2)^2\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}}{30(bc-ad)^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-30*b*d*ArcTan[(Sqrt[b*e - a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])])/(Sqrt[a]*S
qrt[b*e - a*f]) + (30*d^2*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^
2])])/(Sqrt[c]*Sqrt[d*e - c*f]) + (b*(b*c - a*d)*x*Sqrt[e + f*x^2]*(-45*e*S
qrt[(a*(b*e - a*f)*x^2*(e + f*x^2))/(e^2*(a + b*x^2)^2)] - 30*f*x^2*Sqrt[(a
*(b*e - a*f)*x^2*(e + f*x^2))/(e^2*(a + b*x^2)^2)] + 45*e*ArcSin[Sqrt[((b*e
 - a*f)*x^2)/(e*(a + b*x^2))]] + 30*f*x^2*ArcSin[Sqrt[((b*e - a*f)*x^2)/(e*
(a + b*x^2))]] + 16*e*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(5/2)*Sqrt[(a*(e
+ f*x^2))/(e*(a + b*x^2))]*Hypergeometric2F1[2, 3, 7/2, ((b*e - a*f)*x^2)/(
e*(a + b*x^2))] + 16*f*x^2*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(5/2)*Sqrt[(
a*(e + f*x^2))/(e*(a + b*x^2))]*Hypergeometric2F1[2, 3, 7/2, ((b*e - a*f)*x
^2)/(e*(a + b*x^2))]))/(e^2*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(3/2)*(a +
b*x^2)^2*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]))/(30*(b*c - a*d)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1399 vs. $2(177) = 354$.
time = 0.12, size = 1400, normalized size = 6.90

method	result
default	$-\frac{b^2 d^4 \ln\left(\frac{-\frac{2(cf-de)}{d}+\frac{2f\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{-\frac{cf-de}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 f+\frac{2f\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}}}{x-\frac{\sqrt{-cd}}{d}}\right)}{2\left(b\sqrt{-cd}+\sqrt{-ab}\,d\right)^2\left(\sqrt{-ab}\,d-b\sqrt{-cd}\right)^2\sqrt{-cd}\,\sqrt{-\frac{cf-de}{d}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2*d^4/(b*(-c*d)^(1/2)+(-a*b)^(1/2)*d)^2/((-a*b)^(1/2)*d-b*(-c*d)^(1/
2))^2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)
```

```
/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-
c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+1/4
*b^3*d^2*(3*a*d-b*c)/a/(-a*b)^(1/2)/(b*(-c*d)^(1/2)+(-a*b)^(1/2)*d)^2/((-a*
b)^(1/2)*d-b*(-c*d)^(1/2))^2/(-(a*f-b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b+2*f*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*f-b*e)/b)^(1/2)*((x-1/b*(-a*b)^(1/
2))^2*f+2*f*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x-1/b*
(-a*b)^(1/2)))-1/4*b^3*d^2*(3*a*d-b*c)/a/(-a*b)^(1/2)/(b*(-c*d)^(1/2)+(-a*b
)^(1/2)*d)^2/((-a*b)^(1/2)*d-b*(-c*d)^(1/2))^2/(-(a*f-b*e)/b)^(1/2)*ln((-2*
(a*f-b*e)/b-2*f*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*f-b*e)/b)^(1/2)*
((x+1/b*(-a*b)^(1/2))^2*f-2*f*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*f-b*e)
/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/2*b^2*d^4/(b*(-c*d)^(1/2)+(-a*b)^(1/2)*d
)^2/((-a*b)^(1/2)*d-b*(-c*d)^(1/2))^2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln(
(-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2
)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d
)^(1/2))/(x+(-c*d)^(1/2)/d))-1/4*b*d/a/(b*(-c*d)^(1/2)+(-a*b)^(1/2)*d)/((-a
*b)^(1/2)*d-b*(-c*d)^(1/2))*(1/(a*f-b*e)*b/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a
*b)^(1/2))^2*f+2*f*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2)-f
*(-a*b)^(1/2)/(a*f-b*e)/(-(a*f-b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b+2*f*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*f-b*e)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2
*f+2*f*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x-1/b*(-a*b
)^(1/2))))-1/4*b*d/a/(b*(-c*d)^(1/2)+(-a*b)^(1/2)*d)/((-a*b)^(1/2)*d-b*(-c*
d)^(1/2))*(1/(a*f-b*e)*b/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*f-2*f
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2)+f*(-a*b)^(1/2)/(a*f
-b*e)/(-(a*f-b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b-2*f*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))+2*(-(a*f-b*e)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*f-2*f*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**2*(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(188) = 376.
time = 7.96, size = 479, normalized size = 2.36

$$-\frac{1}{2}\left(\frac{2\,d^2\arctan\left(\frac{\left(\sqrt{f}\,x-\sqrt{fx^2+e}\right)^2 d+2\,cf-de}{2\,\sqrt{-c^2f^2+cdfe}}\right)}{\left(b^2c^2f^2-2\,abcdf^2+a^2d^2f^2\right)\sqrt{-c^2f^2+cdfe}}+\frac{\left(2\,ab^2cf-4\,a^2bdf-b^3ce+3\,ab^2de\right)\arctan\left(\frac{\left(\sqrt{f}\,x-\sqrt{fx^2+e}\right)^2 b+2\,af-be}{2\,\sqrt{-a^2f^2+abfe}}\right)}{\left(a^2b^2c^2f^3-2\,a^3bcdf^3+a^4d^2f^3-ab^3c^2f^2e+2\,a^2b^2cdf^2e-a^3bd^2f^2e\right)\sqrt{-a^2f^2+abfe}}+\frac{2\left(2\left(\sqrt{f}\,x-\sqrt{fx^2+e}\right)^2 abf-\left(\sqrt{f}\,x-\sqrt{fx^2+e}\right)^2 b^2e+b^2e^2\right)}{\left(a^2bcf^3-a^3df^3-ab^2cf^2e+a^2bdf^2e\right)\left(\left(\sqrt{f}\,x-\sqrt{fx^2+e}\right)^4 b+4\left(\sqrt{f}\,x-\sqrt{fx^2+e}\right)^2 af-2\left(\sqrt{f}\,x-\sqrt{fx^2+e}\right)^2 be+be^2\right)}\right)f^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*d^2*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d + 2*c*f - d*e)/sq
rt(-c^2*f^2 + c*d*f*e))/((b^2*c^2*f^2 - 2*a*b*c*d*f^2 + a^2*d^2*f^2)*sqrt(-
c^2*f^2 + c*d*f*e)) + (2*a*b^2*c*f - 4*a^2*b*d*f - b^3*c*e + 3*a*b^2*d*e)*a
rctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*b + 2*a*f - b*e)/sqrt(-a^2*f^2 +
 a*b*f*e))/((a^2*b^2*c^2*f^3 - 2*a^3*b*c*d*f^3 + a^4*d^2*f^3 - a*b^3*c^2*f^
2*e + 2*a^2*b^2*c*d*f^2*e - a^3*b*d^2*f^2*e)*sqrt(-a^2*f^2 + a*b*f*e)) + 2*
(2*(sqrt(f)*x - sqrt(f*x^2 + e))^2*a*b*f - (sqrt(f)*x - sqrt(f*x^2 + e))^2*
b^2*e + b^2*e^2)/((a^2*b*c*f^3 - a^3*d*f^3 - a*b^2*c*f^2*e + a^2*b*d*f^2*e)
*((sqrt(f)*x - sqrt(f*x^2 + e))^4*b + 4*(sqrt(f)*x - sqrt(f*x^2 + e))^2*a*f
 - 2*(sqrt(f)*x - sqrt(f*x^2 + e))^2*b*e + b*e^2)))*f^(5/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^2\,\left(d\,x^2+c\right)\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^2)^2*(c + d*x^2)*(e + f*x^2)^(1/2)), x)
```

3.64 $\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=608

$$\frac{d\left(7ce-\frac{2de^2}{f}+\frac{3c^2f}{d}\right)x\sqrt{c+dx^2}}{15b\sqrt{e+fx^2}}+\frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^3\sqrt{e+fx^2}}+\frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+j}}{3b^2}$$

```
[Out] 1/5*d^2*x*(f*x^2+e)^(3/2)*(d*x^2+c)^(1/2)/b/f+1/15*d*(7*c*e-2*d*e^2/f+3*c^2
*f/d)*x*(d*x^2+c)^(1/2)/b/(f*x^2+e)^(1/2)+1/3*(-a*d+b*c)*(-3*a*d*f+4*b*c*f+
b*d*e)*x*(d*x^2+c)^(1/2)/b^3/(f*x^2+e)^(1/2)+1/15*d*e^(3/2)*(-40*a*b*c*d*f+
15*a^2*d^2*f+b^2*c*(34*c*f-d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*El
lipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1
/2)/b^3/c/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/15*(15*
a^2*d^2*f^2-5*a*b*d*f*(7*c*f+d*e)+b^2*(23*c^2*f^2+12*c*d*e*f-2*d^2*e^2))*(1
/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/
e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b^3/f^(3/2)/(e*(d*x^2+c
)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+(-a*d+b*c)^3*e^(3/2)*(1/(1+f*x^2/e))^(
1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e
/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/b^3/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x
^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d*(-a*d+b*c)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(
1/2)/b^2-2/15*d*(-3*c*f+d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/f
```

Rubi [A]
time = 0.52, antiderivative size = 776, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {559, 427, 542, 545, 429, 506, 422, 557, 553}

$$\frac{de^{3/2}\sqrt{c+dx^2}(5bc-3ad)(bc-ad)F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)^3\Pi\left(1-\frac{be}{af};\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^3c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(-3adf+4bcf+bde)E\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}(bc-ad)(-3adf+4bcf+bde)}{3b^3\sqrt{e+fx^2}}+\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)}{3b^2}+\frac{\sqrt{e}\sqrt{c+dx^2}(-3c^2f^2-7cdef+2d^2e^2)E\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15bf^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{de^{3/2}\sqrt{c+dx^2}(de-9cf)F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15bf^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{dx\sqrt{c+dx^2}\left(\frac{3c^2f}{d}+7ce-\frac{2de^2}{f}\right)}{15b\sqrt{e+fx^2}}+\frac{d^2x\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5bf}-\frac{2dx\sqrt{c+dx^2}\sqrt{e+fx^2}(de-3cf)}{15bf}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x^2)^(5/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] (d*(7*c*e - (2*d*e^2)/f + (3*c^2*f)/d)*x*Sqrt[c + d*x^2])/(15*b*Sqrt[e + f*
x^2]) + ((b*c - a*d)*(b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^3*
Sqrt[e + f*x^2]) + (d*(b*c - a*d)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b^2
) - (2*d*(d*e - 3*c*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*b*f) + (d^2*x
*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(5*b*f) - ((b*c - a*d)*Sqrt[e]*(b*d*e +
 4*b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
1 - (d*e)/(c*f)])/(3*b^3*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2]) + (Sqrt[e]*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2)*Sqrt[c + d*x^2]
*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b*f^(3/2)*Sqr
```

```
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*(5*b*c - 3*a*d)*(b
*c - a*d)*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
- (d*e)/(c*f)])/(3*b^3*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2]) - (d*e^(3/2)*(d*e - 9*c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(S
qrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c
*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)^3*e^(3/2)*Sqrt[c + d*x^2]*El
lipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a
*b^3*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

<u>Rule 422</u>

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

<u>Rule 427</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

<u>Rule 429</u>

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

<u>Rule 506</u>

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

<u>Rule 542</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 557

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[(2*b*c - a*d + b*d*x^2)*(Sqrt
[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rule 559

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rubi steps

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \frac{d\int (c+dx^2)^{3/2}\sqrt{e+fx^2}\,dx}{b} + \frac{(bc-ad)\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx}{b}$$

$$= \frac{d^2x\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}{5bf} + \frac{(d(bc-ad))\int \frac{(2bc-ad+bdx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx}{b^3} + $$

$$= \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} - \frac{2d(de-3cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15bf}$$

$$= \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} - \frac{2d(de-3cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15bf}$$

$$= \frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^3\sqrt{e+fx^2}} - \frac{(2d^2e^2-7cdef-3c^2f^2)\,x\sqrt{c}}{15bf\sqrt{e+fx^2}}$$

$$= \frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^3\sqrt{e+fx^2}} - \frac{(2d^2e^2-7cdef-3c^2f^2)\,x\sqrt{c}}{15bf\sqrt{e+fx^2}}$$

<u>**Mathematica**</u> **[C]** Result contains complex when optimal does not.
time = 5.62, size = 456, normalized size = 0.75

$$\frac{-iabde(15a^2d^2f^2-5abdf(de+7cf)+b^2(-2d^2e^2+12cdef+23c^2f^2))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ia(45a^2bcd^2f^3-15a^3d^3f^3+5ab^2df(d^2e^2-cdef-9c^2f^2)+b^3(2d^3e^3-13cd^2e^2f+11c^2def^2+15c^3f^3))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+f\left(ab^2d\sqrt{\frac{d}{c}}x(c+dx^2)(e+fx^2)(11bcf-5adf+bd(e+3fx^2))-15i(bc-ad)^3f(be-af)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)}{15ab^4\sqrt{\frac{d}{c}}f^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x^2)^(5/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] ((-I)*a*b*d*e*(15*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + 7*c*f) + b^2*(-2*d^2*e^2 +
 12*c*d*e*f + 23*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elliptic
E[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(45*a^2*b*c*d^2*f^3 - 15*a^3*d
^3*f^3 + 5*a*b^2*d*f*(d^2*e^2 - c*d*e*f - 9*c^2*f^2) + b^3*(2*d^3*e^3 - 13*
c*d^2*e^2*f + 11*c^2*d*e*f^2 + 15*c^3*f^3))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f
*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + f*(a*b^2*d*Sqrt[d
```

```
/c]*x*(c + d*x^2)*(e + f*x^2)*(11*b*c*f - 5*a*d*f + b*d*(e + 3*f*x^2)) - (1
5*I)*(b*c - a*d)^3*f*(b*e - a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*El
lipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(15*a*b^4*Sqrt
[d/c]*f^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1890 vs. $2(649) = 1298$.
time = 0.23, size = 1891, normalized size = 3.11

method	result
risch	$-\frac{dx\left(-3bd\,x^{2}f+5adf-11bcf-bde\right)\sqrt{d\,x^{2}+c}\,\sqrt{f\,x^{2}+e}}{15f\,b^{2}}+\frac{\left(-\frac{d\left(15a^{2}d^{2}f^{2}-35abcd\,f^{2}-5ab\,d^{2}ef+23b^{2}c^{2}f^{2}+12b^{2}cdef-2b^{2}d^{2}e\right.}{}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(5*(-d/c)^(1/2)*a^2*b^2*d^3*f^3*x^5-1
5*(-d/c)^(1/2)*a*b^3*c*d^2*e*f^2*x^3-45*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*b*c*d^2*f^3+45*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^
2*b^2*c^2*d*f^3-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*d^3*e^2*f-23*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*c^2*d*e*f^2-14*(
-d/c)^(1/2)*a*b^3*c*d^2*f^3*x^5-4*(-d/c)^(1/2)*a*b^3*d^3*e*f^2*x^5+5*(-d/c)
^(1/2)*a^2*b^2*c*d^2*f^3*x^3+5*(-d/c)^(1/2)*a^2*b^2*d^3*e*f^2*x^3-11*(-d/c)
^(1/2)*a*b^3*c^2*d*f^3*x^3-15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellip
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*c^3*f^3-2*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*d^3*e^3+2
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*a*b^3*d^3*e^3+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi
(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^3*c^3*f^3-15*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(
1/2)/(-d/c)^(1/2))*b^4*c^3*e*f^2-(-d/c)^(1/2)*a*b^3*d^3*e^2*f*x^3+15*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a^4*d^3*f^3-15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)
^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^4*d^3*f^3-11*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*c^2*d
```

```
*e*f^2+13*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*a*b^3*c*d^2*e^2*f+35*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*c*d^2*e*f^2-11*(-d/c)^
(1/2)*a*b^3*c^2*d*e*f^2*x-(-d/c)^(1/2)*a*b^3*c*d^2*e^2*f*x+5*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2
*d^3*e^2*f+45*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(
1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*b*c*d^2*f^3+15*((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/
c)^(1/2))*a^3*b*d^3*e*f^2-45*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipt
icPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b^2*c^2*d*f^3-3*
(-d/c)^(1/2)*a*b^3*d^3*f^3*x^7+5*(-d/c)^(1/2)*a^2*b^2*c*d^2*e*f^2*x-15*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*a^3*b*d^3*e*f^2+5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*c*d^2*e*f^2-12*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*c*d^2*e^2*f
-45*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a
/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b^2*c*d^2*e*f^2+45*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/
2))*a*b^3*c^2*d*e*f^2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/b^4/f^2/(-d/c)^(1/2)/a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{5}{2}}\sqrt{e+fx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)*sqrt(e + f*x**2)/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x^2+c)^{5/2}\,\sqrt{f\,x^2+e}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2))/(a + b*x^2),x)
```

```
[Out] int(((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2))/(a + b*x^2), x)
```

3.65 $\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=400

$$\frac{(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^2\sqrt{e+fx^2}}+\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b}-\frac{\sqrt{e}\,(bde+4bcf-3adf)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\right.\right.}{3b^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx}}$$

```
[Out] 1/3*(-3*a*d*f+4*b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)/b^2/(f*x^2+e)^(1/2)+1/3*d*(-
3*a*d+5*b*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^
(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/b^2/c/f^
(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+(-a*d+b*c)^2*e^(3/2)*
(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(1/2)/(1+f*x
^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/b^2/c/f^(1/2)/(e
*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(-3*a*d*f+4*b*c*f+b*d*e)*
(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^
2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b^2/f^(1/2)/(e*(d*x^2
+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1
/2)/b
```

Rubi [A]
time = 0.20, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {557, 553, 542, 545, 429, 506, 422}

$$\frac{de^{3/2}\sqrt{c+dx^2}\,(5bc-3ad)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{c+dx^2}\,(bc-ad)^2\Pi\left(1-\frac{be}{af};\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^2c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\sqrt{c+dx^2}\,(-3adf+4bcf+bde)E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}\,(-3adf+4bcf+bde)}{3b^2\sqrt{e+fx^2}}+\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] ((b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^2*Sqrt[e + f*x^2]) + (
d*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b) - (Sqrt[e]*(b*d*e + 4*b*c*f - 3*
a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(3*b^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
 + (d*(5*b*c - 3*a*d)*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(3*b^2*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*
x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi
[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*b^2*c*S
qrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 557

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[(2*b*c - a*d + b*d*x^2)*(Sqrt
```

```
[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rubi steps

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \frac{d\int \frac{(2bc-ad+bdx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx}{b^2} + \frac{(bc-ad)^2\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b^2}$$

$$= \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} + \frac{(bc-ad)^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{J}}{\sqrt{}}\right.\right.}{ab^2c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

$$= \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} + \frac{(bc-ad)^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{J}}{\sqrt{}}\right.\right.}{ab^2c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

$$= \frac{(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^2\sqrt{e+fx^2}} + \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} + \frac{d(5bc-3ac}{3}$$

$$= \frac{(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^2\sqrt{e+fx^2}} + \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} - \frac{\sqrt{e}(bde+}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.06, size = 346, normalized size = 0.86

$$\frac{-iabde(bde+4bcf-3adf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ia(-6abcdf^2+3a^2d^2f^2+b^2(-d^2e^2+cdef+3c^2f^2))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+f\left(ab^2d\sqrt{\frac{d}{c}}x(c+dx^2)(e+fx^2)-3i(bc-ad)^2(be-af)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)}{3ab^3\sqrt{\frac{d}{c}}f\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] ((-I)*a*b*d*e*(b*d*e + 4*b*c*f - 3*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x
^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-6*a*b*c*d*f^2
 + 3*a^2*d^2*f^2 + b^2*(-(d^2*e^2) + c*d*e*f + 3*c^2*f^2))*Sqrt[1 + (d*x^2)
/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + f*
```

```
(a*b^2*d*Sqrt[d/c]*x*(c + d*x^2)*(e + f*x^2) - (3*I)*(b*c - a*d)^2*(b*e - a
*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSi
nh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*a*b^3*Sqrt[d/c]*f*Sqrt[c + d*x^2]*Sqrt[e
 + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1058 vs. $2(459) = 918$.
time = 0.19, size = 1059, normalized size = 2.65 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((-d/c)^(1/2)*a*b^2*d^2*f^2*x^5+(-d/c)^
(1/2)*a*b^2*c*d*f^2*x^3+(-d/c)^(1/2)*a*b^2*d^2*e*f*x^3+3*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*d^2*f^2
-6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*a^2*b*c*d*f^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c^2*f^2+((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d*e*f-((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
a*b^2*d^2*e^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*a^2*b*d^2*e*f+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d*e*f+((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*d^
2*e^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b
*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*d^2*f^2+6*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a
^2*b*c*d*f^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^
(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b*d^2*e*f-3*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)
^(1/2))*a*b^2*c^2*f^2-6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(
x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c*d*e*f+3*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/
2)/(-d/c)^(1/2))*b^3*c^2*e*f+(-d/c)^(1/2)*a*b^2*c*d*e*f*x)/(d*f*x^4+c*f*x^2
+d*e*x^2+c*e)/b^3/(-d/c)^(1/2)/f/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
[Out] Integral((c + d*x**2)**(3/2)*sqrt(e + f*x**2)/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="giac")
[Out] integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x^2+c\right)}^{3/2}\,\sqrt{f\,x^2+e}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2))/(a + b*x^2),x)
[Out] int(((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2))/(a + b*x^2), x)
```

3.66 $\int \frac{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=321

$$\frac{fx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}}-\frac{\sqrt{e}\,\sqrt{f}\,\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}}+\frac{de^{3/2}\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right.}{bc\sqrt{f}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}}$$

```
[Out] f*x*(d*x^2+c)^(1/2)/b/(f*x^2+e)^(1/2)+d*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*
x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2
))*(d*x^2+c)^(1/2)/b/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1
/2)+(-a*d+b*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x
*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(
1/2)/a/b/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(1/(1+f*
x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/
2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/b/(e*(d*x^2+c)/c/(f*x
^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {549, 433, 429, 506, 422, 553}

$$\frac{e^{3/2}\sqrt{c+dx^2}\,(bc-ad)\Pi\left(1-\frac{be}{af};\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\,\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{de^{3/2}\sqrt{c+dx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bc\sqrt{f}\,\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\,\sqrt{f}\,\sqrt{c+dx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{fx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] (f*x*Sqrt[c + d*x^2])/(b*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2
]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*Sqrt[(e*(c +
d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*e^(3/2)*Sqrt[c + d*x^2]*Elli
pticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*c*Sqrt[f]*Sqrt[(e*(
c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)*e^(3/2)*Sqrt[c
 + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(a*b*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x
^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 549

```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Dist[d/b, Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
Dist[(b*c - a*d)/b, Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-f/e, -d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{a+bx^2}\,dx = \frac{d\int \frac{\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx}{b} + \frac{(bc-ad)\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b}$$

$$= \frac{(bc-ad)e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}} + \frac{(de)\int \ldots}{\sqrt{c+d\ldots}}$$

$$= \frac{fx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}} + \frac{de^{3/2}\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}} + \frac{(bc-ad)e^{3/2}\ldots}{a\ldots}$$

$$= \frac{fx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}} - \frac{\sqrt{e}\,\sqrt{f}\,\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}} + \frac{de^{3/2}\sqrt{c+\ldots}}{bc\sqrt{\ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.72, size = 184, normalized size = 0.57

$$-\frac{i\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\left(abdeE\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+(bc-ad)\left(afF\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+(be-af)\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)\right)\right)}{ab^2\sqrt{\frac{d}{c}}\,\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*b*d*e*EllipticE[I*ArcSinh[
Sqrt[d/c]*x], (c*f)/(d*e)] + (b*c - a*d)*(a*f*EllipticF[I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)] + (b*e - a*f)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)])))/(a*b^2*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.14, size = 340, normalized size = 1.06

method	result

default	$\frac{\sqrt{d x^2+c}\,\sqrt{f x^2+e}\,\sqrt{\frac{d x^2+c}{c}}\,\sqrt{\frac{f x^2+e}{e}}\left(\text{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abde+\text{EllipticPi}\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\cdots\right.\right.}{\cdots}$
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(-\frac{\sqrt{1+\frac{d x^2}{c}}\,\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)adf}{b^2\sqrt{-\frac{d}{c}}\,\sqrt{df x^4+cf x^2+de x^2+ce}}+\frac{\sqrt{1+\frac{d x^2}{c}}\cdots}{b\sqrt{\cdots}}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*(El
lipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d*e+EllipticPi(x*(-d/c)^(1/2),b
*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*d*f-EllipticPi(x*(-d/c)^(1/2),b*c/a/d
,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c*f-EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e
)^(1/2)/(-d/c)^(1/2))*a*b*d*e+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2
)/(-d/c)^(1/2))*b^2*c*e-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d*f+E
llipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*f)/(d*f*x^4+c*f*x^2+d*e*x^2+
c*e)/b^2/(-d/c)^(1/2)/a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
[Out] Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2),x)
[Out] int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2), x)
```

3.67 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=102

$$\frac{e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}}$$

```
[Out] e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(1/2
)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/c/f^(1/2
)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {553}

$$\frac{e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\,\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqr
t[e]], 1 - (d*e)/(c*f)])/(a*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]
*Sqrt[e + f*x^2])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.85, size = 143, normalized size = 1.40

$$-\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(afF\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+(be-af)\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)\right)}{ab\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*f*EllipticF[I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)] + (b*e - a*f)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)]))/(a*b*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.13, size = 191, normalized size = 1.87

method	result
default	$\frac{\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)af-\mathrm{EllipticPi}\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)af+\mathrm{EllipticPi}\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)be\right)\sqrt{\frac{fx^2+e}{e}}}{ba\sqrt{-\frac{d}{c}}\,(dfx^4+cfx^2+dex^2+ce)}$
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{f\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{b\sqrt{-\frac{d}{c}}\sqrt{dfx^4+cfx^2+dex^2+ce}}-\frac{\sqrt{1+\frac{dx^2}{c}}\sqrt{1+}}{b\sqrt{-\frac{d}{c}}\sqrt{}}\right.}{\sqrt{dx^2+c}\sqrt{fx^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*f-EllipticPi(x*(-d/c)^(1/2),b*
c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*f+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/
e)^(1/2)/(-d/c)^(1/2))*b*e)*((f*x^2+e)/e)^(1/2)*((d*x^2+c)/c)^(1/2)/b*(d*x^
2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(-d/c)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e + fx^2}}{(a + bx^2)\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(e + f*x**2)/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{f\,x^2+e}}{\left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
[Out] int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

3.68 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{\sqrt{d}\,\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\,(bc-ad)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{be^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] b*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(1
/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/c/(-a*
d+b*c)/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(1/(1+d*x^2/
c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(
1-c*f/d/e)^(1/2))*d^(1/2)*(f*x^2+e)^(1/2)/(-a*d+b*c)/c^(1/2)/(d*x^2+c)^(1/2
)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {555, 553, 422}

$$\frac{be^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\,\sqrt{e+fx^2}\,(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{d}\,\sqrt{e+fx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\,\sqrt{c+dx^2}\,(bc-ad)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((Sqrt[d]*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)
/(d*e)])/(Sqrt[c]*(b*c - a*d)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c +
d*x^2))])) + (b*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[
(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)*Sqrt[f]*Sqrt[(e*(c
 + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 555

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx &= \frac{b\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{bc-ad} - \frac{d\int \frac{\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx}{bc-ad}\\ &= -\frac{\sqrt{d}\,\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\,(bc-ad)\sqrt{c+dx^2}\,\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{be^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan\right.}{ac(bc-ad)\sqrt{f}\,\sqrt{\frac{e(c+}{c(e+}}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.79, size = 347, normalized size = 1.66

$$\frac{\sqrt{\frac{d}{c}}\left(ad\sqrt{\frac{d}{c}}\,ex+ad\sqrt{\frac{d}{c}}\,fx^3+iade\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+ia(-de+cf)\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+ibce\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)-iacf\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)\right)}{ad(-bc+ad)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*(a*d*Sqrt[d/c]*e*x + a*d*Sqrt[d/c]*f*x^3 + I*a*d*e*Sqrt[1 + (d*x
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] +
 I*a*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*Arc
Sinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*b*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x
^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*c
*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSin
h[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*d*(-(b*c) + a*d)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2])
```

Maple [A]
time = 0.16, size = 390, normalized size = 1.87

method	result
default	$\frac{\left(\sqrt{-\frac{d}{c}}\, adf\, x^{3}-\sqrt{\frac{d\, x^{2}+c}{c}}\, \sqrt{\frac{f\, x^{2}+e}{e}}\, \mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}}, \sqrt{\frac{cf}{de}}\right) acf+\sqrt{\frac{d\, x^{2}+c}{c}}\, \sqrt{\frac{f\, x^{2}+e}{e}}\, \mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}}, \sqrt{\cdots}\right.\right.}{\cdots}$
elliptic	$\frac{\sqrt{(d\, x^{2}+c)\,(f\, x^{2}+e)}\left(\frac{(df\, x^{2}+de)\,x}{c(ad-bc)\sqrt{\left(x^{2}+\frac{c}{d}\right)(df\, x^{2}+de)}}-\frac{\sqrt{1+\frac{d\, x^{2}}{c}}\, \sqrt{1+\frac{f\, x^{2}}{e}}\, \mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}}, \cdots\right.}{\sqrt{-\frac{d}{c}}\, \sqrt{df\, x^{4}+cf\, x^{2}+de\, x^{2}+\cdots}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-d/c)^(1/2)*a*d*f*x^3-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e-((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e+((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/
(-d/c)^(1/2))*a*c*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-
d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c*e+(-d/c)^(1/2)*a*d*e*x)*(
d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/c/a/(-d/c)^(1/2)/(a*d-b*c)/(d*f*x^4+c*f*x^2+
d*e*x^2+c*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e + fx^2}}{(a + bx^2)\,(c + dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(e + f*x**2)/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f\,x^2+e}}{\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.69 $\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=401

$$-\frac{dx\sqrt{e+fx^2}}{3c(bc-ad)\left(c+dx^2\right)^{3/2}}-\frac{\sqrt{d}\,(bc(5de-4cf)-ad(2de-cf))\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}(bc-ad)^2(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}+\frac{de}{3}$$

```
[Out] b^2*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^
(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/c/(-
a*d+b*c)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d*e^
(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(
1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^2/(-a*d+b*c)/
(-c*f+d*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*d*x*(f*x^2+e
)^(1/2)/c/(-a*d+b*c)/(d*x^2+c)^(3/2)-1/3*(b*c*(-4*c*f+5*d*e)-a*d*(-c*f+2*d*
e))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+
d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*d^(1/2)*(f*x^2+e)^(1/2)/c^(3/2)/(-a*d+b*c
)^2/(-c*f+d*e)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {560, 553, 540, 539, 429, 422}

$$\frac{b^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\,\sqrt{e+fx^2}\,(bc-ad)^2\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{d}\,\sqrt{e+fx^2}\,(bc(5de-4cf)-ad(2de-cf))E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{c+dx^2}\,(bc-ad)^2(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}+\frac{de^{3/2}\sqrt{f}\,\sqrt{c+dx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}\,(bc-ad)(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{dx\sqrt{e+fx^2}}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/3*(d*x*Sqrt[e + f*x^2])/(c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (Sqrt[d]*(b*
c*(5*d*e - 4*c*f) - a*d*(2*d*e - c*f))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*(b*c - a*d)^2*(d*e - c*f)*S
qrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + (d*e^(3/2)*Sqrt[f]*
Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3
*c^2*(b*c - a*d)*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2]) + (b^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[
(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*Sqrt[(e*
(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{5/2}}\,dx = \frac{b^2 \int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{(bc-ad)^2} - \frac{d \int \frac{(2bc-ad+bdx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{dx\sqrt{e+fx^2}}{3c(bc-ad)\,(c+dx^2)^{3/2}} + \frac{b^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\right.}{ac(bc-ad)^2\sqrt{f}\,\sqrt{\frac{e\,(c+dx^2)}{c\,(e+fx^2)}}\,\sqrt{e+fx^2}}$$

$$= -\frac{dx\sqrt{e+fx^2}}{3c(bc-ad)\,(c+dx^2)^{3/2}} + \frac{b^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\right.}{ac(bc-ad)^2\sqrt{f}\,\sqrt{\frac{e\,(c+dx^2)}{c\,(e+fx^2)}}\,\sqrt{e+fx^2}}$$

$$= -\frac{dx\sqrt{e+fx^2}}{3c(bc-ad)\,(c+dx^2)^{3/2}} - \frac{\sqrt{d}\,(bc(5de-4cf)-ad(2de-cf))\sqrt{e+fx^2}}{3c^{3/2}(bc-ad)^2(de-cf)\sqrt{c+dx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.45, size = 427, normalized size = 1.06

$$\frac{ac\left(\frac{d}{c}\right)^{3/2}x(e+fx^2)\,(bc(6cde-5c^2f+5d^2ex^2-4cdfx^2)+ad(-3cde+2c^2f-2d^2ex^2+cdfx^2)) - iade(ad(2de-cf)+bc(-5de+4cf))\,(c+dx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right) - ia(-de+cf)\,(2ad^2e+bc(-5de+3cf))\,(c+dx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right) - 3ibc^2(be-af)(-de+cf)\,(c+dx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)}{3ac^2\sqrt{\frac{d}{c}}\,(bc-ad)^2(-de+cf)\,(c+dx^2)^{3/2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (a*c*(d/c)^(3/2)*x*(e + f*x^2)*(b*c*(6*c*d*e - 5*c^2*f + 5*d^2*e*x^2 - 4*c*
d*f*x^2) + a*d*(-3*c*d*e + 2*c^2*f - 2*d^2*e*x^2 + c*d*f*x^2)) - I*a*d*e*(a
*d*(2*d*e - c*f) + b*c*(-5*d*e + 4*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sq
rt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-(d
*e) + c*f)*(2*a*d^2*e + b*c*(-5*d*e + 3*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/
c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*
I)*b*c^2*(b*e - a*f)*(-(d*e) + c*f)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/
(3*a*c^2*Sqrt[d/c]*(b*c - a*d)^2*(-(d*e) + c*f)*(c + d*x^2)^(3/2)*Sqrt[e +
f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2067 vs. $2(466) = 932$.
time = 0.16, size = 2068, normalized size = 5.16

method	result	size
elliptic	Expression too large to display	1366
default	Expression too large to display	2068

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*a^2*d^4*e^2*x^2+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*El
lipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d^4*e^2*x^2-4*(-d/c)^(1/2)*a*b*
c^2*d^2*f^2*x^5-2*(-d/c)^(1/2)*a^2*c*d^3*e*f*x^3-5*(-d/c)^(1/2)*a*b*c^3*d*f
^2*x^3+5*(-d/c)^(1/2)*a*b*c*d^3*e^2*x^3+2*(-d/c)^(1/2)*a^2*c^2*d^2*e*f*x+6*
(-d/c)^(1/2)*a*b*c^2*d^2*e^2*x-5*(-d/c)^(1/2)*a*b*c^3*d*e*f*x-3*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)
/(-d/c)^(1/2))*b^2*c^2*d^2*e^2*x^2+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c^2*d^2*e*f+5*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^
2*d^2*e^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*a^2*c^2*d^2*e*f-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^2*d^2*e^2+5*(-d/c)^(1/2)*a*
b*c*d^3*e*f*x^5+2*(-d/c)^(1/2)*a*b*c^2*d^2*e*f*x^3+(-d/c)^(1/2)*a^2*c*d^3*f
^2*x^5-2*(-d/c)^(1/2)*a^2*d^4*e*f*x^5+2*(-d/c)^(1/2)*a^2*c^2*d^2*f^2*x^3-3*
(-d/c)^(1/2)*a^2*c*d^3*e^2*x+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^4*e*f-3*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e
)^(1/2)/(-d/c)^(1/2))*b^2*c^3*d*e^2-2*(-d/c)^(1/2)*a^2*d^4*e^2*x^3-2*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a^2*c*d^3*e^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*a*b*c^4*f^2+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^3*e^2-3*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-
d/c)^(1/2))*a*b*c^4*f^2-8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^2*d^2*e*f*x^2+4*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^2*d^2*e
*f*x^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),
b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c^2*d^2*e*f*x^2+2*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^3*e
*f*x^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*a*b*c^3*d*f^2*x^2+5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*d^3*e^2*x^2-((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d
^3*e*f*x^2-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a*b*c*d^3*e^2*x^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
```

```
1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c^3*d
*f^2*x^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2
),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^3*d*e*f*x^2-8*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^3*d*e
*f+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*a*b*c^3*d*e*f+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipti
cPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c^3*d*e*f)/(f*x^2
+e)^(1/2)/(a*d-b*c)^2/c^2/(c*f-d*e)/(-d/c)^(1/2)/a/(d*x^2+c)^(3/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(5/2),x)
[Out] Integral(sqrt(e + f*x**2)/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f\,x^2+e}}{\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

3.70 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=630

$$-\frac{dx\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}}-\frac{d(bc(9de-8cf)-ad(4de-3cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(de-cf)\left(c+dx^2\right)^{3/2}}-\frac{b^2\sqrt{d}\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\right.\right.}{\sqrt{c}\,(bc-ad)^3\sqrt{c+dx^2}\sqrt{}}$$

```
[Out] b^3*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^
(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/c/(-
a*d+b*c)^3/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/15*d*e
^(3/2)*(b*c*(-11*c*f+9*d*e)-2*a*d*(-3*c*f+2*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+
f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1
/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^3/(-a*d+b*c)^2/(-c*f+d*e)^2/(e*(d*x^2+c)/c/(
f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/5*d*x*(f*x^2+e)^(1/2)/c/(-a*d+b*c)/(d*x^2
+c)^(5/2)-1/15*d*(b*c*(-8*c*f+9*d*e)-a*d*(-3*c*f+4*d*e))*x*(f*x^2+e)^(1/2)/
c^2/(-a*d+b*c)^2/(-c*f+d*e)/(d*x^2+c)^(3/2)+1/15*(a*d*(3*c^2*f^2-13*c*d*e*f
+8*d^2*e^2)-2*b*c*(4*c^2*f^2-14*c*d*e*f+9*d^2*e^2))*(1/(1+d*x^2/c))^(1/2)*(
1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^
(1/2))*d^(1/2)*(f*x^2+e)^(1/2)/c^(5/2)/(-a*d+b*c)^2/(-c*f+d*e)^2/(d*x^2+c)^
(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)-b^2*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)
^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*d^(
1/2)*(f*x^2+e)^(1/2)/(-a*d+b*c)^3/c^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d
*x^2+c))^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {560, 555, 553, 422, 540, 541, 539, 429}

$$\frac{b^3e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{b^2\sqrt{d}\sqrt{e+fx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{de^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc(9de-11cf)-2ad(2de-3cf))F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{d}\sqrt{e+fx^2}\left(ad(3c^2f^2-13cdef+8d^2e^2)-2bc(4c^2f^2-14cdef+9d^2e^2)\right)E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{c+dx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{dx\sqrt{e+fx^2}(bc(9de-8cf)-ad(4de-3cf))}{15c^2(c+dx^2)^{3/2}(bc-ad)^2(de-cf)}-\frac{dx\sqrt{e+fx^2}}{5c(c+dx^2)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

```
[Out] -1/5*(d*x*Sqrt[e + f*x^2])/(c*(b*c - a*d)*(c + d*x^2)^(5/2)) - (d*(b*c*(9*d
*e - 8*c*f) - a*d*(4*d*e - 3*c*f))*x*Sqrt[e + f*x^2])/(15*c^2*(b*c - a*d)^2
*(d*e - c*f)*(c + d*x^2)^(3/2)) - (b^2*Sqrt[d]*Sqrt[e + f*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*(b*c - a*d)^3*Sqrt[c
+ d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + (Sqrt[d]*(a*d*(8*d^2*e^2
- 13*c*d*e*f + 3*c^2*f^2) - 2*b*c*(9*d^2*e^2 - 14*c*d*e*f + 4*c^2*f^2))*Sqr
```

```
t[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*c
^(5/2)*(b*c - a*d)^2*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*
(c + d*x^2))]) + (d*e^(3/2)*Sqrt[f]*(b*c*(9*d*e - 11*c*f) - 2*a*d*(2*d*e -
3*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c
*f)])/(15*c^3*(b*c - a*d)^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^
2))]*Sqrt[e + f*x^2]) + (b^3*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(
a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^3*Sqr
t[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
```

```
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 555

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \frac{b^2\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx}{(bc-ad)^2} - \frac{d\int \frac{(2bc-ad+bdx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{dx\sqrt{e+fx^2}}{5c(bc-ad)(c+dx^2)^{5/2}} + \frac{b^3\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{(bc-ad)^3} - \frac{(b^2d)\int \frac{\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,d}{(bc-ad)^3}$$

$$= -\frac{dx\sqrt{e+fx^2}}{5c(bc-ad)(c+dx^2)^{5/2}} - \frac{d(bc(9de-8cf)-ad(4de-3cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(de-cf)(c+dx^2)^{3/2}} -$$

$$= -\frac{dx\sqrt{e+fx^2}}{5c(bc-ad)(c+dx^2)^{5/2}} - \frac{d(bc(9de-8cf)-ad(4de-3cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(de-cf)(c+dx^2)^{3/2}} -$$

$$= -\frac{dx\sqrt{e+fx^2}}{5c(bc-ad)(c+dx^2)^{5/2}} - \frac{d(bc(9de-8cf)-ad(4de-3cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(de-cf)(c+dx^2)^{3/2}} -$$

<u>**Mathematica**</u> [**C**] Result contains complex when optimal does not.
time = 6.06, size = 584, normalized size = 0.93

$$\frac{-ad\sqrt{\frac{d}{c}}x(e+fx^2)\left(3c^2(bc-ad)^2(de-cf)^2+c(bc-ad)(-de+cf)(ad(4de-3cf)+bc(-9de+8cf))(c+dx^2)+(abcd(-26d^2e^2+41cdef-11c^2f^2)+a^2d^2(8d^2e^2-13cdef+3c^2f^2)+b^2c^2(33d^2e^2-58cdef+23c^2f^2))(c+dx^2)^2\right)-i(c+dx^2)^2\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(ade(abcd(-26d^2e^2+41cdef-11c^2f^2)+a^2d^2(8d^2e^2-13cdef+3c^2f^2)+b^2c^2(33d^2e^2-58cdef+23c^2f^2))E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-(de-cf)\left(-a(2abcd^2e(13de-14cf)+a^2d^3e(-8de+9cf)+b^2c^2(-33d^2e^2+49cdef-15c^2f^2))F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+15b^2c^3(be-af)(-de+cf)\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)\right)}{15ac^3\sqrt{\frac{d}{c}}(bc-ad)^3(de-cf)^2(c+dx^2)^{5/2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

```
[Out] (-(a*d*Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f)^2 + c*(b*c
- a*d)*(-(d*e) + c*f)*(a*d*(4*d*e - 3*c*f) + b*c*(-9*d*e + 8*c*f))*(c + d*x
^2) + (a*b*c*d*(-26*d^2*e^2 + 41*c*d*e*f - 11*c^2*f^2) + a^2*d^2*(8*d^2*e^2
 - 13*c*d*e*f + 3*c^2*f^2) + b^2*c^2*(33*d^2*e^2 - 58*c*d*e*f + 23*c^2*f^2)
)*(c + d*x^2)^2)) - I*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]
*(a*d*e*(a*b*c*d*(-26*d^2*e^2 + 41*c*d*e*f - 11*c^2*f^2) + a^2*d^2*(8*d^2*e
^2 - 13*c*d*e*f + 3*c^2*f^2) + b^2*c^2*(33*d^2*e^2 - 58*c*d*e*f + 23*c^2*f^
2))*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (d*e - c*f)*(-(a*(2*a*
b*c*d^2*e*(13*d*e - 14*c*f) + a^2*d^3*e*(-8*d*e + 9*c*f) + b^2*c^2*(-33*d^2
*e^2 + 49*c*d*e*f - 15*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d
*e)]) + 15*b^2*c^3*(b*e - a*f)*(-(d*e) + c*f)*EllipticPi[(b*c)/(a*d), I*Arc
```

```
Sinh[Sqrt[d/c]*x], (c*f)/(d*e)])))/(15*a*c^3*Sqrt[d/c]*(b*c - a*d)^3*(d*e -
 c*f)^2*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6244 vs. $2(714) = 1428$.
time = 0.18, size = 6245, normalized size = 9.91

method	result	size
elliptic	Expression too large to display	3138
default	Expression too large to display	6245

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e + fx^2}}{(a + bx^2)\,(c + dx^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(7/2),x)
```

```
[Out] Integral(sqrt(e + f*x**2)/((a + b*x**2)*(c + d*x**2)**(7/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="giac")
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f\,x^2+e}}{\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(7/2)),x)
[Out] int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(7/2)), x)
```

3.71 $\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=659

$$\frac{(bc-ad)^2 f^2 x\sqrt{c+dx^2}}{b^3 d\sqrt{e+fx^2}}+\frac{2(bc-ad)f(2de-cf)x\sqrt{c+dx^2}}{3b^2 d\sqrt{e+fx^2}}+\frac{(3d^2e^2+7cdef-2c^2f^2)\,x\sqrt{c+dx^2}}{15bd\sqrt{e+fx^2}}+\frac{(bc-$$

```
[Out] (-a*d+b*c)^2*f^2*x*(d*x^2+c)^(1/2)/b^3/d/(f*x^2+e)^(1/2)+2/3*(-a*d+b*c)*f*(
-c*f+2*d*e)*x*(d*x^2+c)^(1/2)/b^2/d/(f*x^2+e)^(1/2)+1/15*(-2*c^2*f^2+7*c*d*
e*f+3*d^2*e^2)*x*(d*x^2+c)^(1/2)/b/d/(f*x^2+e)^(1/2)+1/15*e^(3/2)*(15*a^2*d
^2*f+3*b^2*c*(3*c*f+8*d*e)-5*a*b*d*(5*c*f+3*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+
f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1
/2))*(d*x^2+c)^(1/2)/b^3/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e
)^(1/2)+(-a*d+b*c)^2*e^(3/2)*(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(
1/2)*EllipticPi(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(
1/2))*(d*x^2+c)^(1/2)/a/b^3/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^
2+e)^(1/2)-1/15*(15*a^2*d^2*f^2-20*a*b*d*f*(c*f+d*e)+3*b^2*(c^2*f^2+9*c*d*e
*f+d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^
(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b^3/d/f^
(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/5*f*x*(d*x^2+c)^(3/
2)*(f*x^2+e)^(1/2)/b+1/3*(-a*d+b*c)*f*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b^2
+2/15*(-c*f+3*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b
```

Rubi [A]
time = 0.50, antiderivative size = 784, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {559, 427, 542, 545, 429, 506, 422, 557, 553}

$$\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)(be-af)^2\Pi\left(1-\frac{bc}{ad};\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^3\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(bc-ad)(5be-3af)F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(bc-ad)(-3adf+bcf+4bde)E\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3d\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{fx\sqrt{c+dx^2}(bc-ad)(-3adf+bcf+4bde)}{3b^3d\sqrt{e+fx^2}}+\frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)}{3b^2}-\frac{\sqrt{e}\sqrt{c+dx^2}(-2c^2f^2+7cdef+3d^2e^2)E\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15bd\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{c+dx^2}(9de-cf)F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15b\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}(-2c^2f^2+7cdef+3d^2e^2)}{15bd\sqrt{e+fx^2}}+\frac{fx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5b}+\frac{2x\sqrt{c+dx^2}\sqrt{e+fx^2}(3de-cf)}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*f*(4*b*d*e + b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^3*d*Sqrt
[e + f*x^2]) + ((3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2)*x*Sqrt[c + d*x^2])/(15*
b*d*Sqrt[e + f*x^2]) + ((b*c - a*d)*f*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3
*b^2) + (2*(3*d*e - c*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*b) + (f*x*(
c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*b) - ((b*c - a*d)*Sqrt[e]*Sqrt[f]*(4*b
*d*e + b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(3*b^3*d*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
 + f*x^2]) - (Sqrt[e]*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2)*Sqrt[c + d*x^2]*E
```

```
llipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b*d*Sqrt[f]*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)*Sqrt[e]*
Sqrt[f]*(5*b*e - 3*a*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(3*b^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2]) + (e^(3/2)*(9*d*e - c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]
*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*(b*e - a*f)^2*Sqrt[e + f*x
^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e
)])/(a*b^3*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
```

```
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 557

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[(2*b*c - a*d + b*d*x^2)*(Sqrt
[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rule 559

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rubi steps

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \frac{d\int \sqrt{c+dx^2}\,(e+fx^2)^{3/2}\,dx}{b} + \frac{(bc-ad)\int \frac{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}{a+bx^2}\,dx}{b}$$

$$= \frac{fx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5b} + \frac{\int \frac{\sqrt{c+dx^2}\,(e(5de-cf)+2f(3de-cf)x^2)}{\sqrt{e+fx^2}}\,dx}{5b} + \frac{((bc}{}$$

$$= \frac{(bc-ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} + \frac{2(3de-cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15b} +$$

$$= \frac{(bc-ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} + \frac{2(3de-cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15b} +$$

$$= \frac{(bc-ad)f(4bde+bcf-3adf)x\sqrt{c+dx^2}}{3b^3d\sqrt{e+fx^2}} + \frac{(3d^2e^2+7cdef-2c^2f^2)\,x\sqrt{}}{15bd\sqrt{e+fx^2}}$$

$$= \frac{(bc-ad)f(4bde+bcf-3adf)x\sqrt{c+dx^2}}{3b^3d\sqrt{e+fx^2}} + \frac{(3d^2e^2+7cdef-2c^2f^2)\,x\sqrt{}}{15bd\sqrt{e+fx^2}}$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 6.82, size = 445, normalized size = 0.68

$$\frac{-iabe(15a^2d^2f^2-20abdf(de+cf)+3b^2(d^2e^2+9cdef+c^2f^2))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ia(-15a^3d^2f^3+15a^2bdf^2(de+2cf)-3b^3e(d^2e^2+cdef-7c^2f^2)+5ab^2f(d^2e^2-7cdef-3c^2f^2))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+f\left(ab^2\sqrt{\frac{d}{c}}x(c+dx^2)(e+fx^2)(-5adf+3b(2de+2cf+dfx^2))-15i(bc-ad)^2(be-af)^2\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)}{15ab^4\sqrt{\frac{d}{c}}f\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((-I)*a*b*e*(15*a^2*d^2*f^2 - 20*a*b*d*f*(d*e + c*f) + 3*b^2*(d^2*e^2 + 9*c
*d*e*f + c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcS
inh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-15*a^3*d^2*f^3 + 15*a^2*b*d*f^2*(d*e
 + 2*c*f) - 3*b^3*e*(d^2*e^2 + c*d*e*f - 7*c^2*f^2) + 5*a*b^2*f*(d^2*e^2 -
7*c*d*e*f - 3*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I
*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + f*(a*b^2*Sqrt[d/c]*x*(c + d*x^2)*(e +
 f*x^2)*(-5*a*d*f + 3*b*(2*d*e + 2*c*f + d*f*x^2)) - (15*I)*(b*c - a*d)^2*(
```

```
b*e - a*f)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d)
, I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(15*a*b^4*Sqrt[d/c]*f*Sqrt[c + d*x
^2]*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1938 vs. $2(696) = 1392$.

time = 0.23, size = 1939, normalized size = 2.94

method	result
risch	$-\frac{x\left(-3bd\,x^{2}f+5adf-6bcf-6bde\right)\sqrt{d\,x^{2}+c}\,\sqrt{f\,x^{2}+e}}{15b^{2}}+\frac{\left(-\frac{\left(15a^{2}d^{2}f^{2}-20abcd\,f^{2}-20ab\,d^{2}ef+3b^{2}c^{2}f^{2}+27b^{2}cdef+3b^{2}d^{2}e^{2}\right.}{}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(-15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^4*d^2*f^3+15*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)
/(-d/c)^(1/2))*a^4*d^2*f^3-35*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellip
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*c*d*e*f^2-3*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*c*d*e
^2*f-20*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*a^2*b^2*c*d*e*f^2+27*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*c*d*e^2*f+60*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(
-d/c)^(1/2))*a^2*b^2*c*d*e*f^2+15*(-d/c)^(1/2)*a*b^3*c*d*e*f^2*x^3+15*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)
^(1/2)/(-d/c)^(1/2))*a^2*b^2*d^2*e^2*f-5*(-d/c)^(1/2)*a^2*b^2*d^2*e*f^2*x^3
+6*(-d/c)^(1/2)*a*b^3*d^2*e^2*f*x^3+6*(-d/c)^(1/2)*a*b^3*c^2*e*f^2*x+3*(-d/
c)^(1/2)*a*b^3*d^2*f^3*x^7+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipt
icE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*c^2*e*f^2-30*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(
1/2))*a^3*b*c*d*f^3-30*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x
*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*b*d^2*e*f^2-30*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(
1/2)/(-d/c)^(1/2))*a*b^3*c*d*e^2*f-5*(-d/c)^(1/2)*a^2*b^2*d^2*f^3*x^5+6*(-d
/c)^(1/2)*a*b^3*c^2*f^3*x^3+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
```

```
pticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^4*c^2*e^2*f-30*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-
f/e)^(1/2)/(-d/c)^(1/2))*a*b^3*c^2*e*f^2-15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*c^2*f^3-3*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a*b^3*d^2*e^3-5*(-d/c)^(1/2)*a^2*b^2*c*d*e*f^2*x+6*(-d/c)^(1/2)*a*b^3*c*d*
e^2*f*x+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*a*b^3*d^2*e^3+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b^2*c^2*f^3
+30*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d
/e)^(1/2))*a^3*b*c*d*f^3+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipti
cF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*b*d^2*e*f^2+5*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*d^2*e^2
*f+21*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*a*b^3*c^2*e*f^2+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ell
ipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*b*d^2*e*f^2-20*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*d^
2*e^2*f+9*(-d/c)^(1/2)*a*b^3*c*d*f^3*x^5+9*(-d/c)^(1/2)*a*b^3*d^2*e*f^2*x^5
-5*(-d/c)^(1/2)*a^2*b^2*c*d*f^3*x^3)/f/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/b^4/(-
d/c)^(1/2)/a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{3}{2}}\,(e+fx^2)^{\frac{3}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x^2+c\right)}^{3/2}\,{\left(f\,x^2+e\right)}^{3/2}}{b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2),x)
```

```
[Out] int(((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2), x)
```

3.72 $\int \frac{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=403

$$\frac{f(4bde+bcf-3adf)x\sqrt{c+dx^2}}{3b^2d\sqrt{e+fx^2}}+\frac{fx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3b}-\frac{\sqrt{e}\,\sqrt{f}\,(4bde+bcf-3adf)\sqrt{c+dx^2}\,E\Big(\tan}{3b^2d\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+f}}$$

```
[Out] 1/3*f*(-3*a*d*f+b*c*f+4*b*d*e)*x*(d*x^2+c)^(1/2)/b^2/d/(f*x^2+e)^(1/2)-1/3*
(-3*a*d*f+b*c*f+4*b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(
x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x
^2+c)^(1/2)/b^2/d/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*(-3*a
*f+5*b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/
2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/b^2
/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*f*x*(d*x^2+c)^(1/2)*(f
*x^2+e)^(1/2)/b+c^(3/2)*(-a*f+b*e)^2*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2
)*EllipticPi(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2
))*(f*x^2+e)^(1/2)/a/b^2/e/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c)
)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {557, 553, 542, 545, 429, 506, 422}

$$\frac{c^{3/2}\sqrt{e+fx^2}\,(be-af)^2\Pi\left(1-\frac{bc}{ad};\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}\,e\sqrt{c+dx^2}\,\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{\sqrt{e}\,\sqrt{f}\,\sqrt{c+dx^2}\,(5be-3af)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\,\sqrt{f}\,\sqrt{c+dx^2}\,(-3adf+bcf+4bde)E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2d\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{fx\sqrt{c+dx^2}\,(-3adf+bcf+4bde)}{3b^2d\sqrt{e+fx^2}}+\frac{fx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (f*(4*b*d*e + b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^2*d*Sqrt[e + f*x^2])
 + (f*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b) - (Sqrt[e]*Sqrt[f]*(4*b*d*e
+ b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
 - (d*e)/(c*f)])/(3*b^2*d*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*
x^2]) + (Sqrt[e]*Sqrt[f]*(5*b*e - 3*a*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b^2*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*e - a*f)^2*Sqrt[e + f*x^2]*Elliptic
Pi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*b^2*S
qrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 557

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[(2*b*c - a*d + b*d*x^2)*(Sqrt
```

```
[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}{a+bx^2}\,dx = \frac{f\int \frac{\sqrt{c+dx^2}\,(2be-af+bfx^2)}{\sqrt{e+fx^2}}\,dx}{b^2} + \frac{(be-af)^2\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{b^2}$$

$$= \frac{fx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3b} + \frac{c^{3/2}(be-af)^2\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}\ldots}{\sqrt{c}}\right.\right.}{ab^2\sqrt{d}\,e\sqrt{c+dx^2}\,\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

$$= \frac{fx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3b} + \frac{c^{3/2}(be-af)^2\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}\ldots}{\sqrt{c}}\right.\right.}{ab^2\sqrt{d}\,e\sqrt{c+dx^2}\,\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

$$= \frac{f(4bde+bcf-3adf)x\sqrt{c+dx^2}}{3b^2d\sqrt{e+fx^2}} + \frac{fx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3b} + \frac{\sqrt{e}\,\sqrt{f}\,(5b\ldots}{\ldots}$$

$$= \frac{f(4bde+bcf-3adf)x\sqrt{c+dx^2}}{3b^2d\sqrt{e+fx^2}} + \frac{fx\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3b} - \frac{\sqrt{e}\,\sqrt{f}\,(4b\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.71, size = 739, normalized size = 1.83

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (a*b^2*c*Sqrt[d/c]*e*f*x + a*b^2*d*Sqrt[d/c]*e*f*x^3 + a*b^2*c*Sqrt[d/c]*f^
2*x^3 + a*b^2*d*Sqrt[d/c]*f^2*x^5 - I*a*b*e*(4*b*d*e + b*c*f - 3*a*d*f)*Sqr
t[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f
)/(d*e)] - I*a*(3*a^2*d*f^2 - 3*a*b*f*(d*e + c*f) + b^2*e*(-(d*e) + 4*c*f))
*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x],
```

```
(c*f)/(d*e)] - (3*I)*b^3*c*e^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elli
pticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (3*I)*a*b^2*d*e^
2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh
[Sqrt[d/c]*x], (c*f)/(d*e)] + (6*I)*a*b^2*c*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] -
 (6*I)*a^2*b*d*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)
/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*a^2*b*c*f^2*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x
], (c*f)/(d*e)] + (3*I)*a^3*d*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*a*b^3*Sqrt[
d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(462) = 924$.

time = 0.18, size = 1028, normalized size = 2.55

method	result
risch	$\frac{f x \sqrt{d x^2+c}\,\sqrt{f x^2+e}}{3b} - \frac{\left(-\frac{\left(3abd f^2-b^2 c f^2-4b^2 def\right) e \sqrt{1+\frac{d x^2}{c}}\,\sqrt{1+\frac{f x^2}{e}}\left(\text{EllipticF}\left(x\sqrt{-\frac{d}{c}}, \sqrt{-1+\cdots}\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{df x^4+cf x^2+de x^2+\cdots}}\right.}{\cdots}$
default	$\frac{\sqrt{d x^2+c}\,\sqrt{f x^2+e}\left(\sqrt{-\frac{d}{c}}\, a b^2 d f^2 x^5+\sqrt{-\frac{d}{c}}\, a b^2 c f^2 x^3+\sqrt{-\frac{d}{c}}\, a b^2 def x^3+3\sqrt{\frac{d x^2+c}{c}}\,\sqrt{\frac{f x^2+e}{e}}\,\text{Elliptic}\cdots\right.}{\cdots}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((-d/c)^(1/2)*a*b^2*d*f^2*x^5+(-d/c)^(1
/2)*a*b^2*c*f^2*x^3+(-d/c)^(1/2)*a*b^2*d*e*f*x^3+3*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*d*f^2-3*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*a^2*b*c*f^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*a^2*b*d*e*f+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*e*f-((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*d*e^2-
3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a^2*b*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-
```

```
d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*e*f+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*d*e^2-3*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/
(-d/c)^(1/2))*a^3*d*f^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticP
i(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b*c*f^2+6*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/
2)/(-d/c)^(1/2))*a^2*b*d*e*f-6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c*e*f-3*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e
)^(1/2)/(-d/c)^(1/2))*a*b^2*d*e^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c*e^2+(-d
/c)^(1/2)*a*b^2*c*e*f*x)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/b^3/(-d/c)^(1/2)/a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(3/2)/(b*x**2+a),x)
[Out] Integral(sqrt(c + d*x**2)*(e + f*x**2)**(3/2)/(a + b*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^{3/2}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/(a + b*x^2),x)
```

```
[Out] int(((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/(a + b*x^2), x)
```

3.73 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=328

$$\frac{f^2x\sqrt{c+dx^2}}{bd\sqrt{e+fx^2}}-\frac{\sqrt{e}\,f^{3/2}\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bd\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}+\frac{e^{3/2}\sqrt{f}\,\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\right.}{bc\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] f^2*x*(d*x^2+c)^(1/2)/b/d/(f*x^2+e)^(1/2)-f^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+
f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1
/2))*e^(1/2)*(d*x^2+c)^(1/2)/b/d/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^
(1/2)+e^(3/2)*(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi
(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)
^(1/2)/a/b/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+e^(3/2
)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*
x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/b/c/(e*(d*x^2+c)/c/
(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {559, 433, 429, 506, 422, 553}

$$\frac{e^{3/2}\sqrt{c+dx^2}\,(be-af)\Pi\left(1-\frac{be}{af};\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\,\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{f}\,\sqrt{c+dx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bc\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\,f^{3/2}\sqrt{c+dx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bd\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{f^2x\sqrt{c+dx^2}}{bd\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x^2)^(3/2)/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (f^2*x*Sqrt[c + d*x^2])/(b*d*Sqrt[e + f*x^2]) - (Sqrt[e]*f^(3/2)*Sqrt[c + d
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*d*Sqrt[(e
*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*Sqrt[f]*Sqrt[c +
 d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*c*Sqrt[
(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(b*e - a*f)*Sq
rt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 -
(d*e)/(c*f)])/(a*b*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))]]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 559

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rubi steps

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{f\int \frac{\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx}{b} + \frac{(be-af)\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b}$$

$$= \frac{e^{3/2}(be-af)\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(ef)\int \frac{}{\sqrt{c+dx}}}{b}$$

$$= \frac{f^2x\sqrt{c+dx^2}}{bd\sqrt{e+fx^2}} + \frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bc\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{e^{3/2}(be-}{}$$

$$= \frac{f^2x\sqrt{c+dx^2}}{bd\sqrt{e+fx^2}} - \frac{\sqrt{e}\,f^{3/2}\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bd\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{e^{3/2}\sqrt{f}\sqrt{}}{b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.44, size = 184, normalized size = 0.56

$$-\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(abefE\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+(be-af)\left(afF\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+(be-af)\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)\right)\right)}{ab^2\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*b*e*f*EllipticE[I*ArcSinh[
Sqrt[d/c]*x], (c*f)/(d*e)] + (b*e - a*f)*(a*f*EllipticF[I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)] + (b*e - a*f)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)])))/(a*b^2*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.14, size = 300, normalized size = 0.91

method	result

default	$$\frac{\left(-\operatorname{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2f^2+\operatorname{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abef+\operatorname{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abef+\operatorname{EllipticPi}\left(x\right.\right.}{}$$
elliptic	$$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\left(-\frac{f^2\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)a}{b^2\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{f\sqrt{1+\frac{d\,x^2}{c}}}{b\sqrt{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*f^2+EllipticF(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*a*b*e*f+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*e*
f+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*f^2-2*El
lipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*e*f+Elliptic
Pi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*e^2)*((f*x^2+e)/e)
^(1/2)*((d*x^2+c)/c)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(-d/c)^(1/2)/b
^2/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e + fx^2)^{\frac{3}{2}}}{(a + bx^2)\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(1/2),x)
[Out] Integral((e + f*x**2)**(3/2)/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f\,x^2 + e)^{3/2}}{(b\,x^2 + a)\,\sqrt{d\,x^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
[Out] int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

3.74 $\int \frac{\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=224

$$-\frac{(de-cf)\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\,\sqrt{d}\,(bc-ad)\sqrt{c+dx^2}\,\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}+\frac{e^{3/2}(be-af)\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\right.}{ac(bc-ad)\sqrt{f}\,\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\,\sqrt{e+fx^2}}$$

```
[Out] e^(3/2)*(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(
1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)
/a/c/(-a*d+b*c)/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(-c
*f+d*e)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)
/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/(-a*d+b*c)/c^(1/2)/d^
(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {556, 553, 422}

$$\frac{e^{3/2}\sqrt{c+dx^2}\,(be-af)\Pi\left(1-\frac{be}{af};\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\,\sqrt{e+fx^2}\,(bc-ad)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{e+fx^2}\,(de-cf)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\,\sqrt{d}\,\sqrt{c+dx^2}\,(bc-ad)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(((d*e - c*f)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^
2))/(e*(c + d*x^2))])) + (e^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1
- (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a
*d)*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 556

```
Int[((e_) + (f_.)*(x_)^2)^(3/2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2
)^(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[Sqrt[e + f*x^2]/((
a + b*x^2)*Sqrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqr
t[e + f*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

Rubi steps

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{(be-af)\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{bc-ad} - \frac{(de-cf)\int \frac{\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx}{bc-ad}$$

$$= -\frac{(de-cf)\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\,\sqrt{d}\,(bc-ad)\sqrt{c+dx^2}\,\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{e^{3/2}(be-af)\sqrt{c+dx^2}}{ac(bc-ad)\sqrt{f}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.35, size = 492, normalized size = 2.20

$$\frac{\sqrt{\frac{d}{c}}\left(abd\sqrt{\frac{d}{c}}\,e^2x-abc\sqrt{\frac{d}{c}}\,efx+abd\sqrt{\frac{d}{c}}\,efx^3-abc\sqrt{\frac{d}{c}}\,f^2x^3-iabe(-de+cf)\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+ia(-acf^2+be(-de+2cf))\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+ib^2ce^2\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)-2iabcef\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+ia^2cf^2\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)\right)}{abd(-bc+ad)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*(a*b*d*Sqrt[d/c]*e^2*x - a*b*c*Sqrt[d/c]*e*f*x + a*b*d*Sqrt[d/c]
*e*f*x^3 - a*b*c*Sqrt[d/c]*f^2*x^3 - I*a*b*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I
*a*(-(a*c*f^2) + b*e*(-(d*e) + 2*c*f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*b^2*c*e^2*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x
], (c*f)/(d*e)] - (2*I)*a*b*c*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*a^2*c*f^2*S
qrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sq
```

```
rt[d/c]*x], (c*f)/(d*e)]))/(a*b*d*(-(b*c) + a*d)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(274) = 548$.
time = 0.16, size = 630, normalized size = 2.81

method	result
default	$\frac{\left(-\sqrt{-\frac{d}{c}}\, abc f^2 x^3+\sqrt{-\frac{d}{c}}\, abdef x^3+\sqrt{\frac{d x^2+c}{c}}\,\sqrt{\frac{f x^2+e}{e}}\,\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2 c f^2-2\sqrt{\frac{d x^2+c}{c}}\,\sqrt{\frac{f x^2+}{e}}\right.}{}$
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(-\frac{(df x^2+de)(cf-de)x}{d(ad-bc)c\sqrt{\left(x^2+\frac{c}{d}\right)(df x^2+de)}}+\frac{\sqrt{1+\frac{d x^2}{c}}\,\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticF}\left(x\sqrt{-\frac{d}{c}}\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{df x^4+cf x^2+de}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d/c)^(1/2)*a*b*c*f^2*x^3+(-d/c)^(1/2)*a*b*d*e*f*x^3+((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*f^2-2*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*a*b*c*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*a*b*d*e^2+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*e*f-((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d*e^2-((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(
1/2)/(-d/c)^(1/2))*a^2*c*f^2+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c*e*f-((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1
/2)/(-d/c)^(1/2))*b^2*c*e^2-(-d/c)^(1/2)*a*b*c*e*f*x+(-d/c)^(1/2)*a*b*d*e^2
*x)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/c/a/(-d/c)^(1/2)/(a*d-b*c)/(d*f*x^4+c
*f*x^2+d*e*x^2+c*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e + fx^2)^{\frac{3}{2}}}{(a + bx^2)\,(c + dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((e + f*x**2)**(3/2)/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f\,x^2+e\right)}^{3/2}}{\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.75 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=391

$$-\frac{(de-cf)x\sqrt{e+fx^2}}{3c(bc-ad)\left(c+dx^2\right)^{3/2}}-\frac{(bc(5de-cf)-2ad(de+cf))\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}\,(bc-ad)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{e^{3/2}\sqrt{\cdots}}{3c\cdots}$$

```
[Out] b*e^(3/2)*(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f
^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/
2)/a/c/(-a*d+b*c)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
+1/3*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^
(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^2/(-a*
d+b*c)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(-c*f+d*e)*x*(f*
x^2+e)^(1/2)/c/(-a*d+b*c)/(d*x^2+c)^(3/2)-1/3*(b*c*(-c*f+5*d*e)-2*a*d*(c*f+
d*e))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(
1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(3/2)/(-a*d+b*c)^2/d^
(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {558, 553, 540, 539, 429, 422}

$$-\frac{\sqrt{e+fx^2}\,(bc(5de-cf)-2ad(cf+de))E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}\,\sqrt{c+dx^2}\,(bc-ad)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{e^{3/2}\sqrt{f}\,\sqrt{c+dx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}\,(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{be^{3/2}\sqrt{c+dx^2}\,(be-af)\Pi\left(1-\frac{be}{af};\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\,\sqrt{e+fx^2}\,(bc-ad)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{e+fx^2}\,(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/3*((d*e - c*f)*x*Sqrt[e + f*x^2])/(c*(b*c - a*d)*(c + d*x^2)^(3/2)) - ((
b*c*(5*d*e - c*f) - 2*a*d*(d*e + c*f))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*Sqrt[d]*(b*c - a*d)^2*Sqrt[
c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + (e^(3/2)*Sqrt[f]*Sqrt[c
 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(
b*c - a*d)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b*e^(3
/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]
*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*Sqrt[(e*(c + d*x
^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 558

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b*((b*e - a*f)/(b*c - a*d)^2), Int[(c + d*x^2)^(q
 + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Dist[1/(b*c - a*d)^2, Int
[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(b*
e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && GtQ[
r, 1]
```

Rubi steps

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = -\frac{\int \frac{\sqrt{e+fx^2}\,(2bcde-ad^2e-bc^2f+d^2(be-af)x^2)}{(c+dx^2)^{5/2}}\,dx}{(bc-ad)^2} + \frac{(b(be-af))\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+d\ldots}}}{(bc-ad)^2}$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{3c(bc-ad)(c+dx^2)^{3/2}} + \frac{be^{3/2}(be-af)\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\right.\right.}{ac(bc-ad)^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+\ldots}}$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{3c(bc-ad)(c+dx^2)^{3/2}} + \frac{be^{3/2}(be-af)\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\right.\right.}{ac(bc-ad)^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+\ldots}}$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{3c(bc-ad)(c+dx^2)^{3/2}} - \frac{(bc(5de-cf)-2ad(de+cf))\sqrt{e+fx^2}\,E\left(\mathrm{ta}\ldots\right.}{3c^{3/2}\sqrt{d}\,(bc-ad)^2\sqrt{c+dx^2}\sqrt{\frac{c(\ldots}{e(\ldots}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.22, size = 999, normalized size = 2.55

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (3*a^2*c*d^2*Sqrt[d/c]*e^2*x - 6*a*b*c^3*(d/c)^(3/2)*e^2*x + 2*a*b*c^3*Sqrt
[d/c]*e*f*x + a^2*c^3*(d/c)^(3/2)*e*f*x - 5*a*b*c*d^2*Sqrt[d/c]*e^2*x^3 + 2
*a^2*d^3*Sqrt[d/c]*e^2*x^3 + 5*a^2*c*d^2*Sqrt[d/c]*e*f*x^3 - 5*a*b*c^3*(d/c
)^(3/2)*e*f*x^3 + 2*a*b*c^3*Sqrt[d/c]*f^2*x^3 + a^2*c^3*(d/c)^(3/2)*f^2*x^3
 - 5*a*b*c*d^2*Sqrt[d/c]*e*f*x^5 + 2*a^2*d^3*Sqrt[d/c]*e*f*x^5 + 2*a^2*c*d^
2*Sqrt[d/c]*f^2*x^5 + a*b*c^3*(d/c)^(3/2)*f^2*x^5 + I*a*e*(b*c*(-5*d*e + c*
f) + 2*a*d*(d*e + c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]
*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-(d*e) + c*f)*(5*b*c
*e - 2*a*d*e - 3*a*c*f)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]
*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*b^2*c^3*e^2*Sqrt[1
+ (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c
]*x], (c*f)/(d*e)] + (6*I)*a*b*c^3*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*a^
2*c^3*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I
```

```
*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*b^2*c^2*d*e^2*x^2*Sqrt[1 + (d*x
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x],
(c*f)/(d*e)] + (6*I)*a*b*c^2*d*e*f*x^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*a^
2*c^2*d*f^2*x^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a
*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*a*c^2*Sqrt[d/c]*(b*c - a*d)^2
*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1875 vs. $2(456) = 912$.
time = 0.16, size = 1876, normalized size = 4.80

method	result	size
elliptic	Expression too large to display	1645
default	Expression too large to display	1876

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*a^2*c^3*f^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipt
icPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*c^3*f^2+3*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^
(1/2)/(-d/c)^(1/2))*b^2*c^3*e^2+2*(-d/c)^(1/2)*a^2*d^3*e^2*x^3+2*(-d/c)^(1/
2)*a*b*c^3*f^2*x^3-5*(-d/c)^(1/2)*a*b*c*d^2*e*f*x^5-5*(-d/c)^(1/2)*a*b*c^2*
d*e*f*x^3-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*a^2*c^2*d*f^2*x^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*c^2*d*
f^2*x^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2)
,b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^2*d*e^2*x^2+((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c^2*d*e*f+
5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a*b*c^3*e*f-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^2*d*e^2-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c^2*d*e*f-((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a
*b*c^3*e*f+5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a*b*c^2*d*e^2-6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c^3*e*f+2
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*a^2*d^3*e^2*x^2-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d^3*e^2*x^2+2*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^2*e^2-2*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*a^2*c*d^2*e^2+2*(-d/c)^(1/2)*a^2*c*d^2*f^2*x^5+2*(-d/c)^(1/2)*a^2*d^3
```

```
*e*f*x^5+(-d/c)^(1/2)*a^2*c^2*d*f^2*x^3+(-d/c)^(1/2)*a*b*c^2*d*f^2*x^5+5*(-
d/c)^(1/2)*a^2*c*d^2*e*f*x^3-5*(-d/c)^(1/2)*a*b*c*d^2*e^2*x^3+(-d/c)^(1/2)*
a^2*c^2*d*e*f*x+2*(-d/c)^(1/2)*a*b*c^3*e*f*x-6*(-d/c)^(1/2)*a*b*c^2*d*e^2*x
+3*(-d/c)^(1/2)*a^2*c*d^2*e^2*x+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ell
ipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^2*e*f*x^2-5*((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*d^2
*e^2*x^2-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*a^2*c*d^2*e*f*x^2+5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*d^2*e^2*x^2+5*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b
*c^2*d*e*f*x^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*a*b*c^2*d*e*f*x^2-6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c^2
*d*e*f*x^2)/(f*x^2+e)^(1/2)/(a*d-b*c)^2/c^2/(-d/c)^(1/2)/a/(d*x^2+c)^(3/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f\,x^2+e)^{3/2}}{(b\,x^2+a)\,(d\,x^2+c)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

3.76 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=639

$$-\frac{(de-cf)x\sqrt{e+fx^2}}{5c(bc-ad)(c+dx^2)^{5/2}}-\frac{(3bc(3de-cf)-2ad(2de+cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(c+dx^2)^{3/2}}-\frac{b\sqrt{d}(be-af)\sqrt{e+fx^2}\,E\Big(\text{ta}}{\sqrt{c}(bc-ad)^3\sqrt{c+dx^2}}$$

```
[Out] b^2*e^(3/2)*(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x
*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(
1/2)/a/c/(-a*d+b*c)^3/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/
2)+1/15*e^(3/2)*(3*b*c*(-2*c*f+3*d*e)-a*d*(-c*f+4*d*e))*(1/(1+f*x^2/e))^(1/
2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c
/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^3/(-a*d+b*c)^2/(-c*f+d*e)/(e*(d*x^2+c)
/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/5*(-c*f+d*e)*x*(f*x^2+e)^(1/2)/c/(-a*
d+b*c)/(d*x^2+c)^(5/2)-1/15*(3*b*c*(-c*f+3*d*e)-2*a*d*(c*f+2*d*e))*x*(f*x^2
+e)^(1/2)/c^2/(-a*d+b*c)^2/(d*x^2+c)^(3/2)+1/15*(a*d*(-2*c^2*f^2-3*c*d*e*f+
8*d^2*e^2)-3*b*c*(c^2*f^2-6*c*d*e*f+6*d^2*e^2))*(1/(1+d*x^2/c))^(1/2)*(1+d*
x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2
))*(f*x^2+e)^(1/2)/c^(5/2)/(-a*d+b*c)^2/(-c*f+d*e)/d^(1/2)/(d*x^2+c)^(1/2)/
(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)-b*(-a*f+b*e)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2
/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*
d^(1/2)*(f*x^2+e)^(1/2)/(-a*d+b*c)^3/c^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e
/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 639, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {558, 555, 553, 422, 540, 541, 539, 429}

$$\frac{b^2e^{3/2}\sqrt{c+dx^2}(be-af)\Pi\left(1-\frac{be}{af};\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(3bc(3de-2cf)-ad(4de-cf))F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3\sqrt{e+fx^2}(bc-ad)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{e+fx^2}(ad(-2c^2f^2-3cdef+8d^2e^2)-3bc(c^2f^2-6cdef+6d^2e^2))E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{b\sqrt{d}\sqrt{e+fx^2}(be-af)E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{x\sqrt{e+fx^2}(3bc(3de-cf)-2ad(cf+2de))}{15c^2(c+dx^2)^{3/2}(bc-ad)^2}-\frac{x\sqrt{e+fx^2}(de-cf)}{5c(c+dx^2)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

```
[Out] -1/5*((d*e - c*f)*x*Sqrt[e + f*x^2])/(c*(b*c - a*d)*(c + d*x^2)^(5/2)) - ((
3*b*c*(3*d*e - c*f) - 2*a*d*(2*d*e + c*f))*x*Sqrt[e + f*x^2])/(15*c^2*(b*c
- a*d)^2*(c + d*x^2)^(3/2)) - (b*Sqrt[d]*(b*e - a*f)*Sqrt[e + f*x^2]*Ellipt
icE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*(b*c - a*d)^3*S
qrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + ((a*d*(8*d^2*e^2 -
3*c*d*e*f - 2*c^2*f^2) - 3*b*c*(6*d^2*e^2 - 6*c*d*e*f + c^2*f^2))*Sqrt[e +
f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*c^(5/2)
```

```
*Sqrt[d]*(b*c - a*d)^2*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*
(c + d*x^2))]) + (e^(3/2)*Sqrt[f]*(3*b*c*(3*d*e - 2*c*f) - a*d*(4*d*e - c*f
))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])
/(15*c^3*(b*c - a*d)^2*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sq
rt[e + f*x^2]) + (b^2*e^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (b
*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^
3*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
```

```
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 555

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rule 558

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b*((b*e - a*f)/(b*c - a*d)^2), Int[(c + d*x^2)^(q
 + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Dist[1/(b*c - a*d)^2, Int
[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(b*
e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && GtQ[
r, 1]
```

Rubi steps

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = -\frac{\int \frac{\sqrt{e+fx^2}\,(2bcde-ad^2e-bc^2f+d^2(be-af)x^2)}{(c+dx^2)^{7/2}}\,dx}{(bc-ad)^2} + \frac{(b(be-af))\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{3/2}}}{(bc-ad)^2}$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{5c(bc-ad)(c+dx^2)^{5/2}} + \frac{\int \frac{-de(bc(9de-4cf)-ad(4de+cf))-df(bc(8de-3cf)-ad(3de+2cf}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}}{5cd(bc-ad)^2}$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{5c(bc-ad)(c+dx^2)^{5/2}} - \frac{(3bc(3de-cf)-2ad(2de+cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(c+dx^2)^{3/2}} -$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{5c(bc-ad)(c+dx^2)^{5/2}} - \frac{(3bc(3de-cf)-2ad(2de+cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(c+dx^2)^{3/2}} -$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{5c(bc-ad)(c+dx^2)^{5/2}} - \frac{(3bc(3de-cf)-2ad(2de+cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(c+dx^2)^{3/2}} -$$

<u>**Mathematica**</u> **[C]** Result contains complex when optimal does not.
time = 7.90, size = 570, normalized size = 0.89

$$\frac{-a\sqrt{\frac{d}{c}}x(e+fx^2)\left(3c^2(bc-ad)^2(de-cf)^2+c(bc-ad)(-de+cf)(3bc(-3de+cf)+2ad(2de+cf))(c+dx^2)+(a^2d^2(8d^2e^2-3cdef-2c^2f^2)+3b^2c^2(11d^2e^2-11cdef+c^2f^2)+2abcd(-13d^2e^2+3cdef+7c^2f^2))(c+dx^2)^2\right)+i(c+dx^2)^2\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(ae(-3b^2c^2(11d^2e^2-11cdef+c^2f^2)+a^2d^2(-8d^2e^2+3cdef+2c^2f^2)-2abcd(-13d^2e^2+3cdef+7c^2f^2))E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+(de-cf)\left(a(3b^2c^2e(11de-8cf)+a^2d^2e(8de+cf)+abc(-26d^2e^2-7cdef+15c^2f^2))F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-15bc^3(be-af)^2\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)\right)}{15ac^3\sqrt{\frac{d}{c}}(bc-ad)^3(de-cf)(c+dx^2)^{5/2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

```
[Out] (-(a*Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f)^2 + c*(b*c -
a*d)*(-(d*e) + c*f)*(3*b*c*(-3*d*e + c*f) + 2*a*d*(2*d*e + c*f))*(c + d*x^2
) + (a^2*d^2*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2) + 3*b^2*c^2*(11*d^2*e^2 -
11*c*d*e*f + c^2*f^2) + 2*a*b*c*d*(-13*d^2*e^2 + 3*c*d*e*f + 7*c^2*f^2))*(c
 + d*x^2)^2)) + I*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*
e*(-3*b^2*c^2*(11*d^2*e^2 - 11*c*d*e*f + c^2*f^2) + a^2*d^2*(-8*d^2*e^2 + 3
*c*d*e*f + 2*c^2*f^2) - 2*a*b*c*d*(-13*d^2*e^2 + 3*c*d*e*f + 7*c^2*f^2))*El
lipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (d*e - c*f)*(a*(3*b^2*c^2*e*
(11*d*e - 8*c*f) + a^2*d^2*e*(8*d*e + c*f) + a*b*c*(-26*d^2*e^2 - 7*c*d*e*f
 + 15*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - 15*b*c^3*(
b*e - a*f)^2*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))
```

```
)/(15*a*c^3*Sqrt[d/c]*(b*c - a*d)^3*(d*e - c*f)*(c + d*x^2)^(5/2)*Sqrt[e +
f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6210 vs. $2(723) = 1446$.
time = 0.16, size = 6211, normalized size = 9.72

method	result	size
elliptic	Expression too large to display	3112
default	Expression too large to display	6211

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="giac")
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x^2+e\right)^{3/2}}{\left(b\,x^2+a\right)\,\left(d\,x^2+c\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(7/2)),x)
[Out] int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(7/2)), x)
```

3.77 $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=621

$$\frac{d(bc-ad)x\sqrt{c+dx^2}}{b^2\sqrt{e+fx^2}}-\frac{2d(de-2cf)x\sqrt{c+dx^2}}{3bf\sqrt{e+fx^2}}+\frac{d^2x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3bf}-\frac{d(bc-ad)\sqrt{e}\,\sqrt{c+dx^2}\,E\big(}{b^2\sqrt{f}\,\sqrt{\frac{e\,(c+d}{c\,(e+f}}}$$

```
[Out] d*(-a*d+b*c)*x*(d*x^2+c)^(1/2)/b^2/(f*x^2+e)^(1/2)-2/3*d*(-2*c*f+d*e)*x*(d*
x^2+c)^(1/2)/b/f/(f*x^2+e)^(1/2)+2/3*d*(-2*c*f+d*e)*(1/(1+f*x^2/e))^(1/2)*(
1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^
(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f
*x^2+e)^(1/2)-1/3*d*(-3*c*f+d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*El
lipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x
^2+c)^(1/2)/b/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-d*(-a
*d+b*c)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)
/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b^2/f^(1/2)/(
e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+d*(-a*d+b*c)*(1/(1+f*x^2/e))
^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d
*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e)
)^(1/2)/(f*x^2+e)^(1/2)+1/3*d^2*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/f+c^(3/
2)*(-a*d+b*c)^2*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2
)/c^(1/2)/(1+d*x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/
b^2/e/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 621, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {559, 427, 545, 429, 506, 422, 433, 553}

$$\frac{c^{3/2}\sqrt{e+fx^2}\,(bc-ad)^2\Pi\left(1-\frac{bc}{ad};\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}\,e\sqrt{c+dx^2}\,\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{d\sqrt{e}\,\sqrt{c+dx^2}\,(bc-ad)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b^2\sqrt{f}\,\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{d\sqrt{e}\,\sqrt{c+dx^2}\,(bc-ad)E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b^2\sqrt{f}\,\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{dx\sqrt{c+dx^2}\,(bc-ad)}{b^2\sqrt{e+fx^2}}-\frac{d\sqrt{e}\,\sqrt{c+dx^2}\,(de-3cf)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3bf^{3/2}\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{2d\sqrt{e}\,\sqrt{c+dx^2}\,(de-2cf)E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3bf^{3/2}\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{d^2x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3bf}-\frac{2dx\sqrt{c+dx^2}\,(de-2cf)}{3bf\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (d*(b*c - a*d)*x*Sqrt[c + d*x^2])/(b^2*Sqrt[e + f*x^2]) - (2*d*(d*e - 2*c*f
)*x*Sqrt[c + d*x^2])/(3*b*f*Sqrt[e + f*x^2]) + (d^2*x*Sqrt[c + d*x^2]*Sqrt[
e + f*x^2])/(3*b*f) - (d*(b*c - a*d)*Sqrt[e]*Sqrt[c + d*x^2]*EllipticE[ArcT
an[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b^2*Sqrt[f]*Sqrt[(e*(c + d*x^2)
)/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (2*d*Sqrt[e]*(d*e - 2*c*f)*Sqrt[c + d
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b*f^(3/2)
```

```
*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*(b*c - a*d)*Sq
rt[e]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f
)])/(b^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (
d*Sqrt[e]*(d*e - 3*c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(3*b*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sq
rt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)^2*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c
)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*b^2*Sqrt[d]*e*Sq
rt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 559

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rubi steps

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \frac{d\int \frac{(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx}{b} + \frac{(bc-ad)\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{b}$$

$$= \frac{d^2x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3bf} + \frac{(d(bc-ad))\int \frac{\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx}{b^2} + \frac{(bc-ad)^2 \int \frac{\sqrt{}}{(a+bx}}{b^2}$$

$$= \frac{d^2x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3bf} + \frac{c^{3/2}(bc-ad)^2\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right.\right.}{ab^2\sqrt{d}\,e\sqrt{c+dx^2}\,\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

$$= \frac{d(bc-ad)x\sqrt{c+dx^2}}{b^2\sqrt{e+fx^2}} - \frac{2d(de-2cf)x\sqrt{c+dx^2}}{3bf\sqrt{e+fx^2}} + \frac{d^2x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3bf}$$

$$= \frac{d(bc-ad)x\sqrt{c+dx^2}}{b^2\sqrt{e+fx^2}} - \frac{2d(de-2cf)x\sqrt{c+dx^2}}{3bf\sqrt{e+fx^2}} + \frac{d^2x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{3bf}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.98, size = 350, normalized size = 0.56

$$\frac{-iabd^2e(-2bde+7bcf-3adf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)\middle|\frac{cf}{de}\right)-iad(3a^2d^2f^2+3abdf(de-3cf)+b^2(2d^2e^2-8cdef+9c^2f^2))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)\middle|\frac{cf}{de}\right)+f\left(ab^2cd\left(\frac{d}{c}\right)^{3/2}x(c+dx^2)(e+fx^2)-3i(bc-ad)^3f\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)\middle|\frac{cf}{de}\right)\right)}{3ab^3\sqrt{\frac{d}{c}}\,f^2\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*a*b*d^2*e*(-2*b*d*e + 7*b*c*f - 3*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
 (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*d*(3*a^2*d
^2*f^2 + 3*a*b*d*f*(d*e - 3*c*f) + b^2*(2*d^2*e^2 - 8*c*d*e*f + 9*c^2*f^2))
*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x],
(c*f)/(d*e)] + f*(a*b^2*c*d*(d/c)^(3/2)*x*(c + d*x^2)*(e + f*x^2) - (3*I)*(
b*c - a*d)^3*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*
d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*a*b^3*Sqrt[d/c]*f^2*Sqrt[c +
d*x^2]*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.19, size = 988, normalized size = 1.59

method	result
risch	$\frac{d^2 x\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{3bf}-\frac{\left(d\left(-\frac{\left(3ab\,d^2f-7b^2cdf+2b^2d^2e\right)e\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1}\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x}}\right.\right.}{}$
default	$\frac{\left(\sqrt{-\frac{d}{c}}\,a\,b^2d^3f^2x^5+\sqrt{-\frac{d}{c}}\,a\,b^2c\,d^2f^2x^3+\sqrt{-\frac{d}{c}}\,a\,b^2d^3ef\,x^3+3\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^3\right.}{}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-d/c)^(1/2)*a*b^2*d^3*f^2*x^5+(-d/c)^(1/2)*a*b^2*c*d^2*f^2*x^3+(-d/c)
^(1/2)*a*b^2*d^3*e*f*x^3+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
F(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*d^3*f^2-9*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c*d^2*f^2+3*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*a^2*b*d^3*e*f+9*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c^2*d*f^2-8*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d^2*e*f+2*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*a*b^2*d^3*e^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d^3*e*f+7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d^2*e*f-2*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a
*b^2*d^3*e^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^
(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*d^3*f^2+9*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(
1/2))*a^2*b*c*d^2*f^2-9*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(
x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c^2*d*f^2+3*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(
1/2)/(-d/c)^(1/2))*b^3*c^3*f^2+(-d/c)^(1/2)*a*b^2*c*d^2*e*f*x)*(f*x^2+e)^(1
/2)*(d*x^2+c)^(1/2)/a/(-d/c)^(1/2)/f^2/b^3/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{5}{2}}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)
[Out] Integral((c + d*x**2)**(5/2)/((a + b*x**2)*sqrt(e + f*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x^2+c\right)}^{5/2}}{\left(b\,x^2+a\right)\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(1/2)),x)
[Out] int((c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(1/2)), x)
```

3.78 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=319

$$\frac{dx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}}-\frac{d\sqrt{e}\,\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}}+\frac{d\sqrt{e}\,\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}}$$

```
[Out] d*x*(d*x^2+c)^(1/2)/b/(f*x^2+e)^(1/2)-d*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(
1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/
2)*(d*x^2+c)^(1/2)/b/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2
)+d*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+
f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b/f^(1/2)/(e*(d*x
^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+c^(3/2)*(-a*d+b*c)*(1/(1+d*x^2/c))
^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),1-b
*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/b/e/d^(1/2)/(d*x^2+c)^(1/2)/(c*
(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {559, 433, 429, 506, 422, 553}

$$\frac{c^{3/2}\sqrt{e+fx^2}\,(bc-ad)\Pi\left(1-\frac{bc}{ad};\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab\sqrt{d}\,e\sqrt{c+dx^2}\,\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{d\sqrt{e}\,\sqrt{c+dx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\,\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{d\sqrt{e}\,\sqrt{c+dx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\,\sqrt{e+fx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{dx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (d*x*Sqrt[c + d*x^2])/(b*Sqrt[e + f*x^2]) - (d*Sqrt[e]*Sqrt[c + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*Sqrt[f]*Sqrt[(e*(c
+ d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*Sqrt[e]*Sqrt[c + d*x^2]*El
lipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*Sqrt[f]*Sqrt[(e*(
c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*Sqrt[e
 + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f
)/(d*e)])/(a*b*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2
))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 559

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rubi steps

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \frac{d\int \frac{\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx}{b} + \frac{(bc-ad)\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{b}$$

$$= \frac{c^{3/2}(bc-ad)\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab\sqrt{d}\,e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{(cd)\int}{\sqrt{c+d}}$$

$$= \frac{dx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}} + \frac{d\sqrt{e}\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{c^{3/2}(bc-ad)}{}$$

$$= \frac{dx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}} - \frac{d\sqrt{e}\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{d\sqrt{e}\sqrt{c+d}}{b\sqrt{f}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.51, size = 197, normalized size = 0.62

$$-\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(abd^2eE\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)-ad(bde-2bcf+adf)F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+(bc-ad)^2f\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)\right)}{ab^2\sqrt{\frac{d}{c}}\,f\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*b*d^2*e*EllipticE[I*ArcSin
h[Sqrt[d/c]*x], (c*f)/(d*e)] - a*d*(b*d*e - 2*b*c*f + a*d*f)*EllipticF[I*Ar
cSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (b*c - a*d)^2*f*EllipticPi[(b*c)/(a*d),
I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*b^2*Sqrt[d/c]*f*Sqrt[c + d*x^2]*S
qrt[e + f*x^2])
```

Maple [A]
time = 0.14, size = 341, normalized size = 1.07

method	result

default	$\frac{\left(-\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2d^2f+2\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abcdf-\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ab\,d^2e+\mathrm{EllipticE}\left(\right.\right.}{}$
elliptic	$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\left(-\frac{d^2\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)a}{b^2\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{2d\sqrt{1+\frac{d\,x^2}{c}}}{b\sqrt{-}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d^2*f+2*EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*a*b*c*d*f-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
a*b*d^2*e+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d^2*e+EllipticPi(x*
(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*d^2*f-2*EllipticPi(x*(-
d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c*d*f+EllipticPi(x*(-d/c)
^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^2*f)*((f*x^2+e)/e)^(1/2)*((
d*x^2+c)/c)^(1/2)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/a/b^2/f/(-d/c)^(1/2)/(d*f
*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^2)^{\frac{3}{2}}}{(a + bx^2)\sqrt{e + fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)
[Out] Integral((c + d*x**2)**(3/2)/((a + b*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x^2 + c)^{3/2}}{(b\,x^2 + a)\,\sqrt{f\,x^2 + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(1/2)),x)
[Out] int((c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(1/2)), x)
```

3.79 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=102

$$\frac{c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}\,e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2)/c^(1/2
)/(1+d*x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/e/d^(1/2
)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = $\{553\}$

$$\frac{c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}\,e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/
(e*(c + d*x^2))])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \frac{c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}\,e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.82, size = 143, normalized size = 1.40

$$-\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(adF\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+(bc-ad)\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)\right)}{ab\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*d*EllipticF[I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)] + (b*c - a*d)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)]))/(a*b*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.14, size = 191, normalized size = 1.87

method	result
default	$\frac{\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ad-\mathrm{EllipticPi}\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)ad+\mathrm{EllipticPi}\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)bc\right)\sqrt{\frac{fx^2+}{e}}}{ba\sqrt{-\frac{d}{c}}\,(dfx^4+cfx^2+dex^2+ce)}$
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{d\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{b\sqrt{-\frac{d}{c}}\sqrt{dfx^4+cfx^2+dex^2+ce}}-\frac{\sqrt{1+\frac{dx^2}{c}}\sqrt{1}}{b\sqrt{-\frac{d}{c}}}\right.}{\sqrt{dx^2+c}\sqrt{f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] (EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d-EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*d+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c)/b*((f*x^2+e)/e)^(1/2)*((d*x^2+c)/c)^(1/2)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/a/(-d/c)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)

[Out] Integral(sqrt(c + d*x**2)/((a + b*x**2)*sqrt(e + f*x**2)), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{\left(b\,x^2+a\right)\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/((a + b*x^2)*(e + f*x^2)^(1/2)),x)
[Out] int((c + d*x^2)^(1/2)/((a + b*x^2)*(e + f*x^2)^(1/2)), x)
```

3.80 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{\sqrt{-c}\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[Out] EllipticPi(x*d^(1/2)/(-c)^(1/2),b*c/a/d,(c*f/d/e)^(1/2))*(-c)^(1/2)*(1+d*x^
2/c)^(1/2)*(1+f*x^2/e)^(1/2)/a/d^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {552, 551}

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\Pi\left(\frac{bc}{ad};\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[-c]*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), A
rcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[
e + f*x^2])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{1+\frac{dx^2}{c}}\int \frac{1}{(a+bx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{e+fx^2}}\,dx}{\sqrt{c+dx^2}}$$

$$= \frac{\left(\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\right)\int \frac{1}{(a+bx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}}\,dx}{\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$= \frac{\sqrt{-c}\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.25, size = 101, normalized size = 1.01

$$-\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)}{a\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*Arc
Sinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^
2])
```

Maple [A]
time = 0.13, size = 118, normalized size = 1.18

method	result	size
default	$\frac{\mathrm{EllipticPi}\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)\sqrt{\frac{f\,x^2+e}{e}}\sqrt{\frac{d\,x^2+c}{c}}\sqrt{f\,x^2+e}\sqrt{d\,x^2+c}}{a\sqrt{-\frac{d}{c}}\,(df\,x^4+cf\,x^2+de\,x^2+ce)}$	118

elliptic	$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\;\mathrm{EllipticPi}\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\;a\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}$	133

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*((f*x^2+e)/e)^
(1/2)*((d*x^2+c)/c)^(1/2)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/a/(-d/c)^(1/2)/(d
*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}\,\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.81 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=344

$$-\frac{d^{3/2}\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\,(bc-ad)(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{d\sqrt{e}\,(bde-2bcf+adf)\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1\right.}{c(bc-ad)^2\sqrt{f}\,(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] -d*(a*d*f-2*b*c*f+b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(
x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1
/2)/c/(-a*d+b*c)^2/(-c*f+d*e)/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^
2+e)^(1/2)-d^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/(-a*d+b*c)
/(-c*f+d*e)/c^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)+b^2*c^(
3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2)/c^(1/2)/(
1+d*x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/(-a*d+b*c)^
2/e/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {560, 553, 539, 429, 422}

$$\frac{b^2c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}\,e\sqrt{c+dx^2}\,(bc-ad)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{d^{3/2}\sqrt{e+fx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\,\sqrt{c+dx^2}\,(bc-ad)(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{d\sqrt{e}\,\sqrt{c+dx^2}\,(adf-2bcf+bde)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\,\sqrt{e+fx^2}\,(bc-ad)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -((d^(3/2)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)
/(d*e)])/(Sqrt[c]*(b*c - a*d)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^
2))/(e*(c + d*x^2))])) - (d*Sqrt[e]*(b*d*e - 2*b*c*f + a*d*f)*Sqrt[c + d*x^
2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c - a*d)^
2*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]
) + (b^2*c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*(b*c - a*d)^2*e*Sqrt[c + d*x^2]
*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{b^2\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{(bc-ad)^2} - \frac{d\int \frac{2bc-ad+bdx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx}{(bc-ad)^2}$$

$$= \frac{b^2c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}\,(bc-ad)^2e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{d^2\int \frac{\sqrt{e+}}{(c+dx}}{(bc-ad)(}$$

$$= -\frac{d^{3/2}\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\,(bc-ad)(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{d\sqrt{e}\,(bde-}{c(bc-}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.40, size = 365, normalized size = 1.06

$$\frac{\sqrt{\frac{d}{c}}\left(acd\left(\frac{d}{c}\right)^{3/2}ex+acd\left(\frac{d}{c}\right)^{3/2}fx^3+iad^2e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+iad(-de+cf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+ibcde\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)-ibc^2f\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)\right)}{ad(-bc+ad)(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[d/c]*(a*c*d*(d/c)^(3/2)*e*x + a*c*d*(d/c)^(3/2)*f*x^3 + I*a*d^2*e*Sqr
t[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f
)/(d*e)] + I*a*d*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ell
ipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*b*c*d*e*Sqrt[1 + (d*x^2)/c]
*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)] - I*b*c^2*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)
/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*d*(-(b*c) + a*d)*(d*e - c
*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.14, size = 413, normalized size = 1.20

method	result
default	$\frac{\left(-\sqrt{-\frac{d}{c}}\,a\,d^2 f\,x^3+\sqrt{\frac{d\,x^2+c}{c}}\sqrt{\frac{f\,x^2+e}{e}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)acdf-\sqrt{\frac{d\,x^2+c}{c}}\sqrt{\frac{f\,x^2+e}{e}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}}\right.\right.}{}$

elliptic	$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\left(-\frac{\left(df\,x^2+de\right)dx}{c(cf-de)(ad-bc)\sqrt{\left(x^2+\frac{c}{d}\right)(df\,x^2+de)}}+\frac{\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}\,\mathrm{EllipticF}\left(x\sqrt{\right.}{\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+cf\,x^2+d}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d/c)^(1/2)*a*d^2*f*x^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
F(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e+((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e-((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/
e)^(1/2)/(-d/c)^(1/2))*b*c^2*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c*d*e-(-d/c)^(1/
2)*a*d^2*e*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/c/a/(a*d-b*c)/(-d/c)^(1/2)/(c
*f-d*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}\,\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{3/2}\,\sqrt{f\,x^2+e}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)), x)
```

3.82 $\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=435

$$-\frac{d^2x\sqrt{e+fx^2}}{3c(bc-ad)(de-cf)(c+dx^2)^{3/2}}-\frac{d^{3/2}(bc(5de-7cf)-2ad(de-2cf))\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1\right.}{3c^{3/2}(bc-ad)^2(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] -1/3*d*(a*d*(-3*c*f+d*e)-2*b*c*(-3*c*f+2*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x
^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2)
)*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c^2/(-a*d+b*c)^2/(-c*f+d*e)^2/(e*(d*x^2+c
)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*d^2*x*(f*x^2+e)^(1/2)/c/(-a*d+b*c)
/(-c*f+d*e)/(d*x^2+c)^(3/2)-1/3*d^(3/2)*(b*c*(-7*c*f+5*d*e)-2*a*d*(-2*c*f+d
*e))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1
+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(3/2)/(-a*d+b*c)^2/(-c
*f+d*e)^2/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)+b^2*EllipticPi(x*
d^(1/2)/(-c)^(1/2),b*c/a/d,(c*f/d/e)^(1/2))*(-c)^(1/2)*(1+d*x^2/c)^(1/2)*(1
+f*x^2/e)^(1/2)/a/(-a*d+b*c)^2/d^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]

time = 0.37, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {560, 552, 551, 541, 539, 429, 422}

$$\frac{b^2\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\Pi\left(\frac{bc}{ad};\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)^2}-\frac{d^{3/2}\sqrt{e+fx^2}(bc(5de-7cf)-2ad(de-2cf))E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{c+dx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(ad(de-3cf)-2bc(2de-3cf))F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{d^2x\sqrt{e+fx^2}}{3c(c+dx^2)^{3/2}(bc-ad)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -1/3*(d^2*x*Sqrt[e + f*x^2])/(c*(b*c - a*d)*(d*e - c*f)*(c + d*x^2)^(3/2))
- (d^(3/2)*(b*c*(5*d*e - 7*c*f) - 2*a*d*(d*e - 2*c*f))*Sqrt[e + f*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*(b*c - a*d)
^2*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (
d*Sqrt[e]*Sqrt[f]*(a*d*(d*e - 3*c*f) - 2*b*c*(2*d*e - 3*c*f))*Sqrt[c + d*x^
2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a
*d)^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
+ (b^2*Sqrt[-c]*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a
*d), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*(b*c - a*d)^2*S
qrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
```

```
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \frac{b^2\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{(bc-ad)^2} - \frac{d\int \frac{2bc-ad+bdx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{d^2x\sqrt{e+fx^2}}{3c(bc-ad)(de-cf)(c+dx^2)^{3/2}} + \frac{d\int \frac{-bc(5de-6cf)+ad(2de-3cf)-}{(c+dx^2)^{3/2}\sqrt{e+}}}{3c(bc-ad)^2(de-}$$

$$= -\frac{d^2x\sqrt{e+fx^2}}{3c(bc-ad)(de-cf)(c+dx^2)^{3/2}} - \frac{(df(ad(de-3cf)-2bc(2}{3c(bc}$$

$$= -\frac{d^2x\sqrt{e+fx^2}}{3c(bc-ad)(de-cf)(c+dx^2)^{3/2}} - \frac{d^{3/2}(bc(5de-7cf)-2ad(}{3c^{3/2}(bc-ad}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.69, size = 433, normalized size = 1.00

$$\frac{acd\left(\frac{d}{c}\right)^{3/2}x(e+fx^2)\left(bc(-6cde+8c^2f-5d^2ex^2+7cdfx^2)+ad(-5c^2f+2d^2ex^2+cd(3e-4fx^2))\right)+iad^2e(2ad(de-2cf)+bc(-5de+7cf))(c+dx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)\middle|\frac{cf}{de}\right)+iad(-de+cf)(ad(2de-3cf)+bc(-5de+6cf))(c+dx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)\middle|\frac{cf}{de}\right)-3ib^2c^2(de-cf)^2(c+dx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)\middle|\frac{cf}{de}\right)}{3ac^2\sqrt{\frac{d}{c}}(bc-ad)^2(de-cf)^2(c+dx^2)^{3/2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (a*c*d*(d/c)^(3/2)*x*(e + f*x^2)*(b*c*(-6*c*d*e + 8*c^2*f - 5*d^2*e*x^2 + 7
*c*d*f*x^2) + a*d*(-5*c^2*f + 2*d^2*e*x^2 + c*d*(3*e - 4*f*x^2))) + I*a*d^2
*e*(2*a*d*(d*e - 2*c*f) + b*c*(-5*d*e + 7*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I
*a*d*(-(d*e) + c*f)*(a*d*(2*d*e - 3*c*f) + b*c*(-5*d*e + 6*c*f))*(c + d*x^2
)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x],
 (c*f)/(d*e)] - (3*I)*b^2*c^2*(d*e - c*f)^2*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]
*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)])/(3*a*c^2*Sqrt[d/c]*(b*c - a*d)^2*(d*e - c*f)^2*(c + d*x^2)^(3/2)*Sq
rt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2061 vs. $2(460) = 920$.
time = 0.16, size = 2062, normalized size = 4.74

method	result	size
elliptic	Expression too large to display	1325
default	Expression too large to display	2062

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(8*(-d/c)^(1/2)*a*b*c^3*d^2*e*f*x+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c^2*d^3*f^2*x^2+3*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^
(1/2)/(-d/c)^(1/2))*b^2*c^4*d*f^2*x^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^2*d
^3*e^2*x^2-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a^2*c^2*d^3*e*f-6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^4*d*f^2-5*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^2
*d^3*e^2+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*a^2*c^2*d^3*e*f+5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^2*d^3*e^2-6*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-
d/c)^(1/2))*b^2*c^4*d*e*f+2*(-d/c)^(1/2)*a^2*d^5*e^2*x^3-4*(-d/c)^(1/2)*a^2
*c*d^4*f^2*x^5+2*(-d/c)^(1/2)*a^2*d^5*e*f*x^5-5*(-d/c)^(1/2)*a^2*c^2*d^3*f^
2*x^3+3*(-d/c)^(1/2)*a^2*c*d^4*e^2*x+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*f^2*c^
5-5*(-d/c)^(1/2)*a*b*c*d^4*e*f*x^5+(-d/c)^(1/2)*a*b*c^2*d^3*e*f*x^3+2*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*a^2*d^5*e^2*x^2-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d^5*e^2*x^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c^3*d^2*f^2+2*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*a^2*c*d^4*e^2-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^4*e^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^3*
d^2*e^2-(-d/c)^(1/2)*a^2*c*d^4*e*f*x^3+8*(-d/c)^(1/2)*a*b*c^3*d^2*f^2*x^3-5
*(-d/c)^(1/2)*a*b*c*d^4*e^2*x^3-5*(-d/c)^(1/2)*a^2*c^2*d^3*e*f*x-6*(-d/c)^(
1/2)*a*b*c^2*d^3*e^2*x+7*(-d/c)^(1/2)*a*b*c^2*d^3*f^2*x^5+11*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^2
*d^3*e*f*x^2-7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*a*b*c^2*d^3*e*f*x^2+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^4*e*f*x^2+5*((d*
```

```
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*a*b*c*d^4*e^2*x^2-6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x
*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^3*d^2*e*f*x^2+11*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*a*b*c^3*d^2*e*f-7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^3*d^2*e*f-5*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^4*e*f*x^2-6*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*a*b*c^3*d^2*f^2*x^2-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipti
cF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*d^4*e^2*x^2)/(f*x^2+e)^(1/2)/(a*d-
b*c)^2/(c*f-d*e)^2/c^2/(-d/c)^(1/2)/a/(d*x^2+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^2)\,(c + dx^2)^{\frac{5}{2}}\,\sqrt{e + fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(5/2)*sqrt(e + f*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{5/2}\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)), x)
```

3.83 $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=980

$$\frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b(be-af)^2\sqrt{e+fx^2}}+\frac{(be(6d^2e^2-7cdef-c^2f^2)-af(8d^2e^2-13cdef+3c^2f^2))\,x\sqrt{c}}{3ef(be-af)^2\sqrt{e+fx^2}}$$

```
[Out] (-c*f+d*e)*x*(d*x^2+c)^(3/2)/e/(-a*f+b*e)/(f*x^2+e)^(1/2)+1/3*(-a*d+b*c)*(-
3*a*d*f+4*b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)/b/(-a*f+b*e)^2/(f*x^2+e)^(1/2)+1/3
*(b*e*(-c^2*f^2-7*c*d*e*f+6*d^2*e^2)-a*f*(3*c^2*f^2-13*c*d*e*f+8*d^2*e^2))*
x*(d*x^2+c)^(1/2)/e/f/(-a*f+b*e)^2/(f*x^2+e)^(1/2)-1/3*(b*e*(-c^2*f^2-7*c*d
*e*f+6*d^2*e^2)-a*f*(3*c^2*f^2-13*c*d*e*f+8*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)
*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f
)^(1/2))*(d*x^2+c)^(1/2)/f^(3/2)/(-a*f+b*e)^2/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2
+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(2*a*d*f*(-3*c*f+2*d*e)-b*(-3*c^2*f^2-2*c*d*
e*f+3*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)
/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/f^(3/
2)/(-a*f+b*e)^2/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d*(-3*a
*d+5*b*c)*(-a*d+b*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ellipt
icF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/
b/c/(-a*f+b*e)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+(-
a*d+b*c)^3*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(
1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)
/a/b/c/(-a*f+b*e)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
-1/3*(-a*d+b*c)*(-3*a*d*f+4*b*c*f+b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^
(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1
/2)*(d*x^2+c)^(1/2)/b/(-a*f+b*e)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/
(f*x^2+e)^(1/2)+1/3*d*(-a*d+b*c)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(-a*f+b*
e)^2+1/3*d*(a*f*(-3*c*f+4*d*e)-b*e*(-2*c*f+3*d*e))*x*(d*x^2+c)^(1/2)*(f*x^2
+e)^(1/2)/e/f/(-a*f+b*e)^2
```

Rubi [A]
time = 0.75, antiderivative size = 980, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {558, 557, 553, 542, 545, 429, 506, 422, 540}

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((b*c - a*d)*(b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b*(b*e - a*f
)^2*Sqrt[e + f*x^2]) + ((b*e*(6*d^2*e^2 - 7*c*d*e*f - c^2*f^2) - a*f*(8*d^2
*e^2 - 13*c*d*e*f + 3*c^2*f^2))*x*Sqrt[c + d*x^2])/(3*e*f*(b*e - a*f)^2*Sqr
t[e + f*x^2]) + ((d*e - c*f)*x*(c + d*x^2)^(3/2))/(e*(b*e - a*f)*Sqrt[e + f
*x^2]) + (d*(b*c - a*d)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*(b*e - a*f)^2
) + (d*(a*f*(4*d*e - 3*c*f) - b*e*(3*d*e - 2*c*f))*x*Sqrt[c + d*x^2]*Sqrt[e
 + f*x^2])/(3*e*f*(b*e - a*f)^2) - ((b*c - a*d)*Sqrt[e]*(b*d*e + 4*b*c*f -
3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(
c*f)])/(3*b*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqr
t[e + f*x^2]) - ((b*e*(6*d^2*e^2 - 7*c*d*e*f - c^2*f^2) - a*f*(8*d^2*e^2 -
13*c*d*e*f + 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(3*Sqrt[e]*f^(3/2)*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2)
)/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*(5*b*c - 3*a*d)*(b*c - a*d)*e^(3/2
)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/
(3*b*c*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2]) - (Sqrt[e]*(2*a*d*f*(2*d*e - 3*c*f) - b*(3*d^2*e^2 - 2*c*d*e*f - 3
*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/(3*f^(3/2)*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqr
t[e + f*x^2]) + ((b*c - a*d)^3*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)
/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*b*c*Sqrt[f]*(b*e
- a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 557

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[(2*b*c - a*d + b*d*x^2)*(Sqrt
[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rule 558

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b*((b*e - a*f)/(b*c - a*d)^2), Int[(c + d*x^2)^(q
 + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Dist[1/(b*c - a*d)^2, Int
[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(b*
e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && GtQ[
r, 1]
```

Rubi steps

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = -\frac{\int \frac{(c+dx^2)^{3/2}(-bde^2+2bcef-acf^2+(bc-ad)f^2x^2)}{(e+fx^2)^{3/2}}\,dx}{(be-af)^2} + \frac{(b(bc-ad))\int \frac{(c+dx^2)^{3/2}\sqrt{e+j}}{a+bx^2}}{(be-af)^2}$$

$$= \frac{(de-cf)x(c+dx^2)^{3/2}}{e(be-af)\sqrt{e+fx^2}} + \frac{(d(bc-ad))\int \frac{(2bc-ad+bdx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx}{b(be-af)^2} + \frac{(bc-a}{}$$

$$= \frac{(de-cf)x(c+dx^2)^{3/2}}{e(be-af)\sqrt{e+fx^2}} + \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3(be-af)^2} + \frac{d(af(4de-3c}{}$$

$$= \frac{(de-cf)x(c+dx^2)^{3/2}}{e(be-af)\sqrt{e+fx^2}} + \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3(be-af)^2} + \frac{d(af(4de-3c}{}$$

$$= \frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b(be-af)^2\sqrt{e+fx^2}} + \frac{(be(6d^2e^2-7cdef-c^2f^2)-af(}{3ef(be-}$$

$$= \frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b(be-af)^2\sqrt{e+fx^2}} + \frac{(be(6d^2e^2-7cdef-c^2f^2)-af(}{3ef(be-}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.09, size = 352, normalized size = 0.36

$$\frac{-iabde(-ad^2ef+b(2d^2e^2-2cdef+c^2f^2))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-iad^2e(be-af)(-2bde+3bcf-adf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-f\left(ab^2\sqrt{\frac{d}{c}}(de-cf)^2x(c+dx^2)+i(bc-ad)^3ef\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)}{ab^2\sqrt{\frac{d}{c}}ef^2(be-af)\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((-I)*a*b*d*e*(-(a*d^2*e*f) + b*(2*d^2*e^2 - 2*c*d*e*f + c^2*f^2))*Sqrt[1 +
 (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*
e)] - I*a*d^2*e*(b*e - a*f)*(-2*b*d*e + 3*b*c*f - a*d*f)*Sqrt[1 + (d*x^2)/c
]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - f*(a
```

```
*b^2*Sqrt[d/c]*(d*e - c*f)^2*x*(c + d*x^2) + I*(b*c - a*d)^3*e*f*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x
], (c*f)/(d*e)]))/(a*b^2*Sqrt[d/c]*e*f^2*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e
 + f*x^2])
```

Maple [A]
time = 0.16, size = 1063, normalized size = 1.08 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-d/c)^(1/2)*a*b^2*c^2*d*f^3*x^3-2*(-d/c)^(1/2)*a*b^2*c*d^2*e*f^2*x^3+(-d/
c)^(1/2)*a*b^2*d^3*e^2*f*x^3-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipt
icF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*d^3*e*f^2+3*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c*d^2*e*f^
2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a^2*b*d^3*e^2*f-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
F(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d^2*e^2*f+2*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*d^3*e^3+
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*a^2*b*d^3*e^2*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c^2*d*e*f^2+2*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d^2*e^2*f-
2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a*b^2*d^3*e^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x
*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*d^3*e*f^2-3*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2
)/(-d/c)^(1/2))*a^2*b*c*d^2*e*f^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c^2*d*e
*f^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/
a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c^3*e*f^2+(-d/c)^(1/2)*a*b^2*c^3*f^3*x-2
*(-d/c)^(1/2)*a*b^2*c^2*d*e*f^2*x+(-d/c)^(1/2)*a*b^2*c*d^2*e^2*f*x)*(f*x^2+
e)^(1/2)*(d*x^2+c)^(1/2)/a/b^2/(-d/c)^(1/2)/e/(a*f-b*e)/f^2/(d*f*x^4+c*f*x^
2+d*e*x^2+c*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^2)^{\frac{5}{2}}}{(a + bx^2)(e + fx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/((a + b*x**2)*(e + f*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x^2+c)^{5/2}}{(b\,x^2+a)\,(f\,x^2+e)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x)
```

```
[Out] int((c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(3/2)), x)
```

3.84 $\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=223

$$\frac{(de-cf)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\,\sqrt{f}\,(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}}+\frac{c^{3/2}(bc-ad)\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\ldots\right.}{a\sqrt{d}\,e(be-af)\sqrt{c+dx^2}\,\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] (-c*f+d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1
/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/(-a*f+b*e)/e^(1/2)
/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+c^(3/2)*(-a*d+b*c)
*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2)/c^(1/2)/(1+d*
x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/e/(-a*f+b*e)/d^
(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {556, 553, 422}

$$\frac{c^{3/2}\sqrt{e+fx^2}\,(bc-ad)\Pi\left(1-\frac{bc}{ad};\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}\,e\sqrt{c+dx^2}\,(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{\sqrt{c+dx^2}\,(de-cf)E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\,\sqrt{f}\,\sqrt{e+fx^2}\,(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((d*e - c*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*
e)/(c*f)])/(Sqrt[e]*Sqrt[f]*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*Sqrt[e + f*x^2]*EllipticPi[1 - (
b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*(b*
e - a*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 556

```
Int[((e_) + (f_.)*(x_)^2)^(3/2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2
)^(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[Sqrt[e + f*x^2]/((
a + b*x^2)*Sqrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqr
t[e + f*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}\,dx = \frac{(bc-ad)\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{be-af} + \frac{(de-cf)\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{be-af}$$

$$= \frac{(de-cf)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\,\sqrt{f}\,(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}} + \frac{c^{3/2}(bc-ad)\sqrt{e+fx^2}\,\Pi}{a\sqrt{d}\,e(be-af)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.17, size = 304, normalized size = 1.36

$$\frac{ab\sqrt{\frac{d}{c}}\,f(de-cf)x(c+dx^2)-iabde(-de+cf)\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)-iad^2e(be-af)\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)-i(bc-ad)^2ef\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)}{ab\sqrt{\frac{d}{c}}\,ef(be-af)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] (a*b*Sqrt[d/c]*f*(d*e - c*f)*x*(c + d*x^2) - I*a*b*d*e*(-(d*e) + c*f)*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)] - I*a*d^2*e*(b*e - a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elli
pticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*(b*c - a*d)^2*e*f*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x
], (c*f)/(d*e)])/(a*b*Sqrt[d/c]*e*f*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*
x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(273) = 546$.

time = 0.15, size = 594, normalized size = 2.66

method	result
default	$\frac{\left(\sqrt{-\frac{d}{c}}\, abcd\, f^{2} x^{3}-\sqrt{-\frac{d}{c}}\, ab\, d^{2} e f\, x^{3}+\sqrt{\frac{d\, x^{2}+c}{c}}\, \sqrt{\frac{f\, x^{2}+e}{e}}\, \mathrm{EllipticF}\left(x \sqrt{-\frac{d}{c}}, \sqrt{\frac{c f}{d e}}\right) a^{2} d^{2} e f-\sqrt{\frac{d\, x^{2}+c}{c}}\, \sqrt{\frac{f\, x^{2}+e}{e}}\right.}{}$
elliptic	$\frac{\sqrt{\left(d\, x^{2}+c\right)\left(f\, x^{2}+e\right)}\, \left(\frac{\left(d f\, x^{2}+c f\right)(c f-d e) x}{f(a f-b e) e \sqrt{\left(x^{2}+\frac{e}{f}\right)\left(d f\, x^{2}+c f\right)}}+\frac{\sqrt{1+\frac{d x^{2}}{c}}\, \sqrt{1+\frac{f x^{2}}{e}}\, \mathrm{EllipticF}\left(x \sqrt{-\frac{d}{c}}\right.}{\sqrt{-\frac{d}{c}}\, \sqrt{d f\, x^{4}+c f\, x^{2}+d e}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-d/c)^(1/2)*a*b*c*d*f^2*x^3-(-d/c)^(1/2)*a*b*d^2*e*f*x^3+((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d^2*e
*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*a*b*d^2*e^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d^2*e^2-((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d
/c)^(1/2))*a^2*d^2*e*f+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi
(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c*d*e*f-((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/
(-d/c)^(1/2))*b^2*c^2*e*f+(-d/c)^(1/2)*a*b*c^2*f^2*x-(-d/c)^(1/2)*a*b*c*d*e
*f*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/b/a/(-d/c)^(1/2)/e/f/(a*f-b*e)/(d*f*x
^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)
[Out] Integral((c + d*x**2)**(3/2)/((a + b*x**2)*(e + f*x**2)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x^2+c)^{3/2}}{(b\,x^2+a)\,(f\,x^2+e)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x)
[Out] int((c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(3/2)), x)
```

3.85 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{\sqrt{f}\,\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\,(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}}+\frac{bc^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}\,e(be-af)\sqrt{c+dx^2}\,\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] -(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x
^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/(-a*f+b*e)/e^(1/2)/(
e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+b*c^(3/2)*(1/(1+d*x^2/c))^(1
/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),1-b*c/
a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/e/(-a*f+b*e)/d^(1/2)/(d*x^2+c)^(1/
2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {555, 553, 422}

$$\frac{bc^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}\,e\sqrt{c+dx^2}\,(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{\sqrt{f}\,\sqrt{c+dx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\,\sqrt{e+fx^2}\,(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] -((Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/(Sqrt[e]*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2])) + (b*c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*(b*e - a*f)*Sqrt[c + d
*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 555

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\,(e+fx^2)^{3/2}}\,dx = \frac{b\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{be-af} - \frac{f\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{be-af}$$

$$= -\frac{\sqrt{f}\,\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\,(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}} + \frac{bc^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\text{ta}\right.}{a\sqrt{d}\,e(be-af)\sqrt{c+d}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.60, size = 207, normalized size = 0.99

$$\frac{-a\sqrt{\frac{d}{c}}\,fx(c+dx^2)-iade\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)-i(bc-ad)e\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)}{a\sqrt{\frac{d}{c}}\,e(be-af)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] (-(a*Sqrt[d/c]*f*x*(c + d*x^2)) - I*a*d*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x
^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*(b*c - a*d)*e*Sqr
t[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*e*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2])
```

Maple [A]
time = 0.14, size = 285, normalized size = 1.36

method	result
default	$\frac{\left(\sqrt{-\frac{d}{c}}\, adf\, x^{3}-\sqrt{\frac{d\,x^{2}+c}{c}}\, \sqrt{\frac{f\,x^{2}+e}{e}}\, \operatorname{EllipticE}\left(x\sqrt{-\frac{d}{c}}, \sqrt{\frac{cf}{de}}\right) ade+\sqrt{\frac{d\,x^{2}+c}{c}}\, \sqrt{\frac{f\,x^{2}+e}{e}}\, \operatorname{EllipticPi}\left(x\sqrt{-\frac{d}{c}}, \frac{b}{a}\right.\right.}{a\sqrt{-\frac{d}{c}}\, e(af-be)(}$
elliptic	$\frac{\sqrt{(d\,x^{2}+c)\,(f\,x^{2}+e)}\left(\frac{(df\,x^{2}+cf)\,x}{e(af-be)\sqrt{\left(x^{2}+\frac{e}{f}\right)(df\,x^{2}+cf)}}-\frac{d\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{f\,x^{2}}{e}}\, \operatorname{EllipticE}\left(x\sqrt{-\frac{d}{c}}\right.}{(af-be)\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^{4}+cf\,x^{2}+c}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-d/c)^(1/2)*a*d*f*x^3-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*d*e-((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^
(1/2)/(-d/c)^(1/2))*b*c*e+(-d/c)^(1/2)*a*c*f*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(
1/2)/a/(-d/c)^(1/2)/e/(a*f-b*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)
[Out] Integral(sqrt(c + d*x**2)/((a + b*x**2)*(e + f*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^2+c}}{\left(b\,x^2+a\right)\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x)
[Out] int((c + d*x^2)^(1/2)/((a + b*x^2)*(e + f*x^2)^(3/2)), x)
```

3.86 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=344

$$\frac{f^{3/2}\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\,(be-af)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}}-\frac{\sqrt{e}\,\sqrt{f}\,(2bde-bcf-adf)\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right.\right.}{c(be-af)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}}$$

```
[Out] f^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)
/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*e)
/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+b^2*e^(3/2)*(1/(1+
f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^
(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/c/(-a*f+b*e)^2/f^(1/2)
/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(-a*d*f-b*c*f+2*b*d*e)*(1/
(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e
)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c/(-a*f+b*e)^2/(
-c*f+d*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {560, 553, 539, 429, 422}

$$\frac{b^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\,\sqrt{e+fx^2}\,(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{f^{3/2}\sqrt{c+dx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\,\sqrt{e+fx^2}\,(be-af)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\,\sqrt{f}\,\sqrt{c+dx^2}\,(-adf-bcf+2bde)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{e+fx^2}\,(be-af)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] (f^(3/2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(
c*f)])/(Sqrt[e]*(b*e - a*f)*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[f]*(2*b*d*e - b*c*f - a*d*f)*Sqrt[c + d
*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*e - a*
f)^2*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (
b^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/
Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))
/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{b^2\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{(be-af)^2} - \frac{f\int \frac{2be-af+bfx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx}{(be-af)^2}$$

$$= \frac{b^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\,(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}} + \frac{f^2\int \frac{\sqrt{c\cdots}}{(e+\cdots}}{(be-af)\cdots}$$

$$= \frac{f^{3/2}\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\,(be-af)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}} - \frac{\sqrt{e}\,\sqrt{f}\,(2b\cdots}{c(\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.28, size = 221, normalized size = 0.64

$$\frac{-a\sqrt{\frac{d}{c}}\,f^2x(c+dx^2) - iadef\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right) - ibe(-de+cf)\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)}{a\sqrt{\frac{d}{c}}\,e(-be+af)(de-cf)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] (-(a*Sqrt[d/c]*f^2*x*(c + d*x^2)) - I*a*d*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
(f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*b*e*(-(d*e) +
 c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*Arc
Sinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*e*(-(b*e) + a*f)*(d*e - c*f)*
Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.15, size = 303, normalized size = 0.88

method	result
default	$\frac{\left(\sqrt{-\frac{d}{c}}\,adf^2x^3-\sqrt{\frac{dx^2+c}{c}}\,\sqrt{\frac{fx^2+e}{e}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)adef-\sqrt{\frac{dx^2+c}{c}}\,\sqrt{\frac{fx^2+e}{e}}\,\mathrm{EllipticPi}\left(x\sqrt{-\frac{d}{c}}\cdots\right.\right.}{a\sqrt{-\frac{d}{c}}\,e(cf-de)(\cdots}$

elliptic	$$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\left(\frac{\left(df\,x^2+cf\right)fx}{e(cf-de)(af-be)\sqrt{\left(x^2+\frac{e}{f}\right)\left(df\,x^2+cf\right)}}+\frac{\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\mathrm{EllipticF}\left(x\sqrt{\cdots}\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\cdots}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)

[Out] ((-d/c)^(1/2)*a*d*f^2*x^3-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*d*e^2+(-d/c)^(1/2)*a*c*f^2*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/a/(-d/c)^(1/2)/e/(c*f-d*e)/(a*f-b*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

3.87 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=539

$$-\frac{d^2x}{c(bc-ad)(de-cf)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}-\frac{b^2\sqrt{f}\,\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{(bc-ad)^2\sqrt{e}\,(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\sqrt{e+fx^2}}-\frac{d\sqrt{f}\,(2bc^2}{c(b}$$

```
[Out] -d^2*x/c/(-a*d+b*c)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)-d^2*(2*a*d*f
-3*b*c*f+b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)
/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/c/(-a
*d+b*c)^2/(-c*f+d*e)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1
/2)-b^2*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)
/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^2/
(-a*f+b*e)/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-d*(2*b*c
^2*f-a*d*(c*f+d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(
1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c
/(-a*d+b*c)^2/(-c*f+d*e)^2/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e
)^(1/2)+b^3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^
(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2
)/a/(-a*d+b*c)^2/e/(-a*f+b*e)/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2
+c))^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 539, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {560, 555, 553, 422, 541, 539, 429}

$$\frac{b^3c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}\,e\sqrt{c+dx^2}\,(bc-ad)^2(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{b^2\sqrt{f}\,\sqrt{c+dx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\,\sqrt{e+fx^2}\,(bc-ad)^2(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{d\sqrt{f}\,\sqrt{c+dx^2}\,(2bc^2f-ad(cf+de))\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{e}\,\sqrt{e+fx^2}\,(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{d^2\sqrt{e}\,\sqrt{c+dx^2}\,(2adf-3bcf+bde)F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\,\sqrt{e+fx^2}\,(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{d^2x}{c\sqrt{c+dx^2}\,\sqrt{e+fx^2}\,(bc-ad)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] -((d^2*x)/(c*(b*c - a*d)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])) - (b
^2*Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/((b*c - a*d)^2*Sqrt[e]*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*
x^2))]*Sqrt[e + f*x^2]) - (d*Sqrt[f]*(2*b*c^2*f - a*d*(d*e + c*f))*Sqrt[c +
 d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c -
a*d)^2*Sqrt[e]*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2]) - (d^2*Sqrt[e]*(b*d*e - 3*b*c*f + 2*a*d*f)*Sqrt[c + d*x^2]*Ellipti
cF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c - a*d)^2*Sqrt[f]*
(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b^3
```

```
*c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*(b*c - a*d)^2*e*(b*e - a*f)*Sqrt[c + d*
x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 555

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
```

```
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx = \frac{b^2\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx}{(bc-ad)^2} - \frac{d\int \frac{2bc-ad+bdx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{d^2x}{c(bc-ad)(de-cf)\sqrt{c+dx^2}\,\sqrt{e+fx^2}} + \frac{b^3\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+f}}}{(bc-ad)^2(be-}$$

$$= -\frac{d^2x}{c(bc-ad)(de-cf)\sqrt{c+dx^2}\,\sqrt{e+fx^2}} - \frac{b^2\sqrt{f}\,\sqrt{c+dx^2}}{(bc-ad)^2\sqrt{e}\,(be}$$

$$= -\frac{d^2x}{c(bc-ad)(de-cf)\sqrt{c+dx^2}\,\sqrt{e+fx^2}} - \frac{b^2\sqrt{f}\,\sqrt{c+dx^2}}{(bc-ad)^2\sqrt{e}\,(be}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.42, size = 418, normalized size = 0.78

$$\frac{\frac{adx\left(-adf\left(c^2f^2+cdf^2x^2+d^2e(e+fx^2)\right)+b\left(c^3f^3+c^2df^3x^2+d^3e^2(e+fx^2)\right)\right)}{c}+iad\sqrt{\frac{d}{c}}\,e\left(-adf(de+cf)+b(d^2e^2+c^2f^2)\right)\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+iacd\left(\frac{d}{c}\right)^{3/2}e(be-af)(-de+cf)\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)+ib^2c\sqrt{\frac{d}{c}}\,e(de-cf)^2\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{cf}{de}\right)}{ad(-bc+ad)e(be-af)(de-cf)^2\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((a*d*x*(-(a*d*f*(c^2*f^2 + c*d*f^2*x^2 + d^2*e*(e + f*x^2))) + b*(c^3*f^3
+ c^2*d*f^3*x^2 + d^3*e^2*(e + f*x^2))))/c + I*a*d*Sqrt[d/c]*e*(-(a*d*f*(d*
e + c*f)) + b*(d^2*e^2 + c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*
```

```
EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*a*c*d*(d/c)^(3/2)*e*(b*e
 - a*f)*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*
ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*b^2*c*Sqrt[d/c]*e*(d*e - c*f)^2*Sqrt
[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[
d/c]*x], (c*f)/(d*e)])/(a*d*(-(b*c) + a*d)*e*(b*e - a*f)*(d*e - c*f)^2*Sqrt
[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A]
time = 0.17, size = 956, normalized size = 1.77

method	result
default	$\frac{\left(\sqrt{-\frac{d}{c}}\, a^{2} c\, d^{2} f^{3} x^{3}+\sqrt{-\frac{d}{c}}\, a^{2} d^{3} e\, f^{2} x^{3}-\sqrt{-\frac{d}{c}}\, a b\, c^{2} d\, f^{3} x^{3}-\sqrt{-\frac{d}{c}}\, a b\, d^{3} e^{2} f\, x^{3}-\sqrt{\frac{d\, x^{2}+c}{c}}\, \sqrt{\frac{f\, x^{2}+e}{e}}\, \mathrm{EllipticF}\left(x \right.\right.}{}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-d/c)^(1/2)*a^2*c*d^2*f^3*x^3+(-d/c)^(1/2)*a^2*d^3*e*f^2*x^3-(-d/c)^(1/2)
*a*b*c^2*d*f^3*x^3-(-d/c)^(1/2)*a*b*d^3*e^2*f*x^3-((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^2*e*f^2+((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*a^2*d^3*e^2*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*d^2*e^2*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d^3*e^3-((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d
^2*e*f^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*a^2*d^3*e^2*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellip
ticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^2*d*e*f^2+((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d^3*e^3+((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/
e)^(1/2)/(-d/c)^(1/2))*b^2*c^3*e*f^2-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^2*d*
e^2*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c
/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c*d^2*e^3+(-d/c)^(1/2)*a^2*c^2*d*f^3*x+
(-d/c)^(1/2)*a^2*d^3*e^2*f*x-(-d/c)^(1/2)*a*b*c^3*f^3*x-(-d/c)^(1/2)*a*b*d^
3*e^3*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/c/a/(a*d-b*c)/(-d/c)^(1/2)/e/(c*f-
d*e)^2/(a*f-b*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}(e+fx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(b\,x^2+a)\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x)
```

```
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x)
```

3.88 $\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=814

$$-\frac{d^2x}{3c(bc-ad)(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}-\frac{d^2(bc(5de-9cf)-2ad(de-3cf))x}{3c^2(bc-ad)^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}+\frac{b^2f}{(bc-ad)^2}$$

```
[Out] -1/3*d^2*x/c/(-a*d+b*c)/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2)-1/3*d^2*
(b*c*(-9*c*f+5*d*e)-2*a*d*(-3*c*f+d*e))*x/c^2/(-a*d+b*c)^2/(-c*f+d*e)^2/(d*
x^2+c)^(1/2)/(f*x^2+e)^(1/2)+b^2*f^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^
(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x
^2+c)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)/(-c*f+d*e)/e^(1/2)/(e*(d*x^2+c)/c/(f*x^
2+e))^(1/2)/(f*x^2+e)^(1/2)+b^4*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(
1/2)*EllipticPi(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(
1/2))*(d*x^2+c)^(1/2)/a/c/(-a*d+b*c)^2/(-a*f+b*e)^2/f^(1/2)/(e*(d*x^2+c)/c/
(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*d*(b*c*(-6*c^2*f^2-7*c*d*e*f+5*d^2*e^2
)-a*d*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(
1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/
2)*(d*x^2+c)^(1/2)/c^2/(-a*d+b*c)^2/(-c*f+d*e)^3/e^(1/2)/(e*(d*x^2+c)/c/(f*
x^2+e))^(1/2)/(f*x^2+e)^(1/2)-b^2*(-a*d*f-b*c*f+2*b*d*e)*(1/(1+f*x^2/e))^(1
/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/
c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c/(-a*d+b*c)^2/(-a*f+b*e)^2/(-c
*f+d*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d^2*(b*c*(-15*c
*f+7*d*e)-a*d*(-9*c*f+d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ellipti
cF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(
d*x^2+c)^(1/2)/c^2/(-a*d+b*c)^2/(-c*f+d*e)^3/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2
)/(f*x^2+e)^(1/2)
```

Rubi [A]
time = 0.63, antiderivative size = 814, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {560, 553, 539, 429, 422, 541}

$$\frac{e^{3/2}\sqrt{dx^2+c}\,\Pi\left(1-\frac{be}{af};\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)b^4}{ac(bc-ad)^2\sqrt{f}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}+\frac{f^{3/2}\sqrt{dx^2+c}\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)b^2}{(bc-ad)^2\sqrt{e}(be-af)(de-cf)\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}-\frac{\sqrt{e}\sqrt{f}(2bde-bcf-adf)\sqrt{dx^2+c}\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)b^2}{c(bc-ad)^2(be-af)^2(de-cf)\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}-\frac{d\sqrt{f}(bc(5d^2e^2-7cdfe-6c^2f^2)-ad(2d^2e^2-7cdfe-3c^2f^2))\sqrt{dx^2+c}\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2\sqrt{e}(de-cf)^3\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}+\frac{d^2\sqrt{e}\sqrt{f}(bc(7de-15cf)-ad(de-9cf))\sqrt{dx^2+c}\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2(de-cf)^3\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}-\frac{d^2(bc(5de-9cf)-2ad(de-3cf))x}{3c^2(bc-ad)^2(de-cf)^2\sqrt{dx^2+c}\sqrt{fx^2+e}}-\frac{d^2x}{3c(bc-ad)(de-cf)(dx^2+c)^{3/2}\sqrt{fx^2+e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] -1/3*(d^2*x)/(c*(b*c - a*d)*(d*e - c*f)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])
- (d^2*(b*c*(5*d*e - 9*c*f) - 2*a*d*(d*e - 3*c*f))*x)/(3*c^2*(b*c - a*d)^2*
(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]) + (b^2*f^(3/2)*Sqrt[c + d*x^
```

```
2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/((b*c - a*d)^2*
Sqrt[e]*(b*e - a*f)*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[
e + f*x^2]) - (d*Sqrt[f]*(b*c*(5*d^2*e^2 - 7*c*d*e*f - 6*c^2*f^2) - a*d*(2*
d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]
*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)^2*Sqrt[e]*(d*e - c*f)^3*
Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (b^2*Sqrt[e]*Sqrt[
f]*(2*b*d*e - b*c*f - a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c - a*d)^2*(b*e - a*f)^2*(d*e - c*f)*Sqrt[
(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d^2*Sqrt[e]*Sqrt[f]*(b
*c*(7*d*e - 15*c*f) - a*d*(d*e - 9*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)^2*(d*e - c*f)^3*S
qrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b^4*e^(3/2)*Sqrt[c
 + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*
(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \frac{b^2\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx}{(bc-ad)^2} - \frac{d\int \frac{2bc-ad+bdx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{d^2x}{3c(bc-ad)(de-cf)(c+dx^2)^{3/2}\sqrt{e+fx^2}} + \frac{b^4\int \frac{\sqrt{e+\cdots}}{(a+bx^2)\sqrt{c\cdots}}}{(bc-ad)^2(b\cdots}$$

$$= -\frac{d^2x}{3c(bc-ad)(de-cf)(c+dx^2)^{3/2}\sqrt{e+fx^2}} - \frac{d^2(bc(5de\cdots}{3c^2(bc-ad)^2\cdots}$$

$$= -\frac{d^2x}{3c(bc-ad)(de-cf)(c+dx^2)^{3/2}\sqrt{e+fx^2}} - \frac{d^2(bc(5de\cdots}{3c^2(bc-ad)^2\cdots}$$

$$= -\frac{d^2x}{3c(bc-ad)(de-cf)(c+dx^2)^{3/2}\sqrt{e+fx^2}} - \frac{d^2(bc(5de\cdots}{3c^2(bc-ad)^2\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 1645, normalized size = 2.02

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((-I)*a*d*e*(2*a*b*d*(d*e - 3*c*f)*(d*e + c*f)^2 + a^2*d^2*f*(-2*d^2*e^2 +
7*c*d*e*f + 3*c^2*f^2) + b^2*c*(-5*d^3*e^3 + 10*c*d^2*e^2*f + 3*c^3*f^3))*(
c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)] + (Sqrt[d/c]*(6*a*b^2*c^2*d^5*e^4*x - 3*a^2*b*c*d^6*
e^4*x - 11*a*b^2*c^3*d^4*e^3*f*x + 2*a^2*b*c^2*d^5*e^3*f*x + 3*a^3*c*d^6*e^
3*f*x + 11*a^2*b*c^3*d^4*e^2*f^2*x - 8*a^3*c^2*d^5*e^2*f^2*x - 3*a*b^2*c^6*
d*f^4*x + 6*a^2*b*c^5*d^2*f^4*x - 3*a^3*c^4*d^3*f^4*x + 5*a*b^2*c*d^6*e^4*x
^3 - 2*a^2*b*d^7*e^4*x^3 - 4*a*b^2*c^2*d^5*e^3*f*x^3 - a^2*b*c*d^6*e^3*f*x^
3 + 2*a^3*d^7*e^3*f*x^3 - 11*a*b^2*c^3*d^4*e^2*f^2*x^3 + 12*a^2*b*c^2*d^5*e
^2*f^2*x^3 - 4*a^3*c*d^6*e^2*f^2*x^3 + 11*a^2*b*c^3*d^4*e*f^3*x^3 - 8*a^3*c
^2*d^5*e*f^3*x^3 - 6*a*b^2*c^5*d^2*f^4*x^3 + 12*a^2*b*c^4*d^3*f^4*x^3 - 6*a
^3*c^3*d^4*f^4*x^3 + 5*a*b^2*c*d^6*e^3*f*x^5 - 2*a^2*b*d^7*e^3*f*x^5 - 10*a
*b^2*c^2*d^5*e^2*f^2*x^5 + 2*a^2*b*c*d^6*e^2*f^2*x^5 + 2*a^3*d^7*e^2*f^2*x^
5 + 10*a^2*b*c^2*d^5*e*f^3*x^5 - 7*a^3*c*d^6*e*f^3*x^5 - 3*a*b^2*c^4*d^3*f^
4*x^5 + 6*a^2*b*c^3*d^4*f^4*x^5 - 3*a^3*c^2*d^5*f^4*x^5 - I*a*c*d^2*Sqrt[d/
c]*e*(b*e - a*f)*(-(d*e) + c*f)*(2*a*d*(d*e - 3*c*f) + b*c*(-5*d*e + 9*c*f)
)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[S
qrt[d/c]*x], (c*f)/(d*e)] + (3*I)*b^3*c^4*d^3*Sqrt[d/c]*e^4*Sqrt[1 + (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c
*f)/(d*e)] - (9*I)*b^3*c^7*(d/c)^(5/2)*e^3*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (
f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (9
*I)*b^3*c^7*(d/c)^(3/2)*e^2*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ell
ipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*b^3*c^7*S
qrt[d/c]*e*f^3*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*
d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (3*I)*b^3*c^3*d^4*Sqrt[d/c]*e^4*
x^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSi
nh[Sqrt[d/c]*x], (c*f)/(d*e)] - (9*I)*b^3*c^4*d^3*Sqrt[d/c]*e^3*f*x^2*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d
/c]*x], (c*f)/(d*e)] + (9*I)*b^3*c^7*(d/c)^(5/2)*e^2*f^2*x^2*Sqrt[1 + (d*x^
2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (
c*f)/(d*e)] - (3*I)*b^3*c^7*(d/c)^(3/2)*e*f^3*x^2*Sqrt[1 + (d*x^2)/c]*Sqrt[
1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
))/d)/(3*a*c^2*Sqrt[d/c]*(b*c - a*d)^2*e*(b*e - a*f)*(-(d*e) + c*f)^3*(c +
d*x^2)^(3/2)*Sqrt[e + f*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4114 vs. $2(923) = 1846$.
time = 0.19, size = 4115, normalized size = 5.06

method	result	size
elliptic	Expression too large to display	2127

default	Expression too large to display	4115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(11*(-d/c)^(1/2)*a^2*b*c^3*d^3*e^2*f^2*x+2*(-d/c)^(1/2)*a^2*b*c^2*d^4*
e^3*f*x+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*a^2*b*c*d^5*e^4-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c^2*d^4*e^4-9*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(
-d/c)^(1/2))*b^3*c^5*d*e^2*f^2+9*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*El
lipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c^4*d^2*e^3*
f-3*(-d/c)^(1/2)*a*b^2*c^6*f^4*x-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c^3*d^3*e^
4+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a
/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c^6*e*f^3+5*(-d/c)^(1/2)*a*b^2*c*d^5*e^4*
x^3-8*(-d/c)^(1/2)*a^3*c^2*d^4*e^2*f^2*x+3*(-d/c)^(1/2)*a^3*c*d^5*e^3*f*x+6
*(-d/c)^(1/2)*a^2*b*c^5*d*f^4*x-3*(-d/c)^(1/2)*a^2*b*c*d^5*e^4*x+11*(-d/c)^
(1/2)*a^2*b*c^3*d^3*e*f^3*x^3+12*(-d/c)^(1/2)*a^2*b*c^2*d^4*e^2*f^2*x^3-(-d
/c)^(1/2)*a^2*b*c*d^5*e^3*f*x^3-11*(-d/c)^(1/2)*a*b^2*c^3*d^3*e^2*f^2*x^3-4
*(-d/c)^(1/2)*a*b^2*c^2*d^4*e^3*f*x^3+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d^6*e^4*x^2-3*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1
/2)/(-d/c)^(1/2))*b^3*c^2*d^4*e^4*x^2+6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*c^3*d^3*e*f^3-8*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a
^3*c^2*d^4*e^2*f^2+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*c*d^5*e^3*f-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c*d^5*e^4+5*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*a*b^2*c^2*d^4*e^4+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*c^3*d^3*e*f^3+7*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*c^2*d^4*e^2*f^
2-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d
/e)^(1/2))*a^3*c*d^5*e^3*f-3*(-d/c)^(1/2)*a^3*c^2*d^4*f^4*x^5+2*(-d/c)^(1/2
)*a^3*d^6*e^2*f^2*x^5-6*(-d/c)^(1/2)*a^3*c^3*d^3*f^4*x^3+2*(-d/c)^(1/2)*a^3
*d^6*e^3*f*x^3-11*(-d/c)^(1/2)*a*b^2*c^3*d^3*e^3*f*x+2*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*d^6*e^3*f
*x^2-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*a^2*b*d^6*e^4*x^2-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*d^6*e^3*f*x^2-2*(-d/c)^(1/2)*a
^2*b*d^6*e^4*x^3-3*(-d/c)^(1/2)*a^3*c^4*d^2*f^4*x+10*(-d/c)^(1/2)*a^2*b*c^2
*d^4*e*f^3*x^5+2*(-d/c)^(1/2)*a^2*b*c*d^5*e^2*f^2*x^5-10*(-d/c)^(1/2)*a*b^2
*c^2*d^4*e^2*f^2*x^5+5*(-d/c)^(1/2)*a*b^2*c*d^5*e^3*f*x^5-7*(-d/c)^(1/2)*a^
```

```
3*c*d^5*e*f^3*x^5+6*(-d/c)^(1/2)*a^2*b*c^3*d^3*f^4*x^5-2*(-d/c)^(1/2)*a^2*b
*d^6*e^3*f*x^5-3*(-d/c)^(1/2)*a*b^2*c^4*d^2*f^4*x^5-8*(-d/c)^(1/2)*a^3*c^2*
d^4*e*f^3*x^3-4*(-d/c)^(1/2)*a^3*c*d^5*e^2*f^2*x^3+12*(-d/c)^(1/2)*a^2*b*c^
4*d^2*f^4*x^3-6*(-d/c)^(1/2)*a*b^2*c^5*d*f^4*x^3+6*(-d/c)^(1/2)*a*b^2*c^2*d
^4*e^4*x+7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*a^3*c*d^5*e^2*f^2*x^2-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d^5*e^4*x^2+3*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e
)^(1/2)/(-d/c)^(1/2))*b^3*c^5*d*e*f^3*x^2-9*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c
^4*d^2*e^2*f^2*x^2+9*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(
-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c^3*d^3*e^3*f*x^2-9*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*a^2*b*c^4*d^2*e*f^3+8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(
x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c^3*d^3*e^2*f^2+3*((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c^2*d^
4*e^3*f+9*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*a*b^2*c^4*d^2*e^2*f^2-14*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c^3*d^3*e^3*f-6*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*a^2*b*c^4*d^2*e*f^3-10*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c^3*d^3*e^2*f^2-2*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c^2*d
^4*e^3*f+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*a*b^2*c^5*d*e*f^3+10*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c^3*d^3*e^3*f+3*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
a*b^2*c^4*d^2*e*f^3*x^2+10*((d*x^2+c)/c)^(1/2)*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}(e+fx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(5/2)*(e + f*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(b\,x^2+a)\,(d\,x^2+c)^{5/2}\,(f\,x^2+e)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x)
```

```
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)), x)
```

3.89 $\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx$

Optimal. Leaf size=242

$$-\frac{(a-2b)x\sqrt{2+x^2}}{b^2\sqrt{1+x^2}}+\frac{x\sqrt{1+x^2}\,\sqrt{2+x^2}}{3b}+\frac{\sqrt{2}\,(a-2b)\sqrt{2+x^2}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{b^2\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}}-\frac{(3a-7b)\sqrt{2+x^2}\,F(\mathrm{ta}}{3\sqrt{2}\,b^2\sqrt{1+x^2}\,\sqrt{\frac{2}{1}}}$$

```
[Out] -(a-2*b)*x*(x^2+2)^(1/2)/b^2/(x^2+1)^(1/2)+1/3*x*(x^2+1)^(1/2)*(x^2+2)^(1/2
)/b-1/6*(3*a-7*b)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*
(x^2+2)^(1/2)/b^2*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)+1/2*(a-2*b)*(a-b)*(1/(x^2
+1))^(1/2)*EllipticPi(x/(x^2+1)^(1/2),1-b/a,1/2*2^(1/2))*(x^2+2)^(1/2)/a/b^
2*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)+(a-2*b)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^
2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*(x^2+2)^(1/2)/b^2/((x^2+2)/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 239, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {557, 553, 542, 545, 429, 506, 422}

$$-\frac{\sqrt{2}\,\sqrt{x^2+2}\,(3a-5b)F\left(\mathrm{ArcTan}(x)\left|\frac{1}{2}\right.\right)}{3b^2\sqrt{x^2+1}\,\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{2}\,\sqrt{x^2+2}\,(a-2b)E\left(\mathrm{ArcTan}(x)\left|\frac{1}{2}\right.\right)}{b^2\sqrt{x^2+1}\,\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{2\sqrt{x^2+1}\,(a-b)^2\Pi\left(1-\frac{2b}{a};\mathrm{ArcTan}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{ab^2\sqrt{\frac{x^2+1}{x^2+2}}\,\sqrt{x^2+2}}-\frac{x\sqrt{x^2+2}\,(a-2b)}{b^2\sqrt{x^2+1}}+\frac{x\sqrt{x^2+1}\,\sqrt{x^2+2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)^(3/2)*Sqrt[2 + x^2])/(a + b*x^2),x]
```

```
[Out] -(((a - 2*b)*x*Sqrt[2 + x^2])/(b^2*Sqrt[1 + x^2])) + (x*Sqrt[1 + x^2]*Sqrt[
2 + x^2])/(3*b) + (Sqrt[2]*(a - 2*b)*Sqrt[2 + x^2]*EllipticE[ArcTan[x], 1/2
])/(b^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) - (Sqrt[2]*(3*a - 5*b)*Sqr
t[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(3*b^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(
1 + x^2)]) + (2*(a - b)^2*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTan[x/Sq
rt[2]], -1])/(a*b^2*Sqrt[(1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 557

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[(2*b*c - a*d + b*d*x^2)*(Sqrt
[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rubi steps

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx = \frac{\int \frac{\sqrt{2+x^2}\,(-a+2b+bx^2)}{\sqrt{1+x^2}}\,dx}{b^2} + \frac{(a-b)^2\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx}{b^2}$$

$$= \frac{x\sqrt{1+x^2}\,\sqrt{2+x^2}}{3b} + \frac{2(a-b)^2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{ab^2\sqrt{\frac{1+x^2}{2+x^2}}\,\sqrt{2+x^2}} + \frac{\int}{}$$

$$= \frac{x\sqrt{1+x^2}\,\sqrt{2+x^2}}{3b} + \frac{2(a-b)^2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{ab^2\sqrt{\frac{1+x^2}{2+x^2}}\,\sqrt{2+x^2}} - \frac{(}{}$$

$$= -\frac{(a-2b)x\sqrt{2+x^2}}{b^2\sqrt{1+x^2}} + \frac{x\sqrt{1+x^2}\,\sqrt{2+x^2}}{3b} - \frac{\sqrt{2}\,(3a-5b)\sqrt{2+x^2}\,F\left(\tan^{-}\right.}{3b^2\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}}$$

$$= -\frac{(a-2b)x\sqrt{2+x^2}}{b^2\sqrt{1+x^2}} + \frac{x\sqrt{1+x^2}\,\sqrt{2+x^2}}{3b} + \frac{\sqrt{2}\,(a-2b)\sqrt{2+x^2}\,E\left(\tan^{-1}\right.}{b^2\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.22, size = 204, normalized size = 0.84

$$\frac{ab^2x\sqrt{1+x^2}\,\sqrt{2+x^2}+3ia(a-2b)bE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-ia(3a^2-9ab+7b^2)\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+3ia^3\Pi\left(\frac{2b}{a};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-12ia^2b\Pi\left(\frac{2b}{a};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+15iab^2\Pi\left(\frac{2b}{a};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-6ib^3\Pi\left(\frac{2b}{a};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{3ab^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)^(3/2)*Sqrt[2 + x^2])/(a + b*x^2),x]
```

```
[Out] (a*b^2*x*Sqrt[1 + x^2]*Sqrt[2 + x^2] + (3*I)*a*(a - 2*b)*b*EllipticE[I*ArcS
inh[x/Sqrt[2]], 2] - I*a*(3*a^2 - 9*a*b + 7*b^2)*EllipticF[I*ArcSinh[x/Sqrt
[2]], 2] + (3*I)*a^3*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] - (12*I)*
a^2*b*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] + (15*I)*a*b^2*EllipticP
i[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] - (6*I)*b^3*EllipticPi[(2*b)/a, I*ArcSi
nh[x/Sqrt[2]], 2])/(3*a*b^3)
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 370, normalized size = 1.53

method	result

risch	$\frac{x\sqrt{x^2+1}\,\sqrt{x^2+2}}{3b} - \frac{\left(\frac{3i(a-2b)\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)\right)}{2b\sqrt{x^4+3x^2+2}}\right.}{}$
default	$-\frac{\sqrt{x^2+1}\,\sqrt{x^2+2}\,\left(-a\,b^2x^5+3i\sqrt{x^2+1}\,\sqrt{x^2+2}\,\text{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a^3-9i\sqrt{x^2+1}\,\sqrt{x^2+2}\,\text{El}\right.}{}$
elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\,\left(\frac{x\sqrt{x^4+3x^2+2}}{3b}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,a\,\text{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}\,b^2}-\frac{i\sqrt{2}\,\sqrt{2x}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(x^2+1)^(1/2)*(x^2+2)^(1/2)*(-a*b^2*x^5+3*I*(x^2+1)^(1/2)*(x^2+2)^(1/2
)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a^3-9*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*El
lipticF(1/2*I*x*2^(1/2),2^(1/2))*a^2*b+7*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*Elli
pticF(1/2*I*x*2^(1/2),2^(1/2))*a*b^2-3*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*Ellipt
icE(1/2*I*x*2^(1/2),2^(1/2))*a^2*b+6*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*Elliptic
E(1/2*I*x*2^(1/2),2^(1/2))*a*b^2-3*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*EllipticPi
(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a^3+12*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*Ellipt
icPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a^2*b-15*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*
EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a*b^2+6*I*(x^2+1)^(1/2)*(x^2+2)^(
1/2)*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b^3-3*a*b^2*x^3-2*a*b^2*x)/(
x^4+3*x^2+2)/b^3/a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2+1)^{\frac{3}{2}}\sqrt{x^2+2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(3/2)*(x**2+2)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral((x**2 + 1)**(3/2)*sqrt(x**2 + 2)/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(x^2+1\right)}^{3/2}\,\sqrt{x^2+2}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)^(3/2)*(x^2 + 2)^(1/2))/(a + b*x^2),x)
```

```
[Out] int(((x^2 + 1)^(3/2)*(x^2 + 2)^(1/2))/(a + b*x^2), x)
```

3.90 $\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx$

Optimal. Leaf size=192

$$\frac{x\sqrt{2+x^2}}{b\sqrt{1+x^2}}-\frac{\sqrt{2}\sqrt{2+x^2}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{b\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}+\frac{\sqrt{2+x^2}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\,b\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}-\frac{(a-2b)\sqrt{2+x^2}\,\Pi\left(1-\frac{b}{a};\tan^{-1}(x)\right.}{\sqrt{2}\,ab\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

```
[Out] x*(x^2+2)^(1/2)/b/(x^2+1)^(1/2)+1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(
1/2),1/2*2^(1/2))*(x^2+2)^(1/2)/b*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)-1/2*(a-2*
b)*(1/(x^2+1))^(1/2)*EllipticPi(x/(x^2+1)^(1/2),1-b/a,1/2*2^(1/2))*(x^2+2)^
(1/2)/a/b*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)-(1/(x^2+1))^(1/2)*EllipticE(x/(x^
2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*(x^2+2)^(1/2)/b/((x^2+2)/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {548, 433, 429, 506, 422, 553}

$$-\frac{\sqrt{x^2+2}\,(a-2b)\Pi\left(1-\frac{b}{a};\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2}\,ab\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{x^2+2}\,F\left(\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2}\,b\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{x^2+2}\,E\left(\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{b\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{x^2+2}\,x}{b\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x^2]*Sqrt[2 + x^2])/(a + b*x^2),x]
```

```
[Out] (x*Sqrt[2 + x^2])/(b*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + x^2]*EllipticE[ArcT
an[x], 1/2])/(b*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) + (Sqrt[2 + x^2]*E
llipticF[ArcTan[x], 1/2])/(Sqrt[2]*b*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)
]) - ((a - 2*b)*Sqrt[2 + x^2]*EllipticPi[1 - b/a, ArcTan[x], 1/2])/(Sqrt[2]
*a*b*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 548

```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Dist[d/b, Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
Dist[(b*c - a*d)/b, Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f/e, 0] &&  !Simpl
erSqrtQ[d/c, f/e]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2}\,\sqrt{2+x^2}}{a+bx^2}\,dx = \frac{\int \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\,dx}{b} + \frac{(-a+2b)\int \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}\,(a+bx^2)}\,dx}{b}$$
$$= -\frac{(a-2b)\sqrt{2+x^2}\,\Pi\left(1-\frac{b}{a};\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\,ab\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}} + \frac{\int \frac{1}{\sqrt{1+x^2}\,\sqrt{2+x^2}}\,dx}{b} + \frac{\int \frac{}{\sqrt{1+}}}{}$$
$$= \frac{x\sqrt{2+x^2}}{b\sqrt{1+x^2}} + \frac{\sqrt{2+x^2}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\,b\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{(a-2b)\sqrt{2+x^2}\,\Pi\left(1-\frac{b}{a};\tan^{-1}(x)|\right.}{\sqrt{2}\,ab\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}}$$
$$= \frac{x\sqrt{2+x^2}}{b\sqrt{1+x^2}} - \frac{\sqrt{2}\,\sqrt{2+x^2}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{b\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}} + \frac{\sqrt{2+x^2}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\,b\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{(a-}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.90, size = 71, normalized size = 0.37

$$\frac{i\left(-2abE\left(i\sinh^{-1}(x)|\frac{1}{2}\right)+(a-b)\left(aF\left(i\sinh^{-1}(x)|\frac{1}{2}\right)-(a-2b)\Pi\left(\frac{b}{a};i\sinh^{-1}(x)|\frac{1}{2}\right)\right)\right)}{\sqrt{2}\,ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x^2]*Sqrt[2 + x^2])/(a + b*x^2),x]
```

```
[Out] (I*(-2*a*b*EllipticE[I*ArcSinh[x], 1/2] + (a - b)*(a*EllipticF[I*ArcSinh[x]
, 1/2] - (a - 2*b)*EllipticPi[b/a, I*ArcSinh[x], 1/2])))/(Sqrt[2]*a*b^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 121, normalized size = 0.63

method	result
default	$\frac{i\left(\operatorname{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a^2-2\operatorname{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)ba-\operatorname{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)ba-a^2\operatorname{EllipticPi}\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)\right.}{a\,b^2}$
elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\left(\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a}{2\sqrt{x^4+3x^2+2}\,b^2}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{Ell}}{b\sqrt{x^4+3x^2+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] I*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a^2-2*EllipticF(1/2*I*x*2^(1/2),2^(1/
2))*b*a-EllipticE(1/2*I*x*2^(1/2),2^(1/2))*b*a-a^2*EllipticPi(1/2*I*x*2^(1/
2),2*b/a,2^(1/2))+3*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b*a-2*Ellipti
cPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b^2)/a/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}\,\sqrt{x^2+2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)*(x**2+2)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(x**2 + 1)*sqrt(x**2 + 2)/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^2+1}\,\sqrt{x^2+2}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)^(1/2)*(x^2 + 2)^(1/2))/(a + b*x^2),x)
[Out] int(((x^2 + 1)^(1/2)*(x^2 + 2)^(1/2))/(a + b*x^2), x)
```

3.91 $\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{1+x^2}{2+x^2}}\,\sqrt{2+x^2}}$$

```
[Out] 2*(1/(2*x^2+4))^(1/2)*(2*x^2+4)^(1/2)*EllipticPi(x*2^(1/2)/(2*x^2+4)^(1/2),
1-2*b/a,I)*(x^2+1)^(1/2)/a/((x^2+1)/(x^2+2))^(1/2)/(x^2+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {553}

$$\frac{2\sqrt{x^2+1}\,\Pi\left(1-\frac{2b}{a};\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\,\sqrt{x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2]/(Sqrt[1 + x^2]*(a + b*x^2)),x]
```

```
[Out] (2*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTan[x/Sqrt[2]], -1])/(a*Sqrt[(1
 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx=\frac{2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{1+x^2}{2+x^2}}\,\sqrt{2+x^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.87, size = 50, normalized size = 0.86

$$-\frac{i\left(aF\left(i\sinh^{-1}(x)|\frac{1}{2}\right)-(a-2b)\Pi\left(\frac{b}{a};i\sinh^{-1}(x)|\frac{1}{2}\right)\right)}{\sqrt{2}\,ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2]/(Sqrt[1 + x^2]*(a + b*x^2)),x]
[Out] ((-I)*(a*EllipticF[I*ArcSinh[x], 1/2] - (a - 2*b)*EllipticPi[b/a, I*ArcSinh
[x], 1/2]))/(Sqrt[2]*a*b)
```

Maple [A]
time = 0.14, size = 64, normalized size = 1.10

method	result
default	$-\frac{i\left(a\,\mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)-a\,\mathrm{EllipticPi}\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)+2b\,\mathrm{EllipticPi}\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)\right)}{ab}$
elliptic	$\frac{\sqrt{\left(x^{2}+1\right)\left(x^{2}+2\right)}\left(-\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2b\sqrt{x^{4}+3x^{2}+2}}+\frac{i\sqrt{2}\,\sqrt{1+\frac{x^{2}}{2}}\,\sqrt{x^{2}+1}\,\mathrm{Elli}}{b\sqrt{x^{4}+3x^{2}}}\right.}{\sqrt{x^{2}+1}\,\sqrt{x^{2}+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -I*(a*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-a*EllipticPi(1/2*I*x*2^(1/2),2*b/a
,2^(1/2))+2*b*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2)))/a/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*sqrt(x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^4 + (a + b)*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+2}}{(a+bx^2)\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)**(1/2)/(x**2+1)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(x**2 + 2)/((a + b*x**2)*sqrt(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*sqrt(x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}\,\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2)^(1/2)/((x^2 + 1)^(1/2)*(a + b*x^2)),x)
```

```
[Out] int((x^2 + 2)^(1/2)/((x^2 + 1)^(1/2)*(a + b*x^2)), x)
```

3.92 $\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)}\,dx$

Optimal. Leaf size=121

$$\frac{\sqrt{2}\sqrt{2+x^2}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{(a-b)\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}-\frac{2b\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a(a-b)\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}$$

```
[Out] -2*b*(1/(2*x^2+4))^(1/2)*(2*x^2+4)^(1/2)*EllipticPi(x*2^(1/2)/(2*x^2+4)^(1/
2),1-2*b/a,I)*(x^2+1)^(1/2)/a/(a-b)/((x^2+1)/(x^2+2))^(1/2)/(x^2+2)^(1/2)+(
1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*(x^2+2)^(1/
2)/(a-b)/((x^2+2)/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {555, 553, 422}

$$\frac{\sqrt{2}\sqrt{x^2+2}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}\,(a-b)}-\frac{2b\sqrt{x^2+1}\,\Pi\left(1-\frac{2b}{a};\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}\,(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2]/((1 + x^2)^(3/2)*(a + b*x^2)),x]
```

```
[Out] (Sqrt[2]*Sqrt[2 + x^2]*EllipticE[ArcTan[x], 1/2])/((a - b)*Sqrt[1 + x^2]*Sq
rt[(2 + x^2)/(1 + x^2)]) - (2*b*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTa
n[x/Sqrt[2]], -1])/(a*(a - b)*Sqrt[(1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
```

```
[d/c]
```

Rule 555

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rubi steps

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)}\,dx = -\frac{b\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx}{a-b} - \frac{\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}}\,dx}{-a+b}$$

$$= \frac{\sqrt{2}\sqrt{2+x^2}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{(a-b)\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{2b\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a(a-b)\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.22, size = 122, normalized size = 1.01

$$\frac{\frac{2x\sqrt{2+x^2}}{\sqrt{1+x^2}} + 2i\sqrt{2}\,E\left(i\sinh^{-1}(x)\middle|\frac{1}{2}\right) - i\sqrt{2}\,F\left(i\sinh^{-1}(x)\middle|\frac{1}{2}\right) - i\sqrt{2}\,\Pi\left(\frac{b}{a};i\sinh^{-1}(x)\middle|\frac{1}{2}\right) + \frac{2i\sqrt{2}\,b\Pi\left(\frac{b}{a};i\sinh^{-1}(x)\middle|\frac{1}{2}\right)}{a}}{2a-2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2]/((1 + x^2)^(3/2)*(a + b*x^2)),x]
```

```
[Out] ((2*x*Sqrt[2 + x^2])/Sqrt[1 + x^2] + (2*I)*Sqrt[2]*EllipticE[I*ArcSinh[x],
1/2] - I*Sqrt[2]*EllipticF[I*ArcSinh[x], 1/2] - I*Sqrt[2]*EllipticPi[b/a, I
*ArcSinh[x], 1/2] + ((2*I)*Sqrt[2]*b*EllipticPi[b/a, I*ArcSinh[x], 1/2])/a)
/(2*a - 2*b)
```

Maple [A]
time = 0.15, size = 147, normalized size = 1.21

method	result
default	$\frac{\left(i\,\mathrm{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a\sqrt{x^2+1}\,\sqrt{x^2+2}-i\,\mathrm{EllipticPi}\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)a\sqrt{x^2+1}\,\sqrt{x^2+2}+2i\,\mathrm{EllipticPi}\left(\right.\right.}{(x^4+3x^2+2)a(a-b)}$

elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\left(\frac{(x^2+2)x}{(a-b)\sqrt{(x^2+1)(x^2+2)}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2(a-b)\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{}\cdots}{\cdots}\right.}{\sqrt{x^2+1}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] (I*EllipticE(1/2*I*x*2^(1/2),2^(1/2))*a*(x^2+1)^(1/2)*(x^2+2)^(1/2)-I*Ellip
ticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a*(x^2+1)^(1/2)*(x^2+2)^(1/2)+2*I*Elli
pticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b*(x^2+1)^(1/2)*(x^2+2)^(1/2)+a*x^3+2
*a*x)*(x^2+1)^(1/2)*(x^2+2)^(1/2)/(x^4+3*x^2+2)/a/(a-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^6 + (a + 2*b)*x^4 + (2*a + b)*x^2
 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+2}}{(a+bx^2)\,(x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)**(1/2)/(x**2+1)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(x**2 + 2)/((a + b*x**2)*(x**2 + 1)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^2+2}}{\left(x^2+1\right)^{3/2}\,\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2)^(1/2)/((x^2 + 1)^(3/2)*(a + b*x^2)),x)
```

```
[Out] int((x^2 + 2)^(1/2)/((x^2 + 1)^(3/2)*(a + b*x^2)), x)
```

3.93 $\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx$

Optimal. Leaf size=215

$$\frac{x\sqrt{2+x^2}}{3(a-b)\left(1+x^2\right)^{3/2}}+\frac{\sqrt{2}\,(a-2b)\sqrt{2+x^2}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{(a-b)^2\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}}-\frac{\sqrt{2}\,\sqrt{2+x^2}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3(a-b)\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}}+\frac{2b^2\sqrt{1+x^2}\,\Pi\left(\right.}{a(a-b}$$

```
[Out] 2*b^2*(1/(2*x^2+4))^(1/2)*(2*x^2+4)^(1/2)*EllipticPi(x*2^(1/2)/(2*x^2+4)^(1
/2),1-2*b/a,I)*(x^2+1)^(1/2)/a/(a-b)^2/((x^2+1)/(x^2+2))^(1/2)/(x^2+2)^(1/2
)+1/3*x*(x^2+2)^(1/2)/(a-b)/(x^2+1)^(3/2)+(a-2*b)*(1/(x^2+1))^(1/2)*Ellipti
cE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*(x^2+2)^(1/2)/(a-b)^2/((x^2+2)/(x^2
+1))^(1/2)-1/3*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(
1/2)*(x^2+2)^(1/2)/(a-b)/((x^2+2)/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {560, 553, 540, 539, 429, 422}

$$\frac{2b^2\sqrt{x^2+1}\,\Pi\left(1-\frac{2b}{a};\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\,\sqrt{x^2+2}\,(a-b)^2}-\frac{\sqrt{2}\,\sqrt{x^2+2}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{3\sqrt{x^2+1}\,\sqrt{\frac{x^2+2}{x^2+1}}\,(a-b)}+\frac{\sqrt{2}\,\sqrt{x^2+2}\,(a-2b)E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{x^2+1}\,\sqrt{\frac{x^2+2}{x^2+1}}\,(a-b)^2}+\frac{x\sqrt{x^2+2}}{3\left(x^2+1\right)^{3/2}(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2]/((1 + x^2)^(5/2)*(a + b*x^2)),x]
```

```
[Out] (x*Sqrt[2 + x^2])/(3*(a - b)*(1 + x^2)^(3/2)) + (Sqrt[2]*(a - 2*b)*Sqrt[2 +
 x^2]*EllipticE[ArcTan[x], 1/2])/((a - b)^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1
 + x^2)]) - (Sqrt[2]*Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(3*(a - b)*Sq
rt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) + (2*b^2*Sqrt[1 + x^2]*EllipticPi[1
- (2*b)/a, ArcTan[x/Sqrt[2]], -1])/(a*(a - b)^2*Sqrt[(1 + x^2)/(2 + x^2)]*S
qrt[2 + x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = -\frac{\int \frac{\sqrt{2+x^2}\,(-a+2b+bx^2)}{(1+x^2)^{5/2}}\,dx}{(a-b)^2} + \frac{b^2\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx}{(a-b)^2}$$

$$= \frac{x\sqrt{2+x^2}}{3(a-b)(1+x^2)^{3/2}} + \frac{2b^2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a(a-b)^2\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}} + \frac{\int \frac{2(2a}{(1+x^2}$$

$$= \frac{x\sqrt{2+x^2}}{3(a-b)(1+x^2)^{3/2}} + \frac{2b^2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a(a-b)^2\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}} + \frac{(a-2}{}$$

$$= \frac{x\sqrt{2+x^2}}{3(a-b)(1+x^2)^{3/2}} + \frac{\sqrt{2}(a-2b)\sqrt{2+x^2}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{(a-b)^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{\sqrt{2}\sqrt{2+x^2}\,}{3(a-b)\sqrt{1+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.08, size = 357, normalized size = 1.66

$$\frac{8a^2x\sqrt{1+x^2}\sqrt{2+x^2}-14abx\sqrt{1+x^2}\sqrt{2+x^2}+6a^2x^3\sqrt{1+x^2}\sqrt{2+x^2}-12abx^3\sqrt{1+x^2}\sqrt{2+x^2}+6i\sqrt{2}a(a-2b)(1+x^2)^2E\left(i\sinh^{-1}(x)\middle|\frac{1}{2}\right)-i\sqrt{2}a(4a-7b)(1+x^2)^2F\left(i\sinh^{-1}(x)\middle|\frac{1}{2}\right)+3i\sqrt{2}ab\Pi\left(\frac{b}{a};i\sinh^{-1}(x)\middle|\frac{1}{2}\right)-6i\sqrt{2}b^2\Pi\left(\frac{b}{a};i\sinh^{-1}(x)\middle|\frac{1}{2}\right)+6i\sqrt{2}abx^2\Pi\left(\frac{b}{a};i\sinh^{-1}(x)\middle|\frac{1}{2}\right)-12i\sqrt{2}b^2x^2\Pi\left(\frac{b}{a};i\sinh^{-1}(x)\middle|\frac{1}{2}\right)+3i\sqrt{2}abx^4\Pi\left(\frac{b}{a};i\sinh^{-1}(x)\middle|\frac{1}{2}\right)-6i\sqrt{2}b^2x^4\Pi\left(\frac{b}{a};i\sinh^{-1}(x)\middle|\frac{1}{2}\right)}{6a(a-b)^2(1+x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2]/((1 + x^2)^(5/2)*(a + b*x^2)),x]
```

```
[Out] (8*a^2*x*Sqrt[1 + x^2]*Sqrt[2 + x^2] - 14*a*b*x*Sqrt[1 + x^2]*Sqrt[2 + x^2]
 + 6*a^2*x^3*Sqrt[1 + x^2]*Sqrt[2 + x^2] - 12*a*b*x^3*Sqrt[1 + x^2]*Sqrt[2
+ x^2] + (6*I)*Sqrt[2]*a*(a - 2*b)*(1 + x^2)^2*EllipticE[I*ArcSinh[x], 1/2]
 - I*Sqrt[2]*a*(4*a - 7*b)*(1 + x^2)^2*EllipticF[I*ArcSinh[x], 1/2] + (3*I)
*Sqrt[2]*a*b*EllipticPi[b/a, I*ArcSinh[x], 1/2] - (6*I)*Sqrt[2]*b^2*Ellipti
cPi[b/a, I*ArcSinh[x], 1/2] + (6*I)*Sqrt[2]*a*b*x^2*EllipticPi[b/a, I*ArcSi
nh[x], 1/2] - (12*I)*Sqrt[2]*b^2*x^2*EllipticPi[b/a, I*ArcSinh[x], 1/2] + (
3*I)*Sqrt[2]*a*b*x^4*EllipticPi[b/a, I*ArcSinh[x], 1/2] - (6*I)*Sqrt[2]*b^2
*x^4*EllipticPi[b/a, I*ArcSinh[x], 1/2])/(6*a*(a - b)^2*(1 + x^2)^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(234) = 468.
time = 0.16, size = 477, normalized size = 2.22

method	result

elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\left(\frac{x\sqrt{x^4+3x^2+2}}{3(a-b)(x^2+1)^2}+\frac{(x^2+2)x(a-2b)}{(a-b)^2\sqrt{(x^2+1)(x^2+2)}}-\frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\ \mathrm{Elliptic}}{6\sqrt{x^4+3x^2+2}\,(a-}\right.}{}$
default	$\frac{-6i\,\mathrm{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)ab\sqrt{x^2+2}\,\sqrt{x^2+1}-6i\,\mathrm{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)ab\,x^2\sqrt{x^2+2}\,\sqrt{x^2+1}+i\,\mathrm{EllipticF}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(I*EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a*b*(x^2+2)^(1/2)*(x^2+1)^(1/2)-6
*I*EllipticE(1/2*I*x*2^(1/2),2^(1/2))*a*b*(x^2+2)^(1/2)*(x^2+1)^(1/2)-6*I*E
llipticE(1/2*I*x*2^(1/2),2^(1/2))*a*b*x^2*(x^2+2)^(1/2)*(x^2+1)^(1/2)-6*I*E
llipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b^2*(x^2+2)^(1/2)*(x^2+1)^(1/2)+3*
I*EllipticE(1/2*I*x*2^(1/2),2^(1/2))*a^2*(x^2+2)^(1/2)*(x^2+1)^(1/2)-I*Elli
pticF(1/2*I*x*2^(1/2),2^(1/2))*a^2*(x^2+2)^(1/2)*(x^2+1)^(1/2)+3*a^2*x^5-6*
a*b*x^5+3*I*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a*b*x^2*(x^2+2)^(1/2)
*(x^2+1)^(1/2)+I*EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a*b*x^2*(x^2+2)^(1/2)*(
x^2+1)^(1/2)+3*I*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a*b*(x^2+2)^(1/2
)*(x^2+1)^(1/2)+3*I*EllipticE(1/2*I*x*2^(1/2),2^(1/2))*a^2*x^2*(x^2+2)^(1/2
)*(x^2+1)^(1/2)-6*I*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b^2*x^2*(x^2+
2)^(1/2)*(x^2+1)^(1/2)-I*EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a^2*x^2*(x^2+2)
^(1/2)*(x^2+1)^(1/2)+10*a^2*x^3-19*a*b*x^3+8*a^2*x-14*a*b*x)/(x^2+2)^(1/2)/
(a-b)^2/a/(x^2+1)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(5/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^8 + (a + 3*b)*x^6 + 3*(a + b)*x^4
 + (3*a + b)*x^2 + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)**(1/2)/(x**2+1)**(5/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x^2+2}}{\left(x^2+1\right)^{5/2}\,\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2)^(1/2)/((x^2 + 1)^(5/2)*(a + b*x^2)),x)
```

```
[Out] int((x^2 + 2)^(1/2)/((x^2 + 1)^(5/2)*(a + b*x^2)), x)
```

3.94 $\int \frac{\sqrt{2+dx^2}\,\sqrt{3+fx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=298

$$\frac{fx\sqrt{2+dx^2}}{b\sqrt{3+fx^2}}-\frac{\sqrt{2}\,\sqrt{f}\,\sqrt{2+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{b\sqrt{\frac{2+dx^2}{3+fx^2}}\,\sqrt{3+fx^2}}+\frac{3d\sqrt{2+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}\,b\sqrt{f}\,\sqrt{\frac{2+dx^2}{3+fx^2}}\,\sqrt{3+fx^2}}$$

```
[Out] f*x*(d*x^2+2)^(1/2)/b/(f*x^2+3)^(1/2)+3/2*d*(1/(3*f*x^2+9))^(1/2)*(3*f*x^2+
9)^(1/2)*EllipticF(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1/2*(4-6*d/f)^(1/2))
*(d*x^2+2)^(1/2)/b*2^(1/2)/f^(1/2)/((d*x^2+2)/(f*x^2+3))^(1/2)/(f*x^2+3)^(1
/2)+3/2*(-a*d+2*b)*(1/(3*f*x^2+9))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticPi(x*f^(
1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1-3*b/a/f,1/2*(4-6*d/f)^(1/2))*(d*x^2+2)^(1/
2)/a/b*2^(1/2)/f^(1/2)/((d*x^2+2)/(f*x^2+3))^(1/2)/(f*x^2+3)^(1/2)-(1/(3*f*
x^2+9))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticE(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/
2),1/2*(4-6*d/f)^(1/2))*2^(1/2)*f^(1/2)*(d*x^2+2)^(1/2)/b/((d*x^2+2)/(f*x^2
+3))^(1/2)/(f*x^2+3)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {549, 433, 429, 506, 422, 553}

$$\frac{3\sqrt{dx^2+2}\,(2b-ad)\Pi\left(1-\frac{3b}{af};\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}\,ab\sqrt{f}\,\sqrt{fx^2+3}\,\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{3d\sqrt{dx^2+2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}\,b\sqrt{f}\,\sqrt{fx^2+3}\,\sqrt{\frac{dx^2+2}{fx^2+3}}}-\frac{\sqrt{2}\,\sqrt{f}\,\sqrt{dx^2+2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{b\sqrt{fx^2+3}\,\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{fx\sqrt{dx^2+2}}{b\sqrt{fx^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(a + b*x^2),x]
```

```
[Out] (f*x*Sqrt[2 + d*x^2])/(b*Sqrt[3 + f*x^2]) - (Sqrt[2]*Sqrt[f]*Sqrt[2 + d*x^2
]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(b*Sqrt[(2 + d*x
^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) + (3*d*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*b*Sqrt[f]*Sqrt[(2 + d*x^2)/
(3 + f*x^2)]*Sqrt[3 + f*x^2]) + (3*(2*b - a*d)*Sqrt[2 + d*x^2]*EllipticPi[1
 - (3*b)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*a*b
*Sqrt[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 549

```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Dist[d/b, Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
Dist[(b*c - a*d)/b, Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-f/e, -d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx = \frac{d\int \frac{\sqrt{3+fx^2}}{\sqrt{2+dx^2}}\,dx}{b} + \frac{(2b-ad)\int \frac{\sqrt{3+fx^2}}{(a+bx^2)\sqrt{2+dx^2}}\,dx}{b}$$

$$= \frac{3(2b-ad)\sqrt{2+dx^2}\,\Pi\left(1-\frac{3b}{af};\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}\,ab\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} + \frac{(3d)\int \frac{\cdots}{\sqrt{2+dx^2}}}{b}$$

$$= \frac{fx\sqrt{2+dx^2}}{b\sqrt{3+fx^2}} + \frac{3d\sqrt{2+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}\,b\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} + \frac{3(2b-ad)\sqrt{2+\cdots}}{\sqrt{2}\cdots}$$

$$= \frac{fx\sqrt{2+dx^2}}{b\sqrt{3+fx^2}} - \frac{\sqrt{2}\sqrt{f}\sqrt{2+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{b\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} + \frac{3d\sqrt{2+\cdots}}{\sqrt{2}\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.57, size = 134, normalized size = 0.45

$$\frac{i\left(-3abdE\left(i\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+(-2b+ad)\left(afF\left(i\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+(3b-af)\Pi\left(\frac{2b}{ad};i\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)\right)\right)}{\sqrt{3}\,ab^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(a + b*x^2),x]
```

```
[Out] (I*(-3*a*b*d*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (-2*b
 + a*d)*(a*f*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (3*b
- a*f)*EllipticPi[(2*b)/(a*d), I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)]
)))/(Sqrt[3]*a*b^2*Sqrt[d])
```

Maple [A]
time = 0.13, size = 293, normalized size = 0.98

method	result
default	$-\frac{\left(\mathrm{EllipticF}\left(\frac{x\sqrt{3}\sqrt{-f}}{3},\frac{\sqrt{2}\sqrt{3}\sqrt{\frac{d}{f}}}{2}\right)a^2df-a^2\,\mathrm{EllipticPi}\left(\frac{x\sqrt{3}\sqrt{-f}}{3},\frac{3b}{af},\frac{\sqrt{2}\sqrt{-d}\sqrt{3}}{2\sqrt{-f}}\right)df-3\,\mathrm{EllipticF}\left(\cdots\right.}{\cdots}$

elliptic	$\frac{\sqrt{(f x^2+3)(d x^2+2)}\left(-\frac{\sqrt{3f x^2+9}\sqrt{2d x^2+4}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)adf}{2b^2\sqrt{-3f}\sqrt{df x^4+3d x^2+2f x^2+6}}+\frac{3\sqrt{3f x \ldots}}{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*a
^2*d*f-a^2*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/
2)*3^(1/2)/(-f)^(1/2))*d*f-3*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)
*3^(1/2)*(d/f)^(1/2))*d*b*a-2*f*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1
/2)*3^(1/2)*(d/f)^(1/2))*b*a+3*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,
1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*d*b*a+2*EllipticPi(1/3*x*3^(1/2)
*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*f*b*a-6*Elli
pticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)
^(1/2))*b^2)*2^(1/2)/a/(-f)^(1/2)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+2}\,\sqrt{fx^2+3}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+2)**(1/2)*(f*x**2+3)**(1/2)/(b*x**2+a),x)
[Out] Integral(sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^2+2}\,\sqrt{f\,x^2+3}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2))/(a + b*x^2),x)
[Out] int(((d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2))/(a + b*x^2), x)
```

3.95 $\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx$

Optimal. Leaf size=93

$$\frac{2\sqrt{3+fx^2}\,\Pi\left(1-\frac{2b}{ad};\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{2}}\right)|1-\frac{2f}{3d}\right)}{\sqrt{3}\,a\sqrt{d}\,\sqrt{2+dx^2}\,\sqrt{\frac{3+fx^2}{2+dx^2}}}$$

```
[Out] 2/3*(1/(2*d*x^2+4))^(1/2)*(2*d*x^2+4)^(1/2)*EllipticPi(x*d^(1/2)*2^(1/2)/(2
*d*x^2+4)^(1/2),1-2*b/a/d,1/3*(9-6*f/d)^(1/2))*(f*x^2+3)^(1/2)/a*3^(1/2)/d^
(1/2)/(d*x^2+2)^(1/2)/((f*x^2+3)/(d*x^2+2))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = $\{553\}$

$$\frac{2\sqrt{fx^2+3}\,\Pi\left(1-\frac{2b}{ad};\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{2}}\right)|1-\frac{2f}{3d}\right)}{\sqrt{3}\,a\sqrt{d}\,\sqrt{dx^2+2}\,\sqrt{\frac{fx^2+3}{dx^2+2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + d*x^2]/((a + b*x^2)*Sqrt[3 + f*x^2]),x]
```

```
[Out] (2*Sqrt[3 + f*x^2]*EllipticPi[1 - (2*b)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[2]],
 1 - (2*f)/(3*d)])/(Sqrt[3]*a*Sqrt[d]*Sqrt[2 + d*x^2]*Sqrt[(3 + f*x^2)/(2 +
 d*x^2)])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx=\frac{2\sqrt{3+fx^2}\,\Pi\left(1-\frac{2b}{ad};\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{2}}\right)|1-\frac{2f}{3d}\right)}{\sqrt{3}\,a\sqrt{d}\,\sqrt{2+dx^2}\,\sqrt{\frac{3+fx^2}{2+dx^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.63, size = 94, normalized size = 1.01

$$-\frac{i\left(adF\left(i\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+(2b-ad)\Pi\left(\frac{2b}{ad};i\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)\right)}{\sqrt{3}\,ab\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + d*x^2]/((a + b*x^2)*Sqrt[3 + f*x^2]),x]
```

```
[Out] ((-I)*(a*d*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (2*b -
a*d)*EllipticPi[(2*b)/(a*d), I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)]))
/(Sqrt[3]*a*b*Sqrt[d])
```

Maple [A]
time = 0.14, size = 133, normalized size = 1.43

method	result
default	$\frac{\sqrt{2}\left(\text{EllipticF}\left(\frac{x\sqrt{3}\sqrt{-f}}{3},\frac{\sqrt{2}\sqrt{3}\sqrt{\frac{d}{f}}}{2}\right)ad-\text{EllipticPi}\left(\frac{x\sqrt{3}\sqrt{-f}}{3},\frac{3b}{af},\frac{\sqrt{2}\sqrt{-d}\sqrt{3}}{2\sqrt{-f}}\right)ad+2\,\text{EllipticPi}\left(\right.\right.}{2\sqrt{-f}\,ab}$
elliptic	$\frac{\sqrt{(f x^2+3)(d x^2+2)}\left(\frac{\sqrt{3f x^2+9}\sqrt{2d x^2+4}\,\text{EllipticF}\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)d}{2b\sqrt{-3f}\sqrt{df x^4+3d x^2+2f x^2+6}}-\frac{\sqrt{1+\frac{f x^2}{3}}}{b\sqrt{-}}\right.}{\sqrt{f x^2+3}\sqrt{d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(
1/2))*a*d-EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/2
)*3^(1/2)/(-f)^(1/2))*a*d+2*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2
*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*b)/(-f)^(1/2)/a/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + 2)/((b*x^2 + a)*sqrt(f*x^2 + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*f*x^4 + (a*f + 3*b)*x^2 + 3*a),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+2}}{(a+bx^2)\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+2)**(1/2)/(b*x**2+a)/(f*x**2+3)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x**2 + 2)/((a + b*x**2)*sqrt(f*x**2 + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + 2)/((b*x^2 + a)*sqrt(f*x^2 + 3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+2}}{\left(b\,x^2+a\right)\,\sqrt{f\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2 + 2)^(1/2)/((a + b*x^2)*(f*x^2 + 3)^(1/2)),x)
```

```
[Out] int((d*x^2 + 2)^(1/2)/((a + b*x^2)*(f*x^2 + 3)^(1/2)), x)
```

3.96 $\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\Pi\left(\frac{2b}{ad};\sin^{-1}\left(\frac{\sqrt{-d}\,x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)}{\sqrt{3}\,a\sqrt{-d}}$$

```
[Out] 1/3*EllipticPi(1/2*x*(-d)^(1/2)*2^(1/2),2*b/a/d,1/3*6^(1/2)*(f/d)^(1/2))/a*
3^(1/2)/(-d)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {551}

$$\frac{\Pi\left(\frac{2b}{ad};\text{ArcSin}\left(\frac{\sqrt{-d}\,x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)}{\sqrt{3}\,a\sqrt{-d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

```
[Out] EllipticPi[(2*b)/(a*d), ArcSin[(Sqrt[-d]*x)/Sqrt[2]], (2*f)/(3*d)]/(Sqrt[3]
*a*Sqrt[-d])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rubi steps

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \frac{\Pi\left(\frac{2b}{ad};\sin^{-1}\left(\frac{\sqrt{-d}\,x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)}{\sqrt{3}\,a\sqrt{-d}}$$

Mathematica [A]
time = 1.86, size = 49, normalized size = 1.00

$$\frac{\Pi\left(\frac{2b}{ad};\sin^{-1}\left(\frac{\sqrt{-d}\,x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)}{\sqrt{3}\,a\sqrt{-d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

```
[Out] EllipticPi[(2*b)/(a*d), ArcSin[(Sqrt[-d]*x)/Sqrt[2]], (2*f)/(3*d)]/(Sqrt[3]
*a*Sqrt[-d])
```

Maple [A]
time = 0.14, size = 53, normalized size = 1.08

method	result	size
default	$\frac{\sqrt{2}\,\mathrm{EllipticPi}\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{3b}{af},\frac{\sqrt{2}\,\sqrt{-d}\,\sqrt{3}}{2\sqrt{-f}}\right)}{2\sqrt{-f}\,a}$	53
elliptic	$\frac{\sqrt{\left(f\,x^{2}+3\right)\left(d\,x^{2}+2\right)}\,\sqrt{1+\frac{f\,x^{2}}{3}}\,\sqrt{1+\frac{d\,x^{2}}{2}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{f}{3}}\,x,\frac{3b}{af},\frac{\sqrt{-\frac{d}{2}}}{\sqrt{-\frac{f}{3}}}\right)}{\sqrt{f\,x^{2}+3}\,\sqrt{d\,x^{2}+2}\,a\sqrt{-\frac{f}{3}}\,\sqrt{df\,x^{4}+3d\,x^{2}+2f\,x^{2}+6}}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1
/2)*3^(1/2)/(-f)^(1/2))/(-f)^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
[Out] Integral(1/((a + b*x**2)*sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^2+a\right)\,\sqrt{d\,x^2+2}\,\sqrt{f\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2)),x)
[Out] int(1/((a + b*x^2)*(d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2)), x)
```

3.97 $\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=36

$$-\frac{\sqrt{1+\frac{bx^2}{a}}\,F\left(\sin^{-1}(x)\middle|-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

[Out] -EllipticF(x,(-b/a)^(1/2))*(1+b*x^2/a)^(1/2)/(b*x^2+a)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 432, 430}

$$-\frac{\sqrt{\frac{bx^2}{a}+1}\,F\left(\text{ArcSin}(x)\,\middle|-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^2]/((-1 + x^2)*Sqrt[a + b*x^2]),x]

[Out] -((Sqrt[1 + (b*x^2)/a]*EllipticF[ArcSin[x], -(b/a)])/Sqrt[a + b*x^2])

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = -\int \frac{1}{\sqrt{1-x^2}\,\sqrt{a+bx^2}}\,dx$$
$$= -\frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}$$
$$= -\frac{\sqrt{1+\frac{bx^2}{a}}\,F\left(\sin^{-1}(x)\middle|-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.49, size = 37, normalized size = 1.03

$$-\frac{\sqrt{\frac{a+bx^2}{a}}\,F\left(\sin^{-1}(x)\middle|-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/((-1 + x^2)*Sqrt[a + b*x^2]),x]
[Out] -((Sqrt[(a + b*x^2)/a]*EllipticF[ArcSin[x], -(b/a)])/Sqrt[a + b*x^2])
```

Maple [A]
time = 0.16, size = 35, normalized size = 0.97

method	result	size
default	$-\frac{\sqrt{\frac{b\,x^2+a}{a}}\,\mathrm{EllipticF}\left(x,\sqrt{-\frac{b}{a}}\right)}{\sqrt{b\,x^2+a}}$	35
elliptic	$-\frac{\sqrt{-\left(x^2-1\right)\left(b\,x^2+a\right)}\,\sqrt{1+\frac{b\,x^2}{a}}\,\mathrm{EllipticF}\left(x,\sqrt{-1-\frac{-a+b}{a}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{-b\,x^4-a\,x^2+b\,x^2+a}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/(b*x^2+a)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticF(x,(-b/a)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/(sqrt(b*x^2 + a)*(x^2 - 1)), x)
```

Fricas [A]
time = 0.38, size = 13, normalized size = 0.36

$$-\frac{\text{ellipticF}\left(x,-\frac{b}{a}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -ellipticF(x, -b/a)/sqrt(a)
```

Sympy [A]
time = 2.07, size = 19, normalized size = 0.53

$$\begin{cases} -\frac{F\left(\operatorname{asin}{\left(x \right)}\middle| -\frac{b}{a}\right)}{\sqrt{a}} & \text{for}\: x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(x**2-1)/(b*x**2+a)**(1/2),x)
[Out] Piecewise((-elliptic_f(asin(x), -b/a)/sqrt(a), (x > -1) & (x < 1)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + 1)/(sqrt(b*x^2 + a)*(x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{1}{\sqrt{1-x^2}\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((1 - x^2)^(1/2)*(a + b*x^2)^(1/2)),x)
[Out] -int(1/((1 - x^2)^(1/2)*(a + b*x^2)^(1/2)), x)
```

3.98 $\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$

Optimal. Leaf size=113

$$\frac{(be-af)x\sqrt{c+dx^2}}{2e(de-cf)\,(e+fx^2)}-\frac{(bce-2ade+acf)\tanh^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\,\sqrt{c+dx^2}}\right)}{2e^{3/2}(de-cf)^{3/2}}$$

```
[Out] -1/2*(a*c*f-2*a*d*e+b*c*e)*arctanh(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1/
2))/e^(3/2)/(-c*f+d*e)^(3/2)+1/2*(-a*f+b*e)*x*(d*x^2+c)^(1/2)/e/(-c*f+d*e)/
(f*x^2+e)
```

Rubi [A]
time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {541, 12, 385, 214}

$$\frac{x\sqrt{c+dx^2}\,(be-af)}{2e\,(e+fx^2)\,(de-cf)}-\frac{(acf-2ade+bce)\tanh^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{e}\,\sqrt{c+dx^2}}\right)}{2e^{3/2}(de-cf)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

```
[Out] ((b*e - a*f)*x*Sqrt[c + d*x^2])/(2*e*(d*e - c*f)*(e + f*x^2)) - ((b*c*e - 2
*a*d*e + a*c*f)*ArcTanh[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])])/(2*
e^(3/2)*(d*e - c*f)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx &= \frac{(be-af)x\sqrt{c+dx^2}}{2e(de-cf)\,(e+fx^2)}+\frac{\int \frac{-bce+2ade-acf}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx}{2e(de-cf)}\\
&= \frac{(be-af)x\sqrt{c+dx^2}}{2e(de-cf)\,(e+fx^2)}-\frac{(bce-2ade+acf)\int \frac{1}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx}{2e(de-cf)}\\
&= \frac{(be-af)x\sqrt{c+dx^2}}{2e(de-cf)\,(e+fx^2)}-\frac{(bce-2ade+acf)\mathrm{Subst}\left(\int \frac{1}{e-(de-cf)x^2}\,dx,x,\frac{x}{\sqrt{c+\cdots}}\right)}{2e(de-cf)}\\
&= \frac{(be-af)x\sqrt{c+dx^2}}{2e(de-cf)\,(e+fx^2)}-\frac{(bce-2ade+acf)\tanh^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\,\sqrt{c+dx^2}}\right)}{2e^{3/2}(de-cf)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.55, size = 131, normalized size = 1.16

$$\frac{\frac{\sqrt{e}\,(be-af)x\sqrt{c+dx^2}}{(de-cf)(e+fx^2)}-\frac{(bce-2ade+acf)\tan^{-1}\left(\frac{-fx\sqrt{c+dx^2}+\sqrt{d}\,(e+fx^2)}{\sqrt{e}\,\sqrt{-de+cf}}\right)}{(-de+cf)^{3/2}}}{2e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

```
[Out] ((Sqrt[e]*(b*e - a*f)*x*Sqrt[c + d*x^2])/((d*e - c*f)*(e + f*x^2)) - ((b*c*
e - 2*a*d*e + a*c*f)*ArcTan[(-(f*x*Sqrt[c + d*x^2]) + Sqrt[d]*(e + f*x^2))/
(Sqrt[e]*Sqrt[-(d*e) + c*f])])/(-(d*e) + c*f)^(3/2))/(2*e^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 847 vs. 2(97) = 194.
time = 0.12, size = 848, normalized size = 7.50

method	result
default	$\frac{(-af+be)\left(-\frac{f\sqrt{\left(x-\frac{\sqrt{-fe}}{f}\right)^2 d+\frac{2d\sqrt{-fe}\left(x-\frac{\sqrt{-fe}}{f}\right)}{f}+\frac{cf-de}{f}}}{(cf-de)\left(x-\frac{\sqrt{-fe}}{f}\right)}+\frac{d\sqrt{-fe}\,\ln\left(\frac{\frac{2cf-2de}{f}+\frac{2d\sqrt{-fe}\,(}{}}{}\right.}{}\right.}{4ef^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-a*f+b*e)/e/f^2*(-1/(c*f-d*e)*f/(x-(-f*e)^(1/2)/f)*((x-(-f*e)^(1/2)/f)
^2*d+2*d*(-f*e)^(1/2)/f*(x-(-f*e)^(1/2)/f)+(c*f-d*e)/f)^(1/2)+d*(-f*e)^(1/2
)/(c*f-d*e)/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f+2*d*(-f*e)^(1/2)/f*(x-(-f
*e)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x-(-f*e)^(1/2)/f)^2*d+2*d*(-f*e)^(1/2)
/f*(x-(-f*e)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x-(-f*e)^(1/2)/f)))-1/4*(a*f+b*e
)/e/(-f*e)^(1/2)/f/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f+2*d*(-f*e)^(1/2)/f
*(x-(-f*e)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x-(-f*e)^(1/2)/f)^2*d+2*d*(-f*e
)^(1/2)/f*(x-(-f*e)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x-(-f*e)^(1/2)/f))-1/4*(-
a*f-b*e)/e/(-f*e)^(1/2)/f/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f-2*d*(-f*e)^
(1/2)/f*(x+(-f*e)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x+(-f*e)^(1/2)/f)^2*d-2*
d*(-f*e)^(1/2)/f*(x+(-f*e)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x+(-f*e)^(1/2)/f))
+1/4*(-a*f+b*e)/e/f^2*(-1/(c*f-d*e)*f/(x+(-f*e)^(1/2)/f)*((x+(-f*e)^(1/2)/f
)^2*d-2*d*(-f*e)^(1/2)/f*(x+(-f*e)^(1/2)/f)+(c*f-d*e)/f)^(1/2)-d*(-f*e)^(1/
2)/(c*f-d*e)/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f-2*d*(-f*e)^(1/2)/f*(x+(-
f*e)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x+(-f*e)^(1/2)/f)^2*d-2*d*(-f*e)^(1/2
)/f*(x+(-f*e)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x+(-f*e)^(1/2)/f)))
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(101) = 202$.
time = 3.92, size = 523, normalized size = 4.63

$$\left[-\frac{(acf^2x^2 + (bc - 2\,ad)e^2 + ((bc - 2\,ad)fx^2 + acf)e)\sqrt{-cfe + de^2}\log\left(\frac{c^2f^2x^4 - 4\,(cfx^3 - (2\,dx^3 + cx)e)\sqrt{dx^2 + c}\sqrt{-cfe + de^2} + (8\,d^2x^4 + 8\,cdx^2 + c^2)e^2 - 2\,(4\,cdfx^4 + 3\,c^2fx^2)e}{f^2x^4 + 2\,fx^2e + e^2}\right) - 4\,(acf^2xe + bdxe^3 - (bc + ad)fxe^2)\sqrt{dx^2 + c}}{8\,(c^2f^3x^2e^2 + d^2e^5 + (d^2fx^2 - 2\,cdf)e^4 - (2\,cdf^2x^2 - c^2f^2)e^3)}, \frac{(acf^2x^2 + (bc - 2\,ad)e^2 + ((bc - 2\,ad)fx^2 + acf)e)\sqrt{cfe - de^2}\arctan\left(-\frac{(cfx^2 - (2\,dx^2 + c)e)\sqrt{dx^2 + c}\sqrt{cfe - de^2}}{2\,((d^2x^3 + cdx)e^2 - (cdfx^3 + c^2fx)e)}\right) + 2\,(acf^2xe + bdxe^3 - (bc + ad)fxe^2)\sqrt{dx^2 + c}}{4\,(c^2f^3x^2e^2 + d^2e^5 + (d^2fx^2 - 2\,cdf)e^4 - (2\,cdf^2x^2 - c^2f^2)e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((a*c*f^2*x^2 + (b*c - 2*a*d)*e^2 + ((b*c - 2*a*d)*f*x^2 + a*c*f)*e)*
sqrt(-c*f*e + d*e^2)*log((c^2*f^2*x^4 - 4*(c*f*x^3 - (2*d*x^3 + c*x)*e)*sqr
t(d*x^2 + c)*sqrt(-c*f*e + d*e^2) + (8*d^2*x^4 + 8*c*d*x^2 + c^2)*e^2 - 2*(
4*c*d*f*x^4 + 3*c^2*f*x^2)*e)/(f^2*x^4 + 2*f*x^2*e + e^2)) - 4*(a*c*f^2*x*e
 + b*d*x*e^3 - (b*c + a*d)*f*x*e^2)*sqrt(d*x^2 + c))/(c^2*f^3*x^2*e^2 + d^2
*e^5 + (d^2*f*x^2 - 2*c*d*f)*e^4 - (2*c*d*f^2*x^2 - c^2*f^2)*e^3), 1/4*((a*
c*f^2*x^2 + (b*c - 2*a*d)*e^2 + ((b*c - 2*a*d)*f*x^2 + a*c*f)*e)*sqrt(c*f*e
 - d*e^2)*arctan(-1/2*(c*f*x^2 - (2*d*x^2 + c)*e)*sqrt(d*x^2 + c)*sqrt(c*f*
e - d*e^2)/((d^2*x^3 + c*d*x)*e^2 - (c*d*f*x^3 + c^2*f*x)*e)) + 2*(a*c*f^2*
x*e + b*d*x*e^3 - (b*c + a*d)*f*x*e^2)*sqrt(d*x^2 + c))/(c^2*f^3*x^2*e^2 +
d^2*e^5 + (d^2*f*x^2 - 2*c*d*f)*e^4 - (2*c*d*f^2*x^2 - c^2*f^2)*e^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2}{\sqrt{c + dx^2}\,(e + fx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(f*x**2+e)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(101) = 202$.
time = 2.32, size = 336, normalized size = 2.97

$$-\frac{\left(ac\sqrt{d}\,f + bc\sqrt{d}\,e - 2\,ad^{\frac{3}{2}}e\right)\arctan\left(\frac{\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^2 f - cf + 2\,de}{2\,\sqrt{cdfe - d^2e^2}}\right)}{2\,\sqrt{cdfe - d^2e^2}\,(cfe - de^2)} - \frac{\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^2 ac\sqrt{d}\,f^2 - \left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^2 bc\sqrt{d}\,fe - 2\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^2 ad^{\frac{3}{2}}fe - ac^2\sqrt{d}\,f^2 + 2\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^2 bd^{\frac{3}{2}}e^2 + bc^2\sqrt{d}\,fe}{\left(\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^4 f - 2\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^2 cf + 4\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^2 de + c^2f\right)(cf^2e - dfe^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(a*c*sqrt(d)*f + b*c*sqrt(d)*e - 2*a*d^(3/2)*e)*arctan(1/2*((sqrt(d)*x
 - sqrt(d*x^2 + c))^2*f - c*f + 2*d*e)/sqrt(c*d*f*e - d^2*e^2))/(sqrt(c*d*f
```

```
*e - d^2*e^2)*(c*f*e - d*e^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*sqrt(
d)*f^2 - (sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c*sqrt(d)*f*e - 2*(sqrt(d)*x - s
qrt(d*x^2 + c))^2*a*d^(3/2)*f*e - a*c^2*sqrt(d)*f^2 + 2*(sqrt(d)*x - sqrt(d
*x^2 + c))^2*b*d^(3/2)*e^2 + b*c^2*sqrt(d)*f*e)/(((sqrt(d)*x - sqrt(d*x^2 +
 c))^4*f - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*c*f + 4*(sqrt(d)*x - sqrt(d*x^
2 + c))^2*d*e + c^2*f)*(c*f^2*e - d*f*e^2))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b\,x^2+a}{\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

```
[Out] int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

3.99 $\int \frac{\sqrt{c-dx^2}\,\sqrt{e+fx^2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=359

$$\frac{x\sqrt{c-dx^2}\,\sqrt{e+fx^2}}{2a\left(a+bx^2\right)}+\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{1-\frac{dx^2}{c}}\,\sqrt{e+fx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2ab\sqrt{c-dx^2}\,\sqrt{1+\frac{fx^2}{e}}}-\frac{\sqrt{c}\,\sqrt{d}\,(be+af)\sqrt{1-}}{2ab}$$

```
[Out] 1/2*x*(-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(b*x^2+a)+1/2*EllipticE(x*d^(1/2)/
c^(1/2),(-c*f/d/e)^(1/2))*c^(1/2)*d^(1/2)*(1-d*x^2/c)^(1/2)*(f*x^2+e)^(1/2)
/a/b/(-d*x^2+c)^(1/2)/(1+f*x^2/e)^(1/2)+1/2*(a^2*d*f+b^2*c*e)*EllipticPi(x*
d^(1/2)/c^(1/2),-b*c/a/d,(-c*f/d/e)^(1/2))*c^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x
^2/e)^(1/2)/a^2/b^2/d^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)-1/2*(a*f+b*e)*
EllipticF(x*d^(1/2)/c^(1/2),(-c*f/d/e)^(1/2))*c^(1/2)*d^(1/2)*(1-d*x^2/c)^(
1/2)*(1+f*x^2/e)^(1/2)/a/b^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [**A**]
time = 0.21, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {562, 552, 551, 538, 438, 437, 435, 432, 430}

$$\frac{\sqrt{c}\,\sqrt{1-\frac{dx^2}{c}}\,\sqrt{\frac{fx^2}{e}+1}\,\left(a^2df+b^2ce\right)\Pi\left(-\frac{bc}{ad};\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\,\sqrt{c-dx^2}\,\sqrt{e+fx^2}}-\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{1-\frac{dx^2}{c}}\,\sqrt{\frac{fx^2}{e}+1}\,(af+be)F\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2ab^2\sqrt{c-dx^2}\,\sqrt{e+fx^2}}+\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{1-\frac{dx^2}{c}}\,\sqrt{e+fx^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2ab\sqrt{c-dx^2}\,\sqrt{\frac{fx^2}{e}+1}}+\frac{x\sqrt{c-dx^2}\,\sqrt{e+fx^2}}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

```
[Out] (x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(a + b*x^2)) + (Sqrt[c]*Sqrt[d]*Sq
rt[1 - (d*x^2)/c]*Sqrt[e + f*x^2]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -(
(c*f)/(d*e))])/(2*a*b*Sqrt[c - d*x^2]*Sqrt[1 + (f*x^2)/e]) - (Sqrt[c]*Sqrt[
d]*(b*e + a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*a*b^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2
]) + (Sqrt[c]*(b^2*c*e + a^2*d*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticPi[-((b*c)/(a*d)), ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*
a^2*b^2*Sqrt[d]*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] &&  !GtQ[c, 0]
```

Rule 562

```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2)^2, x_Symbol] :> Simp[x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(a + b*x^
2))), x] + (Dist[(b^2*c*e - a^2*d*f)/(2*a*b^2), Int[1/((a + b*x^2)*Sqrt[c +
 d*x^2]*Sqrt[e + f*x^2]), x], x] + Dist[d*(f/(2*a*b^2)), Int[(a - b*x^2)/(S
qrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)} - \frac{(df)\int \frac{a-bx^2}{\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx}{2ab^2} + \frac{1}{2}\left(\frac{ce}{a}+\frac{adf}{b^2}\right)$$

$$= \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)} + \frac{d\int \frac{\sqrt{e+fx^2}}{\sqrt{c-dx^2}}\,dx}{2ab} - \frac{(d(be+af))\int \frac{1}{\sqrt{c-dx^2}\sqrt{}}}{2ab^2}$$

$$= \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)} + \frac{\left(d\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{e+fx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{2ab\sqrt{c-dx^2}} - \frac{\left(d(be+af)\right.}{}$$

$$= \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)} + \frac{\sqrt{c}\left(\frac{ce}{a}+\frac{adf}{b^2}\right)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(-\frac{bc}{ad};\sin\right.}{2a\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}}$$

$$= \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)} + \frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\right.}{2ab\sqrt{c-dx^2}\sqrt{1+\frac{fx^2}{e}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.42, size = 422, normalized size = 1.18

$$\frac{\frac{cex}{a+bx^2}-\frac{dex^3}{a+bx^2}+\frac{cfx^3}{a+bx^2}-\frac{dfx^5}{a+bx^2}+\frac{ic\sqrt{-\frac{d}{c}}\,e\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|-\frac{cf}{de}\right)}{b}-\frac{ic\sqrt{-\frac{d}{c}}\,(be+af)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|-\frac{cf}{de}\right)}{b^2}+\frac{ide\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(-\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|-\frac{cf}{de}\right)}{a\left(-\frac{d}{c}\right)^{3/2}}+\frac{iac\sqrt{-\frac{d}{c}}\,f\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(-\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|-\frac{cf}{de}\right)}{b^2}}{2a\sqrt{c-dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((c*e*x)/(a + b*x^2) - (d*e*x^3)/(a + b*x^2) + (c*f*x^3)/(a + b*x^2) - (d*f
*x^5)/(a + b*x^2) + (I*c*Sqrt[-(d/c)]*e*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2
)/e]*EllipticE[I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))])/b - (I*c*Sqrt[-(
d/c)]*(b*e + a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSi
nh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))])/b^2 + (I*d*e*Sqrt[1 - (d*x^2)/c]*Sqrt[
1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f
)/(d*e))])/(a*(-(d/c))^(3/2)) + (I*a*c*Sqrt[-(d/c)]*f*Sqrt[1 - (d*x^2)/c]*S
qrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -(
(c*f)/(d*e))])/b^2)/(2*a*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(302) = 604$.
time = 0.16, size = 773, normalized size = 2.15

method	result
elliptic	$\sqrt{(-d x^2+c)(f x^2+e)}\left(\frac{x\sqrt{-df x^4+cf x^2-de x^2+ce}}{2a(b x^2+a)}-\frac{df\sqrt{1-\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}}\right.}{2b^2\sqrt{\frac{d}{c}}\sqrt{-df x^4+cf x^2-de}}\right.$
default	$-\sqrt{-d x^2+c}\,\sqrt{f x^2+e}\left(\sqrt{\frac{d}{c}}\,a b^2 df x^5+\sqrt{\frac{-d x^2+c}{c}}\sqrt{\frac{f x^2+e}{e}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{cf}{de}}\right)a^2 bdf x^2+\sqrt{\frac{-d x^2}{c}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((d/c)^(1/2)*a*b^2*d*f*x^5+((-d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*a^
2*b*d*f*x^2+((-d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(d/c)^(1/2
),(-c*f/d/e)^(1/2))*a*b^2*d*e*x^2-((-d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
EllipticE(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*a*b^2*d*e*x^2-((-d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^
(1/2))*a^2*b*d*f*x^2-((-d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*
(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*b^3*c*e*x^2-(d/c)^(1/2)*a*b^
2*c*f*x^3+(d/c)^(1/2)*a*b^2*d*e*x^3+((-d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticF(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*a^3*d*f+((-d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticF(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*a^2*b*d*e-((-d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(d/c)^(1/2),(-c*f/d/e)^(1/2
```

```
))*a^2*b*d*e-((-d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1
/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*a^3*d*f-((-d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*a*
b^2*c*e-(d/c)^(1/2)*a*b^2*c*e*x)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)/a^2/(b*x^2+
a)/b^2/(d/c)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima
")
[Out] integrate(sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas
")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - dx^2}\,\sqrt{e + fx^2}}{\left(a + bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**2,x)
[Out] Integral(sqrt(c - d*x**2)*sqrt(e + f*x**2)/(a + b*x**2)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-d\,x^2}\,\,\sqrt{f\,x^2+e}}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^2,x)
[Out] int(((c - d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^2, x)
```

3.100 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=381

$$-\frac{fx\sqrt{c+dx^2}}{2ab\sqrt{e+fx^2}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)}+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2ab\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}}+\frac{d\sqrt{e}\sqrt{f}\sqrt{c}\cdots}{2b^2\cdots}$$

```
[Out] -1/2*f*x*(d*x^2+c)^(1/2)/a/b/(f*x^2+e)^(1/2)+1/2*(1/(1+f*x^2/e))^(1/2)*(1+f
*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/
2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/a/b/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*
x^2+e)^(1/2)+1/2*d*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1
/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^
(1/2)/b^2/c/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/2*x*(d*x^2+c)
^(1/2)*(f*x^2+e)^(1/2)/a/(b*x^2+a)+1/2*(-a^2*d*f+b^2*c*e)*EllipticPi(x*d^(1
/2)/(-c)^(1/2),b*c/a/d,(c*f/d/e)^(1/2))*(-c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x
^2/e)^(1/2)/a^2/b^2/d^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {562, 552, 551, 545, 429, 506, 422}

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(b^2ce-a^2df\right)\Pi\left(\frac{bc}{ad};\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2ab\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)}-\frac{fx\sqrt{c+dx^2}}{2ab\sqrt{e+fx^2}}+\frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -1/2*(f*x*Sqrt[c + d*x^2])/(a*b*Sqrt[e + f*x^2]) + (x*Sqrt[c + d*x^2]*Sqrt[
e + f*x^2])/(2*a*(a + b*x^2)) + (Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[
ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(2*a*b*Sqrt[(e*(c + d*x^2))/
(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*Elli
pticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(2*b^2*c*Sqrt[(e*(c +
d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[-c]*(b^2*c*e - a^2*d*f)*S
qrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), ArcSin[(Sqrt
[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(2*a^2*b^2*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 562

```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2)^2, x_Symbol] :> Simp[x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(a + b*x^
2))), x] + (Dist[(b^2*c*e - a^2*d*f)/(2*a*b^2), Int[1/((a + b*x^2)*Sqrt[c +
 d*x^2]*Sqrt[e + f*x^2]), x], x] + Dist[d*(f/(2*a*b^2)), Int[(a - b*x^2)/(S
qrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{\left(a+bx^2\right)^2}\,dx = \frac{x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} + \frac{(df)\int \frac{a-bx^2}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx}{2ab^2} + \frac{1}{2}\left(\frac{ce}{a} - \frac{adf}{b^2}\right)\int$$

$$= \frac{x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} + \frac{(df)\int \frac{1}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx}{2b^2} - \frac{(df)\int \frac{}{\sqrt{c+dx}}}{2}$$

$$= -\frac{fx\sqrt{c+dx^2}}{2ab\sqrt{e+fx^2}} + \frac{x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} + \frac{d\sqrt{e}\,\sqrt{f}\,\sqrt{c+dx^2}\,F\left(\tan^{-1}\right.}{2b^2c\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\;\sqrt{}}$$

$$= -\frac{fx\sqrt{c+dx^2}}{2ab\sqrt{e+fx^2}} + \frac{x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} + \frac{\sqrt{e}\,\sqrt{f}\,\sqrt{c+dx^2}\,E\left(\tan^{-1}\right.}{2ab\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\;\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.10, size = 401, normalized size = 1.05

$$\frac{\frac{cex}{a+bx^2}+\frac{dex^3}{a+bx^2}+\frac{cfx^3}{a+bx^2}+\frac{dfx^5}{a+bx^2}+\frac{ic\sqrt{\frac{d}{c}}\,e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)\middle|\frac{cf}{de}\right)}{b}-\frac{ic\sqrt{\frac{d}{c}}\,(be+af)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)\middle|\frac{cf}{de}\right)}{b^2}-\frac{ice\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)\middle|\frac{cf}{de}\right)}{a\sqrt{\frac{d}{c}}}+\frac{iac\sqrt{\frac{d}{c}}\,f\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)\middle|\frac{cf}{de}\right)}{b^2}}{2a\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((c*e*x)/(a + b*x^2) + (d*e*x^3)/(a + b*x^2) + (c*f*x^3)/(a + b*x^2) + (d*f
*x^5)/(a + b*x^2) + (I*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e
]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/b - (I*c*Sqrt[d/c]*(b*e +
 a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)])/b^2 - (I*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ell
ipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]) +
(I*a*c*Sqrt[d/c]*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)
/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/b^2)/(2*a*Sqrt[c + d*x^2]*Sqr
t[e + f*x^2])
```

Maple [A]
time = 0.16, size = 765, normalized size = 2.01

method	result
elliptic	$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\,\left(\frac{x\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{2a\left(b\,x^2+a\right)}+\frac{df\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\right.}{2b^2\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2}}\right.}{}$
default	$\frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,\left(\sqrt{-\frac{d}{c}}\,a\,b^2df\,x^5+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2bdf\,x^2+\sqrt{\frac{d\,x^2+c}{c}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((-d/c)^(1/2)*a*b^2*d*f*x^5+((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*
b*d*f*x^2+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*a*b^2*d*e*x^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*d*e*x^2-((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2
))*a^2*b*d*f*x^2+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c
)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c*e*x^2+(-d/c)^(1/2)*a*b^2*c
*f*x^3+(-d/c)^(1/2)*a*b^2*d*e*x^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*d*f+((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d*e-((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^
2*b*d*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b
*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*d*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c
*e+(-d/c)^(1/2)*a*b^2*c*e*x)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/a^2/(b*x^2+a)/b^
2/(-d/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2}\,\sqrt{e + fx^2}}{(a + bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/(a + b*x**2)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^2 + c}\,\sqrt{f\,x^2 + e}}{(b\,x^2 + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^2,x)
```

```
[Out] int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^2, x)
```

3.101 $\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\,\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=426

$$\frac{b^2x\sqrt{c-dx^2}\,\sqrt{e+fx^2}}{2a(bc+ad)(be-af)\,(a+bx^2)}+\frac{b\sqrt{c}\,\sqrt{d}\,\sqrt{1-\frac{dx^2}{c}}\,\sqrt{e+fx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a(bc+ad)(be-af)\sqrt{c-dx^2}\,\sqrt{1+\frac{fx^2}{e}}}-\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{1-\frac{d}{}}}{2a(b}$$

```
[Out] 1/2*b^2*x*(-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(a*d+b*c)/(-a*f+b*e)/(b*x^2+a)
+1/2*b*EllipticE(x*d^(1/2)/c^(1/2),(-c*f/d/e)^(1/2))*c^(1/2)*d^(1/2)*(1-d*x
^2/c)^(1/2)*(f*x^2+e)^(1/2)/a/(a*d+b*c)/(-a*f+b*e)/(-d*x^2+c)^(1/2)/(1+f*x^
2/e)^(1/2)+1/2*(b^2*c*e-3*a^2*d*f+a*b*(-2*c*f+2*d*e))*EllipticPi(x*d^(1/2)/
c^(1/2),-b*c/a/d,(-c*f/d/e)^(1/2))*c^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1
/2)/a^2/(a*d+b*c)/(-a*f+b*e)/d^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)-1/2*E
llipticF(x*d^(1/2)/c^(1/2),(-c*f/d/e)^(1/2))*c^(1/2)*d^(1/2)*(1-d*x^2/c)^(1
/2)*(1+f*x^2/e)^(1/2)/a/(a*d+b*c)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {563, 552, 551, 538, 438, 437, 435, 432, 430}

$$\frac{\sqrt{c}\,\sqrt{1-\frac{dx^2}{c}}\,\sqrt{\frac{fx^2}{e}+1}\,(-3a^2df+ab(2de-2cf)+b^2ce)\,\Pi\left(-\frac{bc}{ad};\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a^2\sqrt{d}\,\sqrt{c-dx^2}\,\sqrt{e+fx^2}\,(ad+bc)(be-af)}-\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{1-\frac{dx^2}{c}}\,\sqrt{\frac{fx^2}{e}+1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a\sqrt{c-dx^2}\,\sqrt{e+fx^2}\,(ad+bc)}+\frac{b\sqrt{c}\,\sqrt{d}\,\sqrt{1-\frac{dx^2}{c}}\,\sqrt{e+fx^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a\sqrt{c-dx^2}\,\sqrt{\frac{fx^2}{e}+1}\,(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\,\sqrt{e+fx^2}}{2a\,(a+bx^2)\,(ad+bc)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (b^2*x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c + a*d)*(b*e - a*f)*(a + b
*x^2)) + (b*Sqrt[c]*Sqrt[d]*Sqrt[1 - (d*x^2)/c]*Sqrt[e + f*x^2]*EllipticE[A
rcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*a*(b*c + a*d)*(b*e - a*f)*S
qrt[c - d*x^2]*Sqrt[1 + (f*x^2)/e]) - (Sqrt[c]*Sqrt[d]*Sqrt[1 - (d*x^2)/c]*
Sqrt[1 + (f*x^2)/e]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])
/(2*a*(b*c + a*d)*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]) + (Sqrt[c]*(b^2*c*e - 3*
a^2*d*f + a*b*(2*d*e - 2*c*f))*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elli
pticPi[-((b*c)/(a*d)), ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*a^2
*Sqrt[d]*(b*c + a*d)*(b*e - a*f)*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] &&  !GtQ[c, 0]
```

Rule 563

```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Dist[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f)), Int[1/((a + b*x^2)*Sqrt[c + d*
x^2]*Sqrt[e + f*x^2]), x], x] - Dist[d*(f/(2*a*(b*c - a*d)*(b*e - a*f))), I
nt[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c
, d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)(a+bx^2)} + \frac{(df)\int \frac{a+bx^2}{\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx}{2a(bc+ad)(be-af)} \cdots$$

$$= \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)(a+bx^2)} - \frac{d\int \frac{1}{\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx}{2a(bc+ad)} + \cdots$$

$$= \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)(a+bx^2)} + \frac{\left(bd\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{e+fx^2}}{\sqrt{1-\frac{dx^2}{c}}} \cdots}{2a(bc+ad)(be-af)\sqrt{c-dx^2} \cdots}$$

$$= \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)(a+bx^2)} + \frac{\sqrt{c}\,(b^2ce-3a^2df+ab(2de-2c \cdots}{2a^2\sqrt{d}\,(bc- \cdots}$$

$$= \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)(a+bx^2)} + \frac{b\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2} \cdots}{2a(bc+ad)(be-af)\sqrt{c- \cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.20, size = 617, normalized size = 1.45

$$\frac{-\frac{b^2cex}{a+bx^2}+\frac{b^2dex^3}{a+bx^2}-\frac{b^2cfx^3}{a+bx^2}+\frac{b^2dfx^5}{a+bx^2}-ibc\sqrt{-\frac{d}{c}}\,e\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|-\frac{cf}{de}\right)+ic\sqrt{-\frac{d}{c}}\,(be-af)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|-\frac{cf}{de}\right)+\frac{ib^2ce\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(-\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|-\frac{cf}{de}\right)}{a\sqrt{-\frac{d}{c}}}-2ibc\sqrt{-\frac{d}{c}}\,e\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(-\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|-\frac{cf}{de}\right)+\frac{2ibdf\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(-\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|-\frac{cf}{de}\right)}{\left(-\frac{d}{c}\right)^{3/2}}+3iac\sqrt{-\frac{d}{c}}\,f\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(-\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|-\frac{cf}{de}\right)}{2a(bc+ad)(-be+af)\sqrt{c-dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (-((b^2*c*e*x)/(a + b*x^2)) + (b^2*d*e*x^3)/(a + b*x^2) - (b^2*c*f*x^3)/(a
+ b*x^2) + (b^2*d*f*x^5)/(a + b*x^2) - I*b*c*Sqrt[-(d/c)]*e*Sqrt[1 - (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e)
)] + I*c*Sqrt[-(d/c)]*(b*e - a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticF[I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))] + (I*b^2*c*e*Sqrt[1 - (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d
/c)]*x], -((c*f)/(d*e))])/(a*Sqrt[-(d/c)]) - (2*I)*b*c*Sqrt[-(d/c)]*e*Sqrt[
1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqr
t[-(d/c)]*x], -((c*f)/(d*e))] + ((2*I)*b*d*f*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (
f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*
e))])/(-(d/c))^(3/2) + (3*I)*a*c*Sqrt[-(d/c)]*f*Sqrt[1 - (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/
(d*e))])/(2*a*(b*c + a*d)*(-(b*e) + a*f)*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(369) = 738$.
time = 0.15, size = 1077, normalized size = 2.53 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-(d/c)^(1/2)*a*b^2*d*f*x^5+((-d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticF(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*a^2*b*d*f*x^2-((-d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)*EllipticF(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*a*b^2*d*e*x^
2+((-d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(d/c)^(1/2),(-c*f/d/
e)^(1/2))*a*b^2*d*e*x^2-3*((-d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
Pi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*a^2*b*d*f*x^2-2*((-d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^
(1/2)/(d/c)^(1/2))*a*b^2*c*f*x^2+2*((-d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*a*b^2*d*e*x^2+
((-d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,
(-f/e)^(1/2)/(d/c)^(1/2))*b^3*c*e*x^2+(d/c)^(1/2)*a*b^2*c*f*x^3-(d/c)^(1/2)
*a*b^2*d*e*x^3+((-d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(d/c)^(
1/2),(-c*f/d/e)^(1/2))*a^3*d*f-((-d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ell
ipticF(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*a^2*b*d*e+((-d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticE(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*a^2*b*d*e-3*((-d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)
^(1/2)/(d/c)^(1/2))*a^3*d*f-2*((-d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*a^2*b*c*f+2*((-d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^
(1/2)/(d/c)^(1/2))*a^2*b*d*e+((-d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellip
ticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*a*b^2*c*e+(d/c)^(1/2
```

```
)*a*b^2*c*e*x)*(f*x^2+e)^(1/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/(b*x^2+a)/a^2/(
a*f-b*e)/(a*d+b*c)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxi
ma")
[Out] integrate(1/((b*x^2 + a)^2*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fric
as")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c-dx^2}\,\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(-d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
[Out] Integral(1/((a + b*x**2)**2*sqrt(c - d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac
")
[Out] integrate(1/((b*x^2 + a)^2*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^2\,\sqrt{c-d\,x^2}\,\sqrt{f\,x^2+e}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c - d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^2*(c - d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.102 $\int \frac{1}{(a+bx^2)^2 \sqrt{c+dx^2} \sqrt{e+fx^2}} \, dx$

Optimal. Leaf size=485

$$-\frac{bfx\sqrt{c+dx^2}}{2a(bc-ad)(be-af)\sqrt{e+fx^2}}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)}+\frac{b\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\right.}{2a(bc-ad)(be-af)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\ \sqrt{}}$$

```
[Out] -1/2*b*f*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(f*x^2+e)^(1/2)+1/2*b*(1
/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/
e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(-
a*f+b*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/2*d*(1/(1+f*x^2/
e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(
1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c/(-a*d+b*c)/(-a*f+b*e)/(
e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/2*b^2*x*(d*x^2+c)^(1/2)*(f
*x^2+e)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)+1/2*(b^2*c*e+3*a^2*d*f-2*a*
b*(c*f+d*e))*EllipticPi(x*d^(1/2)/(-c)^(1/2),b*c/a/d,(c*f/d/e)^(1/2))*(-c)^
(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/a^2/(-a*d+b*c)/(-a*f+b*e)/d^(1/2)
/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {563, 552, 551, 545, 429, 506, 422}

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(3a^2df-2ab(cf+de)+b^2ce\right)\Pi\left(\frac{bc}{ad};\text{ArcSin}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{2a^2\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)(be-af)}-\frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}F\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2c\sqrt{e+fx^2}(bc-ad)(be-af)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{b\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\text{ArcTan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2a\sqrt{e+fx^2}(bc-ad)(be-af)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}-\frac{bfx\sqrt{c+dx^2}}{2a\sqrt{e+fx^2}(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] -1/2*(b*f*x*Sqrt[c + d*x^2])/(a*(b*c - a*d)*(b*e - a*f)*Sqrt[e + f*x^2]) +
(b^2*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c - a*d)*(b*e - a*f)*(a + b
*x^2)) + (b*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sq
rt[e]], 1 - (d*e)/(c*f)])/(2*a*(b*c - a*d)*(b*e - a*f)*Sqrt[(e*(c + d*x^2))
/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (d*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*Ell
ipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(2*c*(b*c - a*d)*(b*e
 - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[-c]*
(b^2*c*e + 3*a^2*d*f - 2*a*b*(d*e + c*f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x
^2)/e]*EllipticPi[(b*c)/(a*d), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/
(2*a^2*Sqrt[d]*(b*c - a*d)*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 563

```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Dist[(b^2*c*e + 3*a^2*d*f - 2*a*b
```

```
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f)), Int[1/((a + b*x^2)*Sqrt[c + d*
x^2]*Sqrt[e + f*x^2]), x], x] - Dist[d*(f/(2*a*(b*c - a*d)*(b*e - a*f))), I
nt[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c
, d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(bc-ad)(be-af)(a+bx^2)} - \frac{(df)\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2a(bc-ad)(be-af)} \cdots$$

$$= \frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(bc-ad)(be-af)(a+bx^2)} - \frac{(df)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2(bc-ad)(be-af)} \cdots$$

$$= -\frac{bfx\sqrt{c+dx^2}}{2a(bc-ad)(be-af)\sqrt{e+fx^2}} + \frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(bc-ad)(be-af)(a+bx^2)} \cdots$$

$$= -\frac{bfx\sqrt{c+dx^2}}{2a(bc-ad)(be-af)\sqrt{e+fx^2}} + \frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(bc-ad)(be-af)(a+bx^2)} \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.47, size = 587, normalized size = 1.21

$$\frac{\frac{b^2cex}{a+bx^2}+\frac{b^2dex^3}{a+bx^2}+\frac{b^2cfx^3}{a+bx^2}+\frac{b^2dfx^5}{a+bx^2}+ibc\sqrt{\frac{d}{c}}e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ic\sqrt{\frac{d}{c}}(be-af)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-\frac{ib^2ce\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)}{a\sqrt{\frac{d}{c}}}+2ibc\sqrt{\frac{d}{c}}e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+\frac{2ibcf\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)}{\sqrt{\frac{d}{c}}}-3iac\sqrt{\frac{d}{c}}f\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)}{2a(-bc+ad)(-be+af)\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] ((b^2*c*e*x)/(a + b*x^2) + (b^2*d*e*x^3)/(a + b*x^2) + (b^2*c*f*x^3)/(a + b
*x^2) + (b^2*d*f*x^5)/(a + b*x^2) + I*b*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c]*S
qrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*c*Sqr
t[d/c]*(b*e - a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcS
inh[Sqrt[d/c]*x], (c*f)/(d*e)] - (I*b^2*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f
*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*S
```

```
qrt[d/c]) + (2*I)*b*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + ((2*I)*b*c*f*
Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[S
qrt[d/c]*x], (c*f)/(d*e)])/Sqrt[d/c] - (3*I)*a*c*Sqrt[d/c]*f*Sqrt[1 + (d*x^
2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (
c*f)/(d*e)])/(2*a*(-(b*c) + a*d)*(-(b*e) + a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*
x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1077 vs. $2(502) = 1004$.

time = 0.15, size = 1078, normalized size = 2.22 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-(-d/c)^(1/2)*a*b^2*d*f*x^5+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d*f*x^2-((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*d*e*x^2
+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*a*b^2*d*e*x^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(
x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b*d*f*x^2+2*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/
2)/(-d/c)^(1/2))*a*b^2*c*f*x^2+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*El
lipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*d*e*x^2-((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f
/e)^(1/2)/(-d/c)^(1/2))*b^3*c*e*x^2-(-d/c)^(1/2)*a*b^2*c*f*x^3-(-d/c)^(1/2)
*a*b^2*d*e*x^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*a^3*d*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellip
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d*e+((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d*e-3*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/
2)/(-d/c)^(1/2))*a^3*d*f+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
Pi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b*c*f+2*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2
)/(-d/c)^(1/2))*a^2*b*d*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticP
i(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c*e-(-d/c)^(1/2)*
a*b^2*c*e*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/(-d/c)^(1/2)/(b*x^2+a)/a^2/(a*
d-b*c)/(a*f-b*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^2 \sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**2*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.103 $\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}},x\right)$$

```
[Out] Unintegrable((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
[Out] Defer[Int][((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2], x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

Mathematica [A]
time = 15.34, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
[Out] Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2], x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{2}\right)^{\frac{3}{2}} \sqrt{c + dx^{2}}}{\sqrt{e + fx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2),x)
[Out] int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2), x)
```

3.104 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=545

$$\frac{dx\sqrt{a+bx^2}\,\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}-\frac{\sqrt{e}\,\sqrt{de-cf}\,\sqrt{a+bx^2}\,\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\,\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2f\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{e+fx^2}}+$$

```
[Out] 1/2*d*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/f/(d*x^2+c)^(1/2)+1/2*b*(-c*f+d*e)*
EllipticF(x*(-a*f+b*e)^(1/2)/e^(1/2)/(b*x^2+a)^(1/2),((-a*d+b*c)*e/c/(-a*f+
b*e))^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)/d/f/(-
a*f+b*e)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/2*c*(-a*d*
f-b*c*f+b*d*e)*EllipticPi(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-
c*f+d*e),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*e^(1/2)*(b*x^2+a)^(1/2)*(c*(f*
x^2+e)/e/(d*x^2+c))^(1/2)/a/d/f/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^
(1/2)/(f*x^2+e)^(1/2)-1/2*EllipticE(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1
/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*e^(1/2)*(-c*f+d*e)^(1/2)*(b*x^2+a)^
(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)/f/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*
x^2+e)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {569, 568, 435, 567, 551, 566, 430}

$$\frac{b\sqrt{e}\,\sqrt{c+dx^2}\,(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\,\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\,\sqrt{be-af}\,\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{e}\,\sqrt{a+bx^2}\,\sqrt{de-cf}\,\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\,\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2f\sqrt{e+fx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c\sqrt{e}\,\sqrt{a+bx^2}\,\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,(-adf-bcf+bde)\Pi\left(\frac{de}{de-cf};\text{ArcSin}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\,\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2adf\sqrt{e+fx^2}\,\sqrt{de-cf}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}\,\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(2*f*Sqrt[c + d*x^2]) - (Sqrt[e]*Sqrt
[d*e - c*f]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticE
[ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*d)*e)/(
a*(d*e - c*f)))])/(2*f*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2
]) + (b*Sqrt[e]*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*
x^2))]*EllipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b
*c - a*d)*e)/(c*(b*e - a*f))])/(2*d*f*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/
(c*(a + b*x^2))]*Sqrt[e + f*x^2]) - (c*Sqrt[e]*(b*d*e - b*c*f - a*d*f)*Sqrt
[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi[(d*e)/(d*e - c
*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*d)*
```

```
e)/(a*(d*e - c*f))]])/(2*a*d*f*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[e + f*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 566

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x], x
, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 567

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Dist[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2
/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 568

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x/
Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 569

```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[d*x*Sqrt[a + b*x^2]*(Sqrt[e + f*x^2]/(2*f*Sqrt[
c + d*x^2])), x] + (-Dist[c*((d*e - c*f)/(2*f)), Int[Sqrt[a + b*x^2]/((c +
d*x^2)^(3/2)*Sqrt[e + f*x^2]), x], x] - Dist[(b*d*e - b*c*f - a*d*f)/(2*d*f
), Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]), x], x] + Dist[b*c
*((d*e - c*f)/(2*d*f)), Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[(d*e - c*f)/c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \frac{dx\sqrt{a+bx^2}\,\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}} - \frac{(c(de-cf))\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx}{2f} + \frac{(bc(de-c\ldots}{\ldots}$$

$$= \frac{dx\sqrt{a+bx^2}\,\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}} + \frac{\left(b(de-cf)\sqrt{c+dx^2}\,\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\right)\text{Subst}\left(\int \ldots\right.}{2df\sqrt{\frac{a(c\ldots}{c(a\ldots}}}$$

$$= \frac{dx\sqrt{a+bx^2}\,\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}} - \frac{\sqrt{e}\,\sqrt{de-cf}\,\sqrt{a+bx^2}\,\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,E\left(\sin^{-1}\left(\ldots\right.\right.}{2f\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{e+f\ldots}}$$

Mathematica [A]
time = 3.52, size = 503, normalized size = 0.92

$$\frac{\frac{x\sqrt{a+bx^2}\,(c+dx^2)}{\sqrt{e+fx^2}} - \frac{\sqrt{c}\,\sqrt{-de+cf}\,\sqrt{a+bx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{-de+cf}\,x}{\sqrt{c}\,\sqrt{e+fx^2}}\right)\middle|\frac{bce-acf}{ade-acf}\right)}{f\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} + \frac{(be-2af)(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,\sqrt{e+fx^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\,\sqrt{a+bx^2}}\right)\middle|\frac{bce-ade}{bce-acf}\right)}{\sqrt{e}\,f^2\sqrt{be-af}\,\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}} + \frac{e(-bde+bcf+adf)\sqrt{a+bx^2}\,\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\Pi\left(\frac{af}{-be+af};\sin^{-1}\left(\frac{\sqrt{-be+af}\,x}{\sqrt{a}\,\sqrt{e+fx^2}}\right)\middle|\frac{ade-acf}{bce-acf}\right)}{\sqrt{a}\,f^2\sqrt{-be+af}\,\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}}{2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] ((x*Sqrt[a + b*x^2]*(c + d*x^2))/Sqrt[e + f*x^2] - (Sqrt[c]*Sqrt[-(d*e) + c
*f]*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*EllipticE[ArcSin[
(Sqrt[-(d*e) + c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], (b*c*e - a*c*f)/(a*d*e -
 a*c*f)])/(f*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]) + ((b*e - 2*a*f)*(d*e -
```

```
c*f)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]*EllipticF[ArcSi
n[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (b*c*e - a*d*e)/(b*c*e -
a*c*f)])/(Sqrt[e]*f^2*Sqrt[b*e - a*f]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]
) + (e*(-(b*d*e) + b*c*f + a*d*f)*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(
e + f*x^2))]*EllipticPi[(a*f)/(-(b*e) + a*f), ArcSin[(Sqrt[-(b*e) + a*f]*x)
/(Sqrt[a]*Sqrt[e + f*x^2])], (a*d*e - a*c*f)/(b*c*e - a*c*f)])/(Sqrt[a]*f^2
*Sqrt[-(b*e) + a*f]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]))/(2*Sqrt[c + d*x
^2])
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}{\sqrt{f\,x^{2}+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^{2}}\,\sqrt{c+dx^{2}}}{\sqrt{e+fx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2),x)
```

```
[Out] int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2), x)
```

3.105 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=163

$$\frac{c\sqrt{e}\,\sqrt{a+bx^2}\,\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\,\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{a\sqrt{de-cf}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{e+fx^2}}$$

```
[Out] c*EllipticPi(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+d*e),(-(-
a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*e^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x
^2+c))^(1/2)/a/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(
1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {567, 551}

$$\frac{c\sqrt{e}\,\sqrt{a+bx^2}\,\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\Pi\left(\frac{de}{de-cf};\text{ArcSin}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\,\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{a\sqrt{e+fx^2}\,\sqrt{de-cf}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (c*Sqrt[e]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi
[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])],
-(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 567

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Dist[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2
/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx = \frac{\left(c\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\right)\text{Subst}\left(\int \frac{1}{(1-dx^2)\sqrt{1-\frac{(-bc+ad)x^2}{a}}\sqrt{1-\frac{(de}{}}}\right.}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$= \frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{a\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

Mathematica [A]
time = 2.82, size = 162, normalized size = 0.99

$$\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{c+dx^2}}\right)|\frac{(-bc+ad)e}{a(de-cf)}\right)}{a\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (c*Sqrt[e]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi
[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])],
((-(b*c) + a*d)*e)/(a*(d*e - c*f))])/(a*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*f*x^4 + a*f*x^2
 + (b*x^2 + a)*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2}}{\sqrt{a + bx^2}\,\sqrt{e + fx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(sqrt(a + b*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/((a + b*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] int((c + d*x^2)^(1/2)/((a + b*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.106 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{e}\,\sqrt{c+dx^2}\,\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\,\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{be-af}\,\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,\sqrt{e+fx^2}}$$

```
[Out] EllipticE(x*(-a*f+b*e)^(1/2)/e^(1/2)/(b*x^2+a)^(1/2),((-a*d+b*c)*e/c/(-a*f+
b*e))^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)/a/(-a*
f+b*e)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {568, 435}

$$\frac{\sqrt{e}\,\sqrt{c+dx^2}\,\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\,\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{e+fx^2}\,\sqrt{be-af}\,\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticE[Ar
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(b*
e - a*f))])/(a*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e
 + f*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 568

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
```

```
[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x/
Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{(bc-ad)x^2}{c}}}{\sqrt{1-\frac{(be-af)x^2}{e}}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{a\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$= \frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

Mathematica [A]
time = 4.90, size = 148, normalized size = 1.00

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticE[Ar
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(b*
e - a*f))])/(a*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e
 + f*x^2])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\,x^2+c}}{(b\,x^2+a)^{\frac{3}{2}}\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*f*x^6 + 2*a*b
*f*x^4 + a^2*f*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)*e), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2}}{(a + bx^2)^{\frac{3}{2}} \sqrt{e + fx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/((a + b*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/((a + b*x^2)^(3/2)*(e + f*x^2)^(1/2)),x)
[Out] int((c + d*x^2)^(1/2)/((a + b*x^2)^(3/2)*(e + f*x^2)^(1/2)), x)
```

3.107 $\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}},x\right)$$

```
[Out] Unintegrable((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
[Out] Defer[Int][((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

Mathematica [A]
time = 17.98, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
[Out] Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}\sqrt{d\,x^2+c}}{(f\,x^2+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^2*x^4 + 2*f*x
^2*e + e^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{d\,x^2+c}}{{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2),x)
```

```
[Out] int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2), x)
```

3.108 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=484

$$-\frac{(de-cf)x\sqrt{a+bx^2}}{ef\sqrt{c+dx^2}\sqrt{e+fx^2}}+\frac{\sqrt{c}\sqrt{de-cf}\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ef\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}-\frac{c^{3/2}(be-a}{}$$

```
[Out] -c^(3/2)*(-a*f+b*e)*(1/(1+x^2*(-c*f+d*e)/c/(f*x^2+e)))^(1/2)*(1+x^2*(-c*f+d
*e)/c/(f*x^2+e))^(1/2)*EllipticF(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2)
/(1+x^2*(-c*f+d*e)/c/(f*x^2+e))^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*(
b*x^2+a)^(1/2)/a/e/f/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^
2+c)^(1/2)+(1/(1+x^2*(-c*f+d*e)/c/(f*x^2+e)))^(1/2)*(1+x^2*(-c*f+d*e)/c/(f*
x^2+e))^(1/2)*EllipticE(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2)/(1+x^2*(
-c*f+d*e)/c/(f*x^2+e))^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*c^(1/2)*(-
c*f+d*e)^(1/2)*(b*x^2+a)^(1/2)/e/f/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c
)^(1/2)-(-c*f+d*e)*x*(b*x^2+a)^(1/2)/e/f/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)+b*
c*EllipticPi(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+d*e),(-(-
a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*e^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x
^2+c))^(1/2)/a/f/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)
^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {571, 567, 551, 568, 433, 429, 506, 422}

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\Pi\left(\frac{de}{de-cf};\text{ArcSin}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)F\left(\text{ArcTan}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{fx^2+e}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{aef\sqrt{c+dx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{de-cf}\,E\left(\text{ArcTan}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{fx^2+e}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}(de-cf)}{ef\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

```
[Out] -(((d*e - c*f)*x*Sqrt[a + b*x^2])/(e*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])) +
(Sqrt[c]*Sqrt[d*e - c*f]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d*e - c*f]*
x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(e*f*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(b*e - a*f)
*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x
^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*e*f*Sqrt[d*e - c*f]*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*c*Sqrt[e]*Sqrt[a + b*x^
2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi[(d*e)/(d*e - c*f), ArcS
```

```
in[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*d)*e)/(a*(d*
e - c*f)))])/(a*f*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 567

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Dist[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2
/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 568

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x/
Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 571

```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/((e_) + (f_.)*(x_
)^2)^(3/2), x_Symbol] :> Dist[b/f, Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqr
t[e + f*x^2]), x], x] - Dist[(b*e - a*f)/f, Int[Sqrt[c + d*x^2]/(Sqrt[a + b
*x^2]*(e + f*x^2)^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \frac{b\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx}{f} - \frac{(be-af)\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx}{f}$$

$$= -\frac{\left((be-af)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1-\frac{(-de+cf)x^2}{c}}}{\sqrt{1-\frac{(-be+af)x^2}{a}}}\,dx,\right.}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$$= \frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{c+dx^2}}\right)\middle|-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$= -\frac{(de-cf)x\sqrt{a+bx^2}}{ef\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{c^{3/2}(be-af)\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{de-cf}}{\sqrt{c}\sqrt{e+f}}\right.\right.}{aef\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+}}$$

$$= -\frac{(de-cf)x\sqrt{a+bx^2}}{ef\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{\sqrt{c}\sqrt{de-cf}\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{de-c}}{\sqrt{c}\sqrt{e+}}\right.\right.}{ef\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [F]
time = 24.96, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
[Out] Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}{\left(f\,x^{2}+e\right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^{2}}\,\sqrt{c+dx^{2}}}{\left(e+fx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2),x)
```

```
[Out] int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2), x)
```

3.109 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=319

$$\frac{(de-cf)x\sqrt{a+bx^2}}{e(be-af)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}-\frac{\sqrt{c}\,\sqrt{de-cf}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\,\sqrt{e+fx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{e(be-af)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}+\frac{c^{3/2}\sqrt{}}{}$$

```
[Out] c^(3/2)*(1/(1+x^2*(-c*f+d*e)/c/(f*x^2+e)))^(1/2)*(1+x^2*(-c*f+d*e)/c/(f*x^2
+e))^(1/2)*EllipticF(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2)/(1+x^2*(-c*
f+d*e)/c/(f*x^2+e))^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*(b*x^2+a)^(1/
2)/a/e/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-(1/
(1+x^2*(-c*f+d*e)/c/(f*x^2+e)))^(1/2)*(1+x^2*(-c*f+d*e)/c/(f*x^2+e))^(1/2)*
EllipticE(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2)/(1+x^2*(-c*f+d*e)/c/(f
*x^2+e))^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*c^(1/2)*(-c*f+d*e)^(1/2)
*(b*x^2+a)^(1/2)/e/(-a*f+b*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/
2)+(-c*f+d*e)*x*(b*x^2+a)^(1/2)/e/(-a*f+b*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2
)
```

Rubi [A]

time = 0.31, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {568, 433, 429, 506, 422}

$$\frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\,\sqrt{fx^2+e}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ae\sqrt{c+dx^2}\,\sqrt{de-cf}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\,\sqrt{a+bx^2}\,\sqrt{de-cf}\,E\left(\text{ArcTan}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\,\sqrt{fx^2+e}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{e\sqrt{c+dx^2}\,(be-af)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\,(de-cf)}{e\sqrt{c+dx^2}\,\sqrt{e+fx^2}\,(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((d*e - c*f)*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x
^2]) - (Sqrt[c]*Sqrt[d*e - c*f]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d*e
- c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/
(e*(b*e - a*f)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^
(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e
+ f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*e*Sqrt[d*e - c*f]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))]]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 568

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x/
Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{(-de+cf)x^2}{c}}}{\sqrt{1-\frac{(-be+af)x^2}{a}}}\,dx, x, \frac{x}{\sqrt{e+f}}\right.}{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{(-be+af)x^2}{a}}\sqrt{1-\frac{(-de+}{c}}}\right.}{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$$= \frac{(de-cf)x\sqrt{a+bx^2}}{e(be-af)\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right.\right.}{ae\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c}}$$

$$= \frac{(de-cf)x\sqrt{a+bx^2}}{e(be-af)\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{\sqrt{c}\sqrt{de-cf}\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{c}}\right.\right.}{e(be-af)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [A]
time = 5.02, size = 148, normalized size = 0.46

$$\frac{\sqrt{a}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{-be+af}\,x}{\sqrt{a}\sqrt{e+fx^2}}\right)\middle|\frac{a(-de+cf)}{c(-be+af)}\right)}{e\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[c + d*x^2]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]*EllipticE[Ar
cSin[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])], (a*(-(d*e) + c*f))/
```

```
(c*(-(b*e) + a*f))])/(e*Sqrt[-(b*e) + a*f]*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x
^2))/(c*(e + f*x^2))])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\,x^{2}+c}}{\sqrt{b\,x^{2}+a}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*f^2*x^6 + a*f^2
*x^4 + (b*x^2 + a)*e^2 + 2*(b*f*x^4 + a*f*x^2)*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^{2}}}{\sqrt{a + bx^{2}}\,\left(e + fx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(sqrt(a + b*x**2)*(e + f*x**2)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}\,{\left(f\,x^2+e\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

```
[Out] int((c + d*x^2)^(1/2)/((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

3.110 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
[Out] Defer[Int][Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx$$

Mathematica [A]
time = 18.55, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
[Out] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\,x^2+c}}{\left(b\,x^2+a\right)^{\frac{3}{2}}\left(f\,x^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*f^2*x^8 + 2*a
*b*f^2*x^6 + a^2*f^2*x^4 + (b^2*x^4 + 2*a*b*x^2 + a^2)*e^2 + 2*(b^2*f*x^6 +
 2*a*b*f*x^4 + a^2*f*x^2)*e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2}}{(a + bx^2)^{\frac{3}{2}} (e + fx^2)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/((a + b*x**2)**(3/2)*(e + f*x**2)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{d\,x^2+c}}{{\left(b\,x^2+a\right)}^{3/2}\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)),x)
```

```
[Out] int((c + d*x^2)^(1/2)/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)), x)
```

3.111 $\int \frac{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=541

$$\frac{x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}-\frac{\sqrt{c}\,\sqrt{bc-ad}\,\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}\,\sqrt{e+fx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c+dx^2}\,\sqrt{\frac{a\left(e+fx^2\right)}{e\left(a+bx^2\right)}}}+\frac{(bc}{}$$

```
[Out] 1/2*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)-1/2*EllipticE(x*(-a*d
+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))*c^(1
/2)*(-a*d+b*c)^(1/2)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*(f*x^2+e)^(1/2)/b/(d*x
^2+c)^(1/2)/(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)-1/2*a*(a*d*f-b*(c*f+d*e))*Ellip
ticPi(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*
e)/(-a*d+b*c)/e)^(1/2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)/b^2
/c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1
/2*(-a*d+b*c)*(-a*f+2*b*e)*EllipticF(x*(-a*f+b*e)^(1/2)/e^(1/2)/(b*x^2+a)^(
1/2),((-a*d+b*c)*e/c/(-a*f+b*e))^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)*(a*(f*x^2+e
)/e/(b*x^2+a))^(1/2)/b^2/c/(-a*f+b*e)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
/(f*x^2+e)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$,
Rules used = {570, 568, 435, 567, 551, 566, 430}

$$\frac{\sqrt{e}\,\sqrt{c+dx^2}\,(bc-ad)(2be-af)\sqrt{\frac{a\left(e+fx^2\right)}{e\left(a+bx^2\right)}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\,\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{e+fx^2}\,\sqrt{be-af}\,\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{a\sqrt{c+dx^2}\,\sqrt{\frac{a\left(e+fx^2\right)}{e\left(a+bx^2\right)}}\,(adf-b(cf+de))\Pi\left(\frac{bc}{bc-ad};\text{ArcSin}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{bx^2+a}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2b^2\sqrt{c}\,\sqrt{e+fx^2}\,\sqrt{bc-ad}\,\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\,\sqrt{e+fx^2}\,\sqrt{bc-ad}\,\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{bx^2+a}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c+dx^2}\,\sqrt{\frac{a\left(e+fx^2\right)}{e\left(a+bx^2\right)}}}+\frac{x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*Sqrt[a + b*x^2]) - (Sqrt[c]*Sqrt[b*c
 - a*d]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]*EllipticE[Arc
Sin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e - a*f))/((b*c -
 a*d)*e)])/(2*b*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]) + ((
b*c - a*d)*Sqrt[e]*(2*b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a
 + b*x^2))]*EllipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])]
, ((b*c - a*d)*e)/(c*(b*e - a*f))])/(2*b^2*c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]) - (a*(a*d*f - b*(d*e + c*f))*Sqrt[
c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(b*c - a*
d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e - a*f))/
```

```
((b*c - a*d)*e)])/(2*b^2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*(a
 + b*x^2))]*Sqrt[e + f*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 566

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x], x
, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 567

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Dist[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2
/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 568

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x/
Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 570

```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[x*Sqrt[a + b*x^2]*(Sqrt[c + d*x^2]/(2*Sqrt[e +
f*x^2])), x] + (Dist[e*((b*e - a*f)/(2*f)), Int[Sqrt[c + d*x^2]/(Sqrt[a + b
*x^2]*(e + f*x^2)^(3/2)), x], x] - Dist[(b*d*e - b*c*f - a*d*f)/(2*f^2), In
t[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[(b*e - a
*f)*((d*e - 2*c*f)/(2*f^2)), Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e
+ f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[(d*e - c*f)/c]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \frac{x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{2\sqrt{a+bx^2}} - \frac{(a(bc-ad))\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx}{2b} + \frac{((bc-ad)(\cdots}{\cdots}$$

$$= \frac{x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{2\sqrt{a+bx^2}} - \frac{\left((bc-ad)\sqrt{\frac{a\,(c+dx^2)}{c\,(a+bx^2)}}\,\sqrt{e+fx^2}\right)\text{Subst}\left(\int \cdots\right.}{2b\sqrt{c+dx^2}\,\sqrt{\frac{a\,(e+\cdots}{e\,(a+\cdots}}}$$

$$= \frac{x\sqrt{c+dx^2}\,\sqrt{e+fx^2}}{2\sqrt{a+bx^2}} - \frac{\sqrt{c}\,\sqrt{bc-ad}\,\sqrt{\frac{a\,(c+dx^2)}{c\,(a+bx^2)}}\,\sqrt{e+fx^2}\,E\left(\sin^{-1}\left(\cdots\right.\right.}{2b\sqrt{c+dx^2}\,\sqrt{\frac{a\,(e+fx^2)}{e\,(a+bx^2)}}}$$

Mathematica [A]
time = 3.71, size = 512, normalized size = 0.95

$$\frac{\sqrt{a+bx^2}\,\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\left(b^2c\sqrt{bc-ad}\,x\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,(e+fx^2)-bc\sqrt{bc-ad}\,\sqrt{e}\,\sqrt{be-af}\,\sqrt{a+bx^2}\,\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\,\sqrt{a+bx^2}}\right)|\frac{bce-ade}{bce-acf}\right)+\sqrt{bc-ad}\,(2bc-ad)\sqrt{e}\,\sqrt{be-af}\,\sqrt{a+bx^2}\,\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\,\sqrt{a+bx^2}}\right)|\frac{bce-ade}{bce-acf}\right)-a\sqrt{c}\,(adf-b(de+cf))\sqrt{a+bx^2}\,\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\Pi\left(\frac{bc}{bc-ad};\sin^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)|\frac{bce-acf}{bce-ade}\right)\right)}{2ab^2\sqrt{bc-ad}\,\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*(b^2*c*Sqrt[b*c - a*
d]*x*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*(e + f*x^2) - b*c*Sqrt[b*c - a*d
]*Sqrt[e]*Sqrt[b*e - a*f]*Sqrt[a + b*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^
2))]*EllipticE[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (b*c*
e - a*d*e)/(b*c*e - a*c*f)] + Sqrt[b*c - a*d]*(2*b*c - a*d)*Sqrt[e]*Sqrt[b*
e - a*f]*Sqrt[a + b*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticF[Ar
```

```
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (b*c*e - a*d*e)/(b*c*e
 - a*c*f)] - a*Sqrt[c]*(a*d*f - b*(d*e + c*f))*Sqrt[a + b*x^2]*Sqrt[(a*(e +
 f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(b*c - a*d), ArcSin[(Sqrt[b*c -
a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (b*c*e - a*c*f)/(b*c*e - a*d*e)]))/(2*a
*b^2*Sqrt[b*c - a*d]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\,x^{2}+c}\;\sqrt{f\,x^{2}+e}}{\sqrt{b\,x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^{2}}\,\sqrt{e+fx^{2}}}{\sqrt{a+bx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/sqrt(a + b*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(1/2),x)
```

```
[Out] int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(1/2), x)
```

3.112 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}},x\right)$$

[Out] Unintegrable((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$$

Verification is not applicable to the result.

[In] Int[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]

[Out] Defer[Int][(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$$

Mathematica [A]
time = 5.13, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]

[Out] Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d*f*x^4 + c*f*x
^2 + (d*x^2 + c)*e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(b\,x^2+a)^{3/2}}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.113 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=159

$$\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\Pi\left(\frac{bc}{bc-ad};\sin^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

```
[Out] a*EllipticPi(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(
-a*f+b*e)/(-a*d+b*c)/e)^(1/2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1
/2)/c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2
)
```

Rubi [A]
time = 0.09, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {567, 551}

$$\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\Pi\left(\frac{bc}{bc-ad};\text{ArcSin}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(
b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e
- a*f))/((b*c - a*d)*e)])/(Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*
(a + b*x^2))]*Sqrt[e + f*x^2])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 567

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Dist[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2
/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx = \frac{\left(a\sqrt{c+dx^2}\,\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\right)\text{Subst}\left(\int \frac{1}{(1-bx^2)\sqrt{1-\frac{(bc-ad)x^2}{c}}\,\sqrt{1-\frac{(be}{}}}\right.}{c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,\sqrt{e+fx^2}}$$

$$= \frac{a\sqrt{c+dx^2}\,\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\Pi\left(\frac{bc}{bc-ad};\sin^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\,\sqrt{bc-ad}\,\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,\sqrt{e+fx^2}}$$

Mathematica [A]
time = 2.79, size = 159, normalized size = 1.00

$$\frac{a\sqrt{c+dx^2}\,\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\Pi\left(\frac{bc}{bc-ad};\sin^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\,\sqrt{bc-ad}\,\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(
b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e
- a*f))/((b*c - a*d)*e)])/(Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*
(a + b*x^2))]*Sqrt[e + f*x^2])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2}}{\sqrt{c + dx^2}\,\sqrt{e + fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
[Out] int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.114 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

```
[Out] EllipticF(x*(-a*f+b*e)^(1/2)/e^(1/2)/(b*x^2+a)^(1/2),((-a*d+b*c)*e/c/(-a*f+
b*e))^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)/c/(-a*
f+b*e)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {566, 430}

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticF[Ar
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(b*
e - a*f))])/(c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e
 + f*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 566

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
```

```
[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x], x
, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx = \frac{\left(\sqrt{c+dx^2}\,\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{(bc-ad)x^2}{c}}\,\sqrt{1-}\right.}{c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,\sqrt{e+fx^2}}$$

$$= \frac{\sqrt{e}\,\sqrt{c+dx^2}\,\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\,\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right.}{c\sqrt{be-af}\,\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,\sqrt{e+fx^2}}$$

Mathematica [A]
time = 3.32, size = 148, normalized size = 1.00

$$\frac{\sqrt{e}\,\sqrt{c+dx^2}\,\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\,\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{be-af}\,\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticF[Ar
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(b*
e - a*f))])/(c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e
 + f*x^2])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
[Out] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="m
axima")
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="f
ricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f*x^6 + (b*c
+ a*d)*f*x^4 + a*c*f*x^2 + (b*d*x^4 + (b*c + a*d)*x^2 + a*c)*e), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx^2}\,\sqrt{c + dx^2}\,\sqrt{e + fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.115 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,\sqrt{e+fx^2}},x\right)$$

```
[Out] Unintegrable(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
[Out] Defer[Int][1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$$

Mathematica [A]
time = 10.13, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
[Out] Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*d*f*x^8 + (b^
2*c + 2*a*b*d)*f*x^6 + a^2*c*f*x^2 + (2*a*b*c + a^2*d)*f*x^4 + (b^2*d*x^6 +
 (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c + a^2*d)*x^2)*e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```